

KALUZA KLEIN TYPE FRW COSMOLOGICAL MODEL WITH EXTENDED CHAPLYGIN GAS

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ABSTRACT. In this paper we consider the extended Chaplygin gas equation of state as a model of dark energy for which it recovers barotropic fluids with quadratic equation of state. We obtain scale factor dependence energy density and Hubble expansion parameter for particular values of n , m and α . Similarly for arbitrary values of n , m and α we have discuss nature of cosmological parameters such as scale factor, Hubble expansion parameter, time dependent dark energy density and deceleration parameter in the framework of Kaluza Klein type FRW cosmological model. Further, we study stability of the model by using speed of sound and observe that the model is stable at late time.

AMS Classification: 54H20

Keywords: Extended Chaplygin gas; Kaluza Klein cosmology

1. INTRODUCTION

Recent discoveries by many cosmologists states that expansion of our universe is accelerated. It provide new area of research in cosmological studies. It is observed that major portion of the universe is filled with dark matter (DM) and dark energy (DE). Therefore DE is one of the important discipline in theoretical physics and cosmology [48]. The most important question is describing the nature of DE. For searching the best possible solutions of this problems number of models are proposed during last few decades for example new exotic forms of matter i.e. quintessence [10, 5]. Another candidate is Einstein's cosmological constant which has two crucial problems so called fine tuning and cosmic coincidence [30]. There are also other interesting models to describe the DE such as k-essence model [2], tachyonic model [45], phantom [6], holographic models [24], inhomogeneous spacetime [23, 8] and higher dimensional space time. While the above mentioned alternatives to explain the observed acceleration of the current phase have both positive and negative aspects, the one that caught the attention of large number of workers is the introduction of a Chaplygin type of gas [51, 32, 33] as new matter field to stimulate a sort of DE.

The concept of higher dimensional model was first introduced by Kaluza-Klein (KK) [22]. According to this theory universe may have five dimensions, where the extra dimension of FRW metric is considered to unify Maxwell's theory of electromagnetism and Einstein's gravity theory. After extensive research many cosmologists found that higher dimensional cosmology provide better platform to understand the interaction of particles. As we know our space-time is explicitly four dimensions, it is believed that the unseen dimensions must be connected to the DM and DE. For various DE and DM, KK cosmological models are studied by number of authors [17, 31, 42, 4, 19, 3]. The original KK theory becomes base for other extra dimensional theories in different perspectives like brane models [11], string theory [35] and super gravity [49]. Therefore, it would be very interesting to develop the Chaplygin gas (CG) model in the KK universe. Many researchers studied CG model in the frame work of KK theory, for example, Khadekar et al. [20] studied role of modified Chaplygin gas (MCG) in Little Rip cosmological models in KK theory, variable modified Chaplygin gas (VMCG) in anisotropic universe with

KK metric discussed by Chayan et al. [9], variable Chaplygin gas (VCG) in KK framework investigated by Salti et al. [44], etc.

An interesting model to describe DE is CG [19, 3]. Pure CG obeys equation of state (EoS) [7] of the form,

$$(1) \quad p = -\frac{B}{\rho},$$

where p and ρ are pressure and energy density respectively and B is positive constant.

CG is not consistent with observational data [16]. Motivated by the desire to investigate the observational loopholes better and better the form of EoS of matter is later generalized by adding arbitrary constant with an exponent over the mass density, which is called as GCG.

The EoS for GCG is given by

$$(2) \quad p = -\frac{B}{\rho^\alpha},$$

where $0 < \alpha \leq 1$.

At high energy GCG behaves almost like a pressure less dust whereas at low energy regime it behaves like a DE, its pressure being negative and almost constant. Thus GCG smoothly interpolates between a non relativistic matter dominated phase in the early universe to a DE dominated phase in the late universe. This interesting property of GCG has motivated cosmologists to consider it as a candidate for DM and DE models. It is also possible to study viscosity in GCG [38, 52, 50, 39, 40, 1]. Later on GCG disqualified because it violates the time honored principal of energy conditions, it is fairly successful to interpret the observational results coming out of gravitational lensing or recent CMBR and SNe data in varied cosmic probes via the fine tuning of the value of the newly introduced arbitrary constant. The form of EoS is again modified through the addition of an ordinary matter field, which is termed in the literature as MCG, claiming even better match with observational results. The MCG equation of state has two parts : the first term gives an ordinary fluid obeying a linear barotropic EoS, and the second term relates pressure to some negative power energy density. So here we are essentially dealing with a two-fluid model.

MCG obeys EoS of the form,

$$(3) \quad p = A\rho - \frac{B}{\rho^\alpha},$$

where $0 < A < 1/3$.

However, it is possible to consider barotropic fluid with quadratic EoS or even with higher order EoS [25, 37]. Therefore, it is interesting to extend MCG EoS which recovers at least barotropic fluid with quadratic EoS. This is called extended Chaplygin gas (ECG), which was first proposed by Pourhassan and Kahya [36].

The EoS for ECG is defined as,

$$(4) \quad p = \sum_n A_n \rho^n - \frac{B}{\rho^\alpha}.$$

But if we reduce $n = 1$ above expression recovers standard MCG [14]. For particular choices of constants, ECG model reduces to GCG [3, 50, 54] and MCG [26] models. An attractive property of this type of model is that two unknown dark sections in the universe i.e. DE and DM can be unified in an exotic EoS. ECG models have been studied widely. For example, Saadat [41] investigated cosmological model of the early universe based on ECG, Salehi [43] studied bouncing universe in the presence of ECG, Khurshudyan [21] discussed interacting ECG in Lyra manifold, etc.

With the motivation of this work, in this paper we have generalized work of Pourhassan and Kahya [36] by assuming B as a function of Hubble parameter of the form $B = B_0 H^{-m}$ in the framework of KK theory of gravitation.

This paper is organized as follows: In section 2 we have consider FRW cosmological model filled with ECG and derived Einstein field equations, to study cosmological parameters. In section 3 we have obtained numerical solution for cosmological parameters and discussed their behavior graphically for different values of n . In section 4 we calculated deceleration parameter and checked its consistency with observational data. In section 5 we find square of speed of sound $C_s^2 = \dot{p}/\dot{\rho} \geq 0$ to test stability of our model in the framework of KK theory of gravitation. In section 6 we give concluding remarks. All numerical calculations and graphical analysis had been performed by using Wolframs Mathematica and Maple software.

2. Field equations and their solutions

The KK type Friedmann-Robertson-Walker (FRW) universe is described by following metric,

$$(5) \quad ds^2 = dt^2 - a^2(t) [dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2) + d\Psi^2],$$

where $a(t)$ is the scale factor.

The Einstein field equations given by,

$$(6) \quad R^\mu_\nu - \frac{1}{2}Rg^\mu_\nu = T^\mu_\nu,$$

where T^μ_ν is energy momentum tensor.

It is assumed that our universe is filled with the ECG which plays role of DE with EoS (4). In this case energy momentum tensor can be written as follows,

$$(7) \quad T^\mu_\nu = (p + \rho)\delta^\mu_0\delta_\nu^0 - p\delta^\mu_\nu,$$

where ρ is the energy density and p is the isotropic pressure.

Also, $u^0 = 1$ and $u^i = 0$ ($i = 1, 2, 3, 4$) with $g^{\mu\nu}u_\mu u_\nu = 1$.

The Einstein field equations for the model (5) with the help of Eq. (7) becomes

$$(8) \quad \frac{\dot{a}^2}{a^2} = \frac{\rho}{6},$$

$$(9) \quad \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = \frac{-p}{3}.$$

We assume the law of conservation of energy ($T_{;j}^{ij} = 0$) which gives

$$(10) \quad \dot{\rho} + 4H(\rho + p) = 0.$$

3. Extended Chaplygin gas cosmology

Pourhassan and Kahya [36] considered the ECG EoS (4) with B is constant. In our work we consider B is inversely proportional to some power of Hubble parameter H of the form,

$$(11) \quad B(H) = B_0 H^{-m},$$

where m is positive constant.

Thus EoS (4) becomes,

$$(12) \quad p = \sum_n A_n \rho^n - \frac{B_0 H^{-m}}{\rho^\alpha}.$$

By using Einstein field equation (8) in Eq. (12) we get

$$(13) \quad p = \sum_n A_n \rho^n - \frac{B_0(6)^{m/2}}{\rho^{\alpha+m/2}}.$$

In the following subsections we have determined energy density as a function of scale factor and Hubble parameter for particular values of m , n and α , from Eqs. (10) and (13) by using method proposed by Pourhassan and Kahya [36].

3.1. **$n = 1$.** For this case if we assume $A_1 = A$, the EoS for ECG defined in Eq. (13) reduces to,

$$(14) \quad p = A\rho - \frac{B_0(6)^{m/2}}{\rho^{\alpha+m/2}}.$$

By using Eq. (14) in Eq. (10), we get following energy density,

$$(15) \quad \rho = \left[\frac{B_0(6)^{m/2}}{1+A} + \frac{C}{a^{4(1+\alpha+m/2)(1+A)}} \right]^{\frac{1}{1+\alpha+m/2}},$$

where C is constant of integration.

By using Eq. (15) in Eq. (8) Hubble parameter is defined as,

$$(16) \quad H = \frac{\dot{a}}{a} = \frac{1}{\sqrt{6}} \left[\frac{B_0(6)^{m/2}}{1+A} + \frac{C}{a^{4(1+\alpha+m/2)(1+A)}} \right]^{\frac{1}{2(1+\alpha+m/2)}}.$$

By using numerical method we have solved the Friedmann equation (16) and obtain the time dependence of scale factor as shown in figs. 1 - 4 for different parameters. Here we have fix A , B_0 and m and observed the behavior of scale factor for different values of α . It has been observed, from fig. 1 shows that scale factor increases with increasing α in early time whereas fig. 2 shows that it decreases with increasing α in late time.

Also in fig. 3 we fixed A , m , α and found that value of B_0 doesn't matter in early universe because density is high at the initial stage, which reduces the second term of EoS to negligible value. Next we take $A = (1 - \alpha)/(1 + \alpha)$, $B_0 = 3$, $m = 1$ and represent behavior of scale factor for different values of α in fig. 4, which shows that scale factor decreases with increasing α .

For comparing this case with observational data we discussed Hubble expansion parameter as a function of redshift [34] graphically in fig. 5 for particular values of arbitrary constants A , B_0 , α and C . The initial value of Hubble expansion parameter for $C = 1$ obtained as $H_0 \sim 70$ which is near several observational data [12]. From

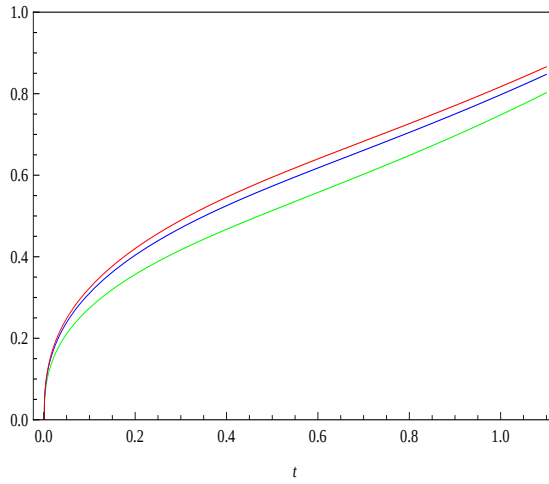


FIGURE 1. Scale factor versus time for $B_0 = 3$, $A = 1/3$, $m = 1$, $\alpha = 0.1$ (green line), $\alpha = 0.7$ (blue line), $\alpha = 1$ (red line).

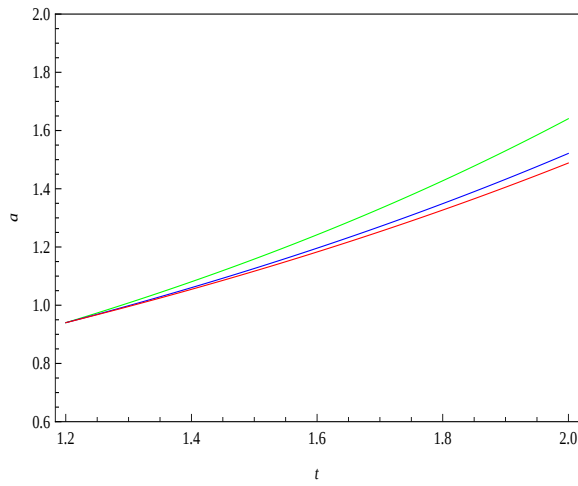


FIGURE 2. Scale factor versus time for $B_0 = 3$, $A = 1/3$, $m = 1$, $\alpha = 0.1$ (green line), $\alpha = 0.7$ (blue line), $\alpha = 1$ (red line).

fig. 5 it is clear that the solid line is near the best fit values [27, 53] i.e. for small values of constant C this case have agreement with observational data.

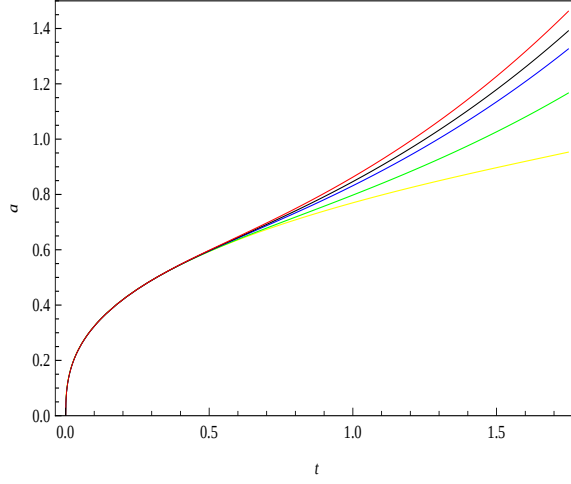


FIGURE 3. Scale factor versus time for $A = 1/3$, $m = 1$, $\alpha = 0.5$, $B_0 = 0$ (yellow line), $B_0 = 0.6$ (green line), $B_0 = 1.8$ (blue line), $B_0 = 2.5$ (black line), $B_0 = 3.4$ (red line).

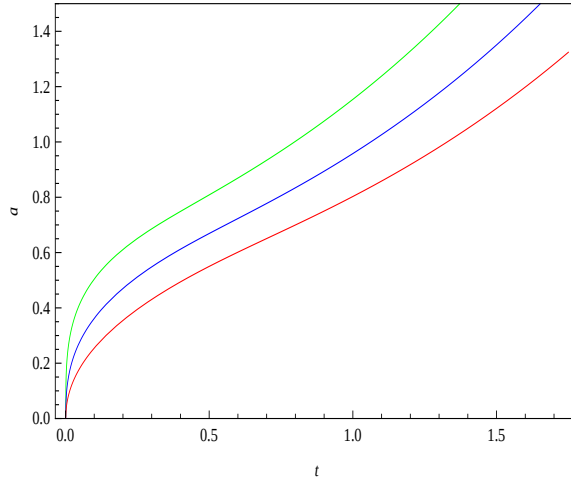


FIGURE 4. Scale factor versus time for $B_0 = 3$, $A = \frac{1-\alpha}{1+\alpha}$, $m = 1$, $\alpha = 0.1$ (green line), $\alpha = 0.5$ (blue line), $\alpha = 0.9$ (red line).

3.2. **$n = 2$, $m = 1$, $\alpha = 1/2$.** For this case if we assume $A_1 = A_2 = A$, Eq. (13) becomes

$$(17) \quad p = A\rho + A\rho^2 - \frac{B_0\sqrt{6}}{\rho}.$$

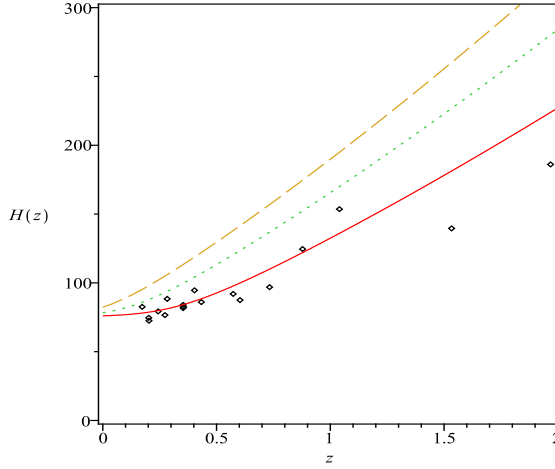


FIGURE 5. Hubble parameter versus redshift for $B_0 = 1$, $A = 1/3$, $m = 1$, $\alpha = 0.5$, $C = 0.1$ (solid line), $C = 0.6$ (dotted line), $C = 0.8$ (dashed line).

By using Eq. (17) in Eq. (10) we get

$$(18) \quad \int \frac{d\rho}{\left[(1+A)\rho + A\rho^2 - \frac{\sqrt{6}B_0}{\rho}\right]} = -4 \int \frac{da}{a}.$$

On solving Eq. (18) we get,

$$(19) \quad \rho = \left[\frac{Xa^8 + C_0a^{-4A(2+3X)}}{a^8} \right],$$

where C_0 is constant of integration and X is root of the equation.

$$(20) \quad (1+A)X^2 + AX^3 - B_0\sqrt{6} = 0.$$

By using Eq. (19) in Eq. (8), we define Hubble parameter as,

$$(21) \quad H = \frac{\dot{a}}{a} = \sqrt{\frac{Xa^8 + C_0a^{-4A(2+3X)}}{6a^8}},$$

For this case variation of Hubble parameter with respect to redshift is plotted in fig. 6 for different values of C_0 and it is found that this case is far away from observational data at any time.

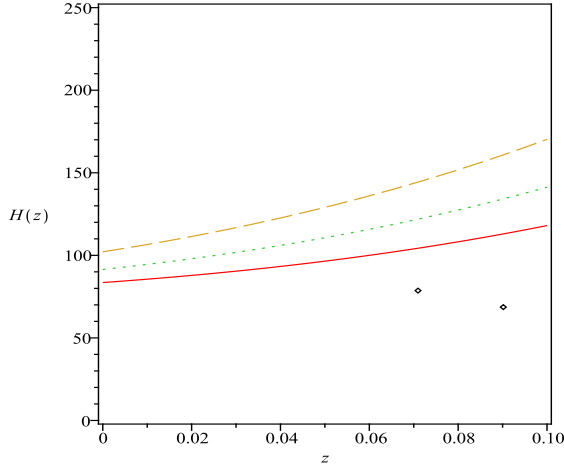


FIGURE 6. Hubble parameter versus redshift for $B_0 = 1$, $A = 1/3$, $m = 1$, $\alpha = 0.5$, $C_0 = 0.6$ (solid line), $C_0 = 1$ (dotted line), $C_0 = 1.6$ (dashed line).

3.3. **$n = 3$, $m = 1$, $\alpha = 1/2$.** For this case if we assume $A_1 = A_2 = A$, Eq. (13) gives

$$(22) \quad p = A\rho + A\rho^2 + A\rho^3 - \frac{\sqrt{6}B_0}{\rho}.$$

By using Eq. (22) in Eq. (10) we get

$$(23) \quad \int \frac{d\rho}{\left[(1+A)\rho + A\rho^2 + A\rho^3 - \frac{\sqrt{6}B_0}{\rho}\right]} = -4 \int \frac{da}{a}.$$

On Solving Eq. (23) we get expression for energy density as,

$$(24) \quad \rho = \left[\frac{Y a^8 + C_1 a^{-4A(2+3Y+4Y^2)}}{a^8} \right],$$

where C_1 is constant of integration and Y is root of the equation,

$$(25) \quad (A+1)Y^2 + AY^3 + AY^4 - B_0\sqrt{6} = 0.$$

By using Eq. (24) in Eq. (8) we get

$$(26) \quad H = \frac{\dot{a}}{a} = \sqrt{\frac{Y a^8 + C_1 a^{-4A(2+4Y^2+3Y)}}{a^8}},$$

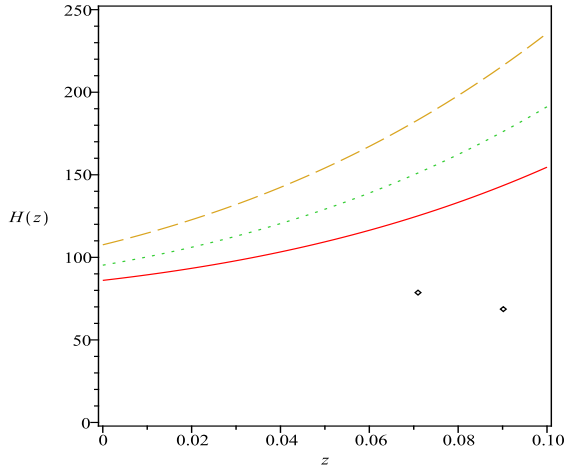


FIGURE 7. Hubble parameter versus redshift for $B_0 = 1$, $A = 1/3$, $m = 1$, $\alpha = 0.5$, $C_1 = 0.6$ (solid line), $C_1 = 1$ (dotted line), $C_1 = 1.6$ (dashed line).

In this case behavior of Hubble parameter versus redshift is shown in fig. 7. From figs. 7 and 8 it is observed that the behavior of Hubble parameter is same for the cases $n = 2$ and $n = 3$, therefore we check the next case for arbitrary value of n .

3.4. $n = \beta$ (arbitrary), $m = 1$ and $\alpha = 1/2$. In previous subsections we discussed some particular cases of n, m and α . By repeating the process used in section 3.2 and 3.3 we get expression for energy density corresponding to $m = 1$ and $\alpha = 0.5$ as

$$(27) \quad \rho = \left[\frac{\zeta a^8 + C_2 a^{-4A(2+\sum(\beta+1)\zeta^{\beta-1})}}{a^8} \right],$$

where C_2 is constant of integration and ζ is root of the equation,

$$(28) \quad (A+1)\zeta^2 + A \sum_{\beta} \zeta^{\beta+1} - B_0 \sqrt{6} = 0.$$

We can calculate $H(z)$ by using relation

$$(29) \quad \Omega = \frac{\rho}{6H_0^2},$$

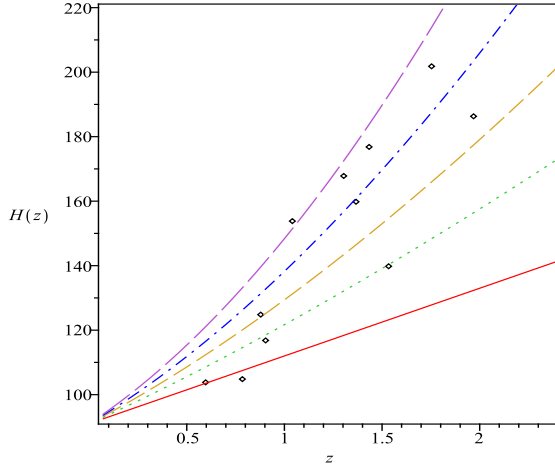


FIGURE 8. Variation of Hubble expansion parameter against redshift for arbitrary value of n and α . We fix $c = 0.3$ and $d = 1$ (solid line), $d = 1.3$ (dotted line), $d = 1.5$ (dashed line), $d = 1.7$ (dash dotted line), $d = 1.9$ (long dashed line): dots denote observational data [46, 28, 47, 29, 13, 15].

where H_0 being the present Hubble parameter in the form,

$$(30) \quad \Omega(z) = \Omega_0[1 + c(1+z)^d]^2,$$

where c and d are the constants related to A , B_0 , C_2 and m .

Since there is only one fluid and spatial section is flat choose $\Omega_0 = 1$. Using Friedmann's Eq. (8), it is possible to obtain $H(z)$ and compare it with the observational data, which is shown in figs. 8, 9 and 10. From figs. 8 and 9, it is clear that for $c = 0.3, d = 1.9$ and $c = 0.5, d = 1.3$ respectively, Hubble parameter is in best fit with the observational data. On the other hand fig. 10 represents for $c = 0.7$ and different values of d , Hubble parameter is not consistent with the observational data. From this, we can conclude that as the value of c increases $H(z)$ shows disagreement with observational data.

3.5. Arbitrary n , m and α . For arbitrary values of n , m and α we define differential equations for different cosmological parameters. Using Eqs. (8) and (13) in Eq. (9) and by assuming $A_n = A$ we get the following second order differential

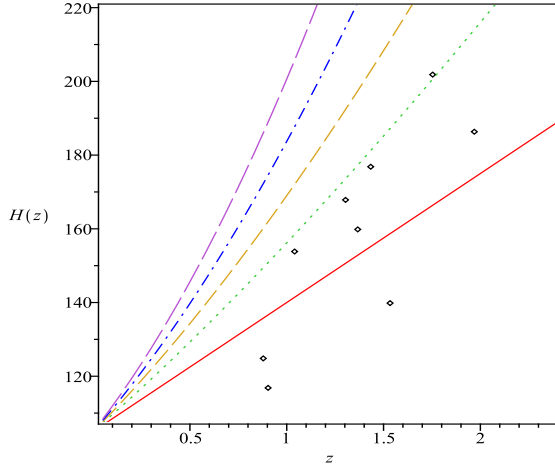


FIGURE 9. Variation of Hubble expansion parameter against redshift for arbitrary value of n and α . We fix $c = 0.5$ and $d = 1$ (solid line), $d = 1.3$ (dotted line), $d = 1.5$ (dashed line), $d = 1.7$ (dash dotted line), $d = 1.9$ (long dashed line): dots denote observational data [46, 28, 47, 29, 13, 15].

equation in terms of scale factor

$$(31) \quad 2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{1}{3}\sum_n (6)^n A \left(\frac{\dot{a}}{a}\right)^{2n} - B_0(6)^{\frac{m}{2}+1} \left(\frac{\dot{a}}{a}\right)^{-2(\alpha+\frac{m}{2})} = 0.$$

We solved Eq. (31) numerically to obtain behavior of scale factor versus time. Fig. 11 shows that general behavior of scale factor. It is observed that increasing n decreases value of scale factor and fig. 12 shows variation of scale factor against varying n at the early time. Fig. 13 shows that scale factor increases with decreasing value of α .

On the other hand we combine Eqs. (8), (10) and (13) to obtain the following first order differential equation of the energy density

$$(32) \quad \dot{\rho} + \frac{4}{\sqrt{6}} \left[\rho^{\frac{3}{2}} + A \left(\sum_n \rho^{n+\frac{1}{2}} \right) - B_0 \sqrt{6^m} \rho^{-(\alpha+\frac{m-1}{2})} \right] = 0.$$

To study the nature of dark energy density we solved Eq. (32) numerically. Results of numerical analysis are represented by figs. 14, 15 and 16. From these figures, it is clear that dark energy density is decreasing function of time. In fig. 14 and fig.

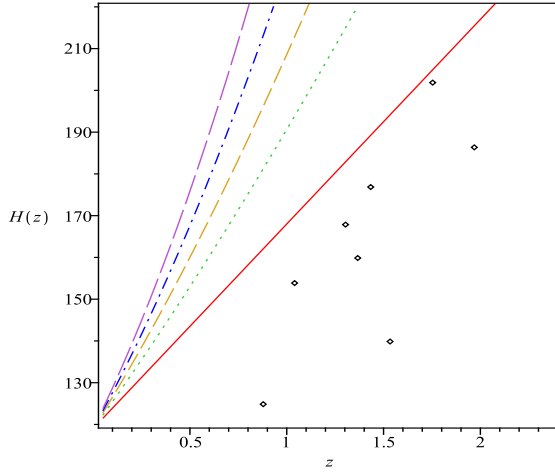


FIGURE 10. Variation of Hubble expansion parameter against redshift for arbitrary value of n and α . We fix $c = 0.7$ and $d = 1$ (solid line), $d = 1.3$ (dotted line), $d = 1.5$ (dashed line), $d = 1.7$ (dash dotted line), $d = 1.9$ (long dashed line): dots denote observational data [46, 28, 47, 29, 13, 15].

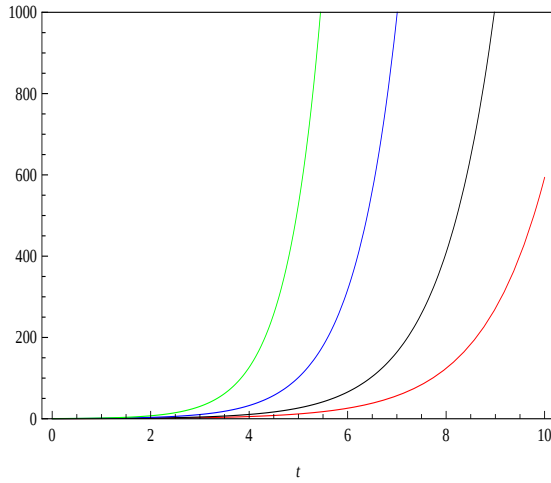


FIGURE 11. Scale factor versus time for $B_0 = 3$, $A = 1/3$, $m=1$, $\alpha = 0.9$, $n = 1$ (green line), $n = 2$ (blue line), $n = 3$ (black line), $n = 4$ (red line).

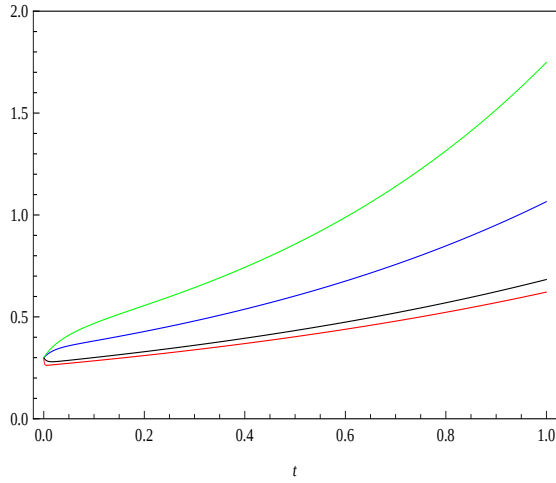


FIGURE 12. Scale factor versus time for $B_0 = 3$, $A = 1/3$, $m=1$, $\alpha = 0.9$, $n = 1$ (green line), $n = 2$ (blue line), $n = 3$ (black line), $n = 4$ (red line).

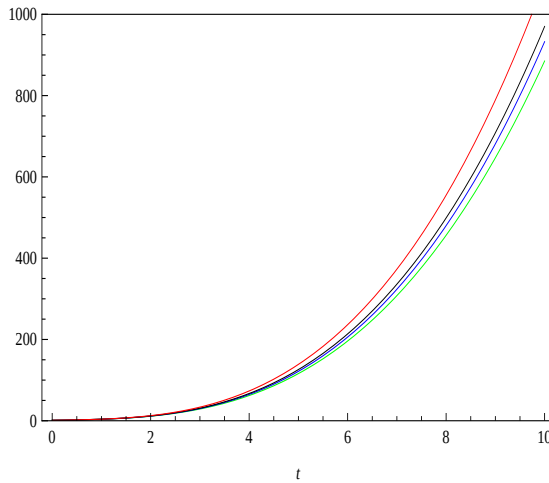


FIGURE 13. Scale factor versus time for $B_0 = 3$, $A = 1/3$, $m=2$, $n=2$, $\alpha = 0.11$ (green line), $\alpha = 0.9$ (blue line), $\alpha = 0.5$ (red line).

15 we can see that energy density decreases with increasing n and m . Fig. 16 shows that energy density increases with increasing α .

Now, by using Eq. (8) we can rewrite Eq. (9) as

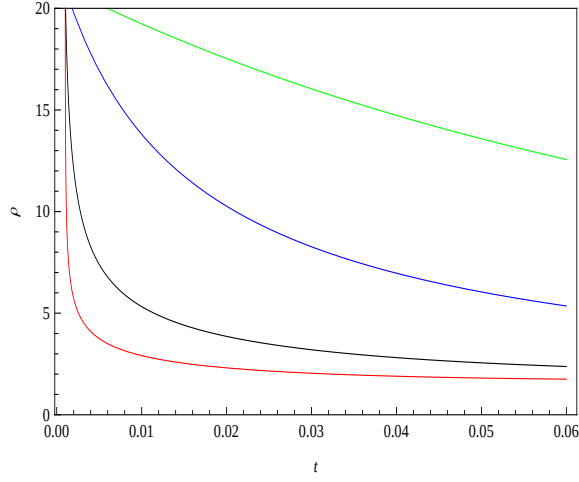


FIGURE 14. Energy density versus time for $B_0 = 3$, $A = 1/3$, $m = 2$, $\alpha = 0.9$, $n = 1$ (green line), $n = 2$ (blue line), $n = 3$ (black line), $n = 4$ (red line).

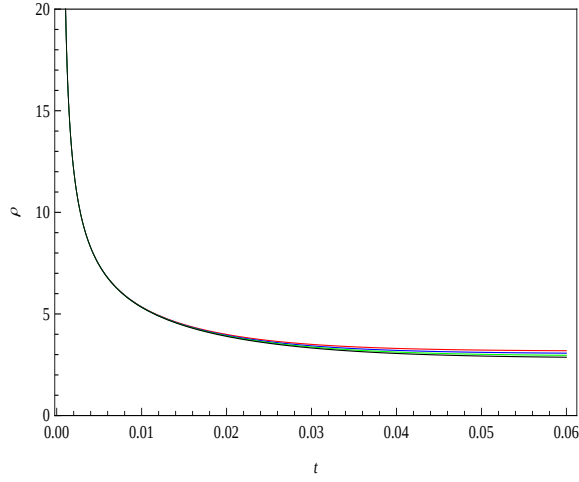


FIGURE 15. Energy density versus time for $B_0 = 3$, $A = 1/3$, $n = 4$, $\alpha = 0.3$, $m = 1$ (red line), $m = 6$ (blue line), $m = 11$ (black line).

$$(33) \quad \dot{H} + 2H^2 = \frac{-p}{3}.$$

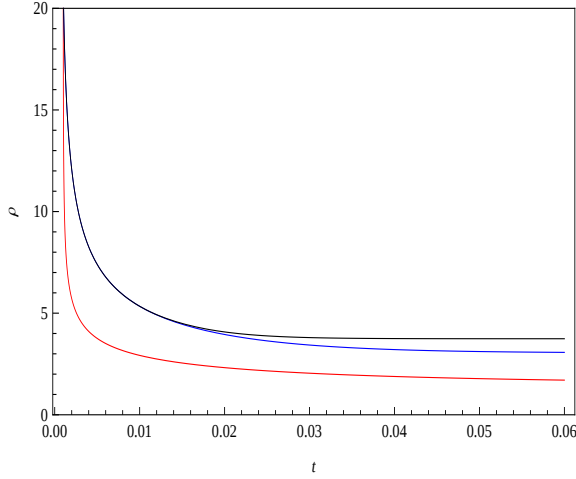


FIGURE 16. Energy density versus time for $B_0 = 3$, $A = 1/3$, $n = 3$, $m = 6$, $\alpha = 0.1$ (red line), $\alpha = 0.3$ (blue line), $\alpha = 0.5$ (green line), $\alpha = 0.7$ (black line).

Using Eqs. (8) and (13) in Eq. (33) we get following differential equation for Hubble expansion parameter

$$(34) \quad \dot{H} + 2H^2 + \frac{1}{3} \left[\sum_n A(6)^n H^{2n} - B_0(6)^{-\alpha} H^{-2(\alpha + \frac{m}{2})} \right] = 0.$$

Numerical solution of above equation is represented in fig. 17 and it is seen that Hubble expansion parameter decreases with increasing n .

4. Deceleration parameter

In section 3, we have discussed behavior of cosmological parameters by using numerical methods. Since from Eqs. (31), (32) and (34) we cannot define analytical behavior of these parameters therefore we use deceleration parameter to get detailed knowledge about the dynamics.

The deceleration parameter plays vital role in cosmology which describes cosmic dynamics for the late time acceleration. A positive deceleration parameter represents a decelerating universe, negative value describes accelerating universe and $q = 0$ shows expansion of universe with constant rate. The deceleration parameter

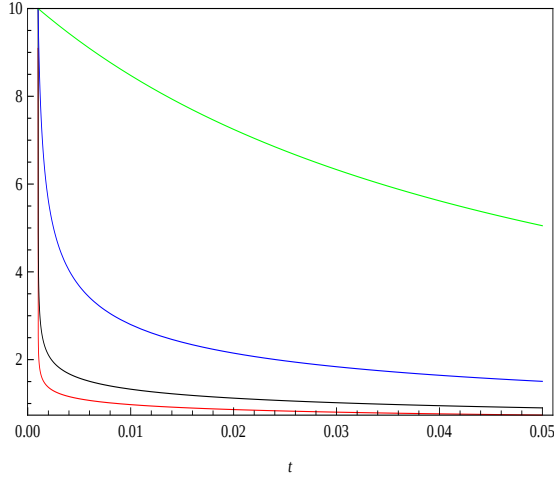


FIGURE 17. Hubble parameter versus time for $B_0 = 3$, $A = 1/3$, $m = 2$, $\alpha = 0.9$, $n = 1$ (green line), $n = 2$ (blue line), $n = 3$ (black line), $n = 4$ (red line).

is given by

$$(35) \quad q = - \left(\frac{\dot{a}}{a} \right)^{-2} \frac{\ddot{a}}{a} = -1 - \frac{\dot{H}}{H^2}.$$

Using Eqs. (8) and (13) in Eq. (35) we get

$$(36) \quad q = 1 + 2 \left[\sum_n A \rho^{n-1} - \frac{B_0(6)^{m/2}}{\rho^{(m/2+\alpha+1)}} \right].$$

At the early universe because of high energy density deceleration parameter may be reduced to the following expression

$$(37) \quad q = 1 + 2 \left[\sum_n A \rho^{n-1} \right].$$

For $n = 1$ and $A = -1$ Eq. (37) gives $q = -1$, i.e., we recover the results of Λ CDM model. On other hand, behavior of deceleration parameter in late time may be described by

$$(38) \quad q = 1 - 2B_0\rho^{-(\alpha+1)}.$$

On putting $\alpha = -1$ and $B_0 = 1$ in Eq. (38) we get $q = -1$. It means for particular choices of arbitrary constant involved in deceleration parameter we get $q = -1$. This shows universe is accelerating.

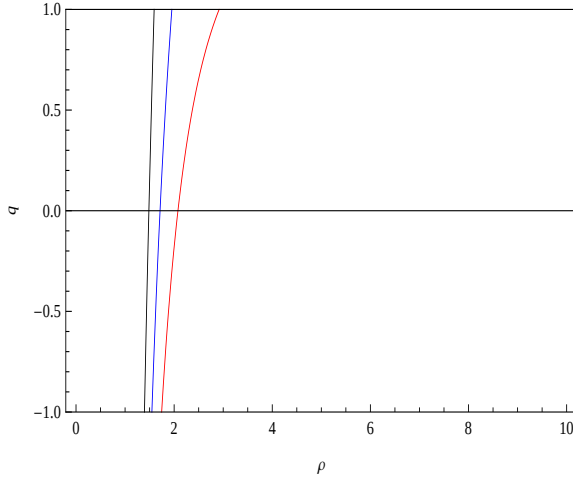


FIGURE 18. Deceleration parameter versus density for $B_0 = 3$, $A = 1/3$, $m = 2$, $\alpha = 0.7$, and $n = 1$ (red line), $n = 2$ (blue line), $n = 3$ (black line).

5. The speed of sound

There are different methods to investigate stability of the theory. In this paper we investigated stability by using squared sound speed.

The speed of sound is given by

$$(39) \quad C_s^2 = \frac{\delta p}{\delta \rho} = \frac{\dot{p}}{\dot{\rho}} \geq 0.$$

This is the important parameter to define the stability of model. We can choose the appropriate constant in the energy density to have stable model. By differentiating Eq. (13) with respect to time, we can obtained expression for squared sound speed as

$$(40) \quad C_s^2 = \frac{\dot{p}}{\dot{\rho}} = \sum_n n A \rho^{n-1} + \frac{(6)^{m/2} (\alpha + m/2) B_0}{\rho^{(\alpha+1+m/2)}},$$

where value of ρ for $n = 2$ and $n = 3$ can be obtained from Eq. (19) and Eq. (24) respectively.

Here the nature of speed of sound in terms of scale factor for varying n is shown in fig. 19 and it is noted that C_s^2 decreases with increasing a and become stable after certain stage of evolution. It is clear that increasing n increases value of sound

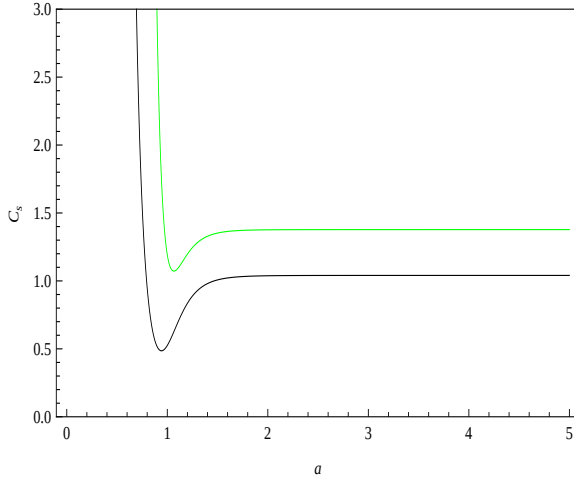


FIGURE 19. Variation of speed of sound versus scale factor for $B_0 = 0.5, A = 0.05, \alpha = 0.5, C = 1, m = 1, n = 2$ (black line), $n = 3$ (green line).

speed. In early time, C_s^2 is decreasing and becomes stable at late time. Therefore our model is stable.

6. Conclusion

In this paper we have presented FRW cosmological model of dark energy filled with ECG and studied the behavior of cosmological parameters for particular and arbitrary values of n, m , and α . Here we assumed the parameter B as a function of Hubble parameter of the form $B(H) = B_0 H^{-m}$. In case 3.1 we have obtained energy density in terms of scale factor, Hubble parameter and scale factor by using numerical methods. For different choices of n, m and α we plot scale factor versus time in figs. 1, 2, 3, 4. From figs. 1 and 2 it is observed that scale factor increases with increasing α in early time and decreases with increasing α in late time. Fig. 3 shows that value of B_0 doesn't affect the scale factor initially because in early time energy density is high, which reduces second term of Eq. (14) to zero. Fig. 4 shows that for $A = (1 - \alpha)/(1 + \alpha)$ scale factor decreases with increasing α . For the case $n = 1$, if we put $m = 0$ our results are consistent with the results obtained by Pourhassan and Kahya [36].

In case 3.2 and 3.3 we have plotted Hubble parameter versus redshift for different choices of α and observed that their behavior is same for both the cases. In section 3.4 we have compared $H(z)$ with the observational data which is shown in figs. 8, 9, 10. From graphs, it is clear that for some particular choices of c Hubble parameter is consistent with observational data and as the value of c increases $H(z)$ shows disagreement with observational data. In section 3.6 we obtained numerical solution for cosmological parameters and represented them graphically. From figs. 11, 12, and 13 it is clear that, scale factor decreases with increasing n and α . Numerical analysis for energy density is represented in figs. 14, 15 and 17. From these figures, it is clear that energy density is a decreasing function of time. Also, it decreases with increasing n and m . Numerical solution for Hubble expansion parameter is plotted in fig. 17 and shows that Hubble expansion parameter is decreases with increasing n .

In section 4, we obtained deceleration parameter and analyzed it's nature in early time and late time. From fig. 18 it is clear that a transition from decelerated $q < 1$ to accelerated $q < 0$ universe is observed when q crosses the zero. Thus, the universe passes from matter dominated era to dark energy dominated phase and undergoes an accelerated expansion. We have also investigated stability of our model by calculating square of speed of sound (C_s^2). Fig. 19 shows the behavior of C_s^2 against scale factor for different values of n . It is observed that speed of sound starts with the positive value and become constant after certain stage of evolution. Thus, we get stability of our model throughout the universe and there is no region of instability.

REFERENCES

- [1] Amani A. R. and Pourhassan B., Viscous generalized Chaplygin gas with arbitrary α , Int. J. Theor. Phys., **52**, 1309-1313 (2013).
- [2] Armendriz-Picon C., Dave R., Mukhanov, V. and Steinhardt P. J., Dynamical solution to the problem of small cosmological constant and late time cosmic acceleration, Phys. Rev. Lett., **85**, 4438-4441 (2000).
- [3] Bento M. C, Bertolami O. and Sen A. A., Generalized Chaplygin gas, accelerated expansion, and dark-energy-matter unification, Phys. Rev. D, **66**, 043507 (2002).
- [4] Bertone G., Servant G. and Sigl G., Indirect detection of Kaluza Klein dark matter, Phys. Rev. D, **68**, 044008 (2003).
- [5] Caldwell R. R., Dave R. and Steinhardt P. J., Cosmological imprint of an energy component with general equation of state, Phys. Rev. Lett. B, **80**, 1582 (1998).

- [6] Caldwell R. R., A phantom menace?, *Phys. Rev. Lett. B*, **545**, 23 (2002).
- [7] Chaplygin S., On gas jets, *Sci. Mem. Moscow Univ. Math. Phys.*, **21**, 1-121 (1904)
- [8] Chatterjee S., In homogeneities in dusty universe - a possible alternative to dark energy?, *JCAP*, **03**, 014 (2011).
- [9] Chayan R., Chakroborty S. and Debnath U., Variable modified Chaplygin gas in anisotropic universe with Kaluza-Klein metric, *Int. J. Theor. Phys.*, **52**, 862876 (2013).
- [10] Chiba T., Sugiyama N. and Nakamura T., Cosmology with x - matter, *Mon. Not. Roy. Astron. Soc. Rev. Mod. Phys. B*, **61**, L5 (1989).
- [11] Clifford J., *D-Branes*, Cambridge Univ. Press, Cambridge (2003).
- [12] Costa S. S., Ujevic M. and Dos Santos A. F., A mathematical analysis of the evolution of perturbations in a modified Chaplygin gas model, *Gen. Rel. and Grav.*, **40**, 16831703 (2008).
- [13] Delubac T., Baryon acoustic oscillations in the Ly α forest
- [14] Elmardi M. and Abede A., Cosmological Chaplygin gas as modified gravity, *J. Phys.*, **905**, 012015 (2017).
- [15] Fonta-Ribera A., Quasar-Lyman forest cross-correlation from BOSS DR11: Baryon Acoustic Oscillations, *JCAP*, **5**, 27 (2014).
- [16] Gorini V., Kamenshchik A., Moschella U. and Pasquier V., The Chaplygin gas as a model for dark energy, *Proceedings of the 10th Marcel Grossmann Meeting*, Rio de Janeiro, Brazil, 840 - 859, (2006).
- [17] Gorini V., Kamenshchik A. and Moschella U., Can the Chaplygin gas be a plausible model for dark energy?, *Phys. Rev.*, **67**, 063509 (2003).
- [18] Jimenez R., Licia V., Tommaso T. and Daniel S., Constraints on the Equation of State of Dark Energy and the Hubble Constant from Stellar Ages and the Cosmic Microwave Background, *Astrophys. J.*, **593**, 622 (2003).
- [19] Kamenshchik A., Moschella U. and Pasquier V., An alternative to quintessence, *Phys. Lett. B*, **511**, 265-268 (2001).
- [20] Khadekar G. S., Shelote R., Gharad N., Role of modified Chaplygin gas in Little Rip cosmological models in Kaluza-Klein theory of gravitation, *Mathematics Today*, **31**, 26-35 (2015).
- [21] Khurshudyan M., Interacting extended Chaplygin gas Cosmology in Lyra Manifold, *Astrophys. Space Sci.*, **44**, 360 (2015).
- [22] Klein O., Quantum theory and five-dimensional theory of relativity, *Z. Phys.*, **37**, 895 (1926).
- [23] Krasinski A., Hellaby C., Bolejko K. and Celerier M. N., Imitating accelerated expansion of the universe by matter inhomogeneities: correction of some misunderstandings, *Gen. Rel. Grav.*, **42**, 2453 (2010).
- [24] Li M., A model of dark energy, *Phys. Rev. Lett. B*, **603**, 1 (2004).
- [25] Linder E. V. and Scherrer R. J., Aetherizing Lambda: barotropic fluids as dark energy, *Phys. Rev. D*, **80**, 023008 (2009).
- [26] Lu J., Xu L., Wu Y. and Liu M., Combined constraints on modified Chaplygin gas model from cosmological observed data: Markov Chain Monte Carlo approach, *Gen. Rel. Grav.*, **43**, 819 (2011).

- [27] Ma C., Zhang T. J., Power of observational Hubble parameter data: a figure of merit exploration, *Astrophys. J.*, **730**, 74 (2011).
- [28] Moresco M., Cimatti A., Jimenez R. and Pozzet L., et al., Improved constraints on the expansion rate of the universe up to $z \sim 1.1$ from the spectroscopic evolution of cosmic chronometers, *JCAP*, **8**, 6 (2012).
- [29] Moresco M., Raising the bar: new constraints on the Hubble parameter with cosmic chronometers at $z=2$, *Mon. Not. R. Astron. Soc.*, **450**, L16 (2015).
- [30] Nobbenhuis S., Categorizing different approaches to the cosmological constant problem, *Foundation of Physics B*, **36**, 613-680 (2006).
- [31] Nojiri S. and Odintsov S. D., Final state and thermodynamics of a dark energy universe, *Phys. Rev. D*, **70**, 103522 (2004).
- [32] Panigrahi D. and Chatterjee S., Spherically symmetric inhomogeneous model with Chaplygin gas, *JCAP*, **10**, 002 (2011).
- [33] Panigrahi D. and Chatterjee S., FRW type of cosmology with a Chaplygin gas, *Int. J. Mod. Phys.*, **21**, 1250079 (2012).
- [34] Paul B. C., Thakur P. and Saha A., Observational constraints on extended Chaplygin gas cosmologies, *Pramana J. Phys.*, **29**, 89 (2017).
- [35] Polchinski J., *String Theory*, Cambridge Univ. Press, Cambridge (1998).
- [36] Pourhassan B. and Kahya E. O., FRW cosmology with extended Chaplygin gas, *Adv. High Energy Phys.*, **2014**, 231452 (2014).
- [37] Rahman F., Jamil M. and Chakraborty K., Revisiting the classical electron model in general relativity, *Astrophys. Space Sci.*, **331**, 191-197 (2011).
- [38] Saadat H. and Pourhassan B., Effect of varying bulk viscosity on generalized Chaplygin gas, *Int. J. Theor. Phys.*, **53**, 1168-1173 (2014).
- [39] Saadat H. and Pourhassan B., FRW bulk viscous cosmology with modified Chaplygin gas in flat space, *Astrophys. Space Sci.*, **343**, 783-786 (2013).
- [40] Saadat H. and Farahani H., Viscous Chaplygin gas in non-flat universe, *Int. J. Theor. Phys.*, **52**, 1160-1164 (2013).
- [41] Saadat H., A cosmological model of the early universe based on ECG with variable Λ -term in Lyra geometry, *Int. J. Theor. Phys.*, **55**, 2364-2375 (2016).
- [42] Sahni V., Saini T. D., Starobinsky A. A. and Alam U., Statefinder - A new geometrical diagnostic of dark energy, *JETP Lett.*, **77**, 201 (2003).
- [43] Salehi A., Bouncing universe in the presence of an extended Chaplygin gas, *Phys. Rev. D.*, **94**, 123519 (2016).
- [44] Salti M., Aydogdu O., Yanar H. and Sogut K., Variable generalized Chaplygin gas in a 5D cosmology, *Annals of Phys.*, **390** 131-142 (2018).
- [45] Sen A., Remarks on techyon driven cosmology, *Physica Scripta*, **T117**, 70-75 (2005).
- [46] Simon J., Licia V. and Jimenez R., Constraints on the redshift dependence of the dark energy potential, *Phys. Rev.*, **71**, 123001 (2005).

- [47] Stern D., Jimenez R., Verde L., Kamionkowski M. and Adam Stanford S., Cosmic chronometers: constraining the equation of state of dark energy. I: $H(z)$ measurements, JCAP, **1002**, 008 (2010).
- [48] Weinberg S., The cosmological constant problem, Rev. Mod. Phys., B **61**, 1 (1989).
- [49] Wess J. and Bagger J., Supersymmetry and Supergravity, Princeton Univ. Press, Princeton (1992).
- [50] Xu Y. D., Huang Z. G. and Zhai X. H., Generalized Chaplygin gas model with or without viscosity in the w - w' plane, Astrophys. Space Sci., **337**, 493-498 (2012).
- [51] Yu A., Kamenshchik A., Moschella U. and Pasquir V., An alternative to quintessence, Phys. Lett. B, **511**, 265 (2010).
- [52] Zhai X. H., Xu Y. D. and Li X. Z., Viscous generalized Chaplygin gas, Int. J. Mod. Phys., **15**, 1151 (2006).
- [53] Zhang T. J., Ma C. and Lan T., Constraints on the dark side of the universe and observational Hubble parameter data, Adv. in Astron., **2010**, 184284 (2010).
- [54] Zhu Z. H., Generalized Chaplygin gas as a unified scenario of dark matter/energy: Observational constraints, Astrophys. Space Sci., **423**, 421-426 (2004). of BOSS DR11 quasars, Astron. Astrophys., **574**, A59 (2015).