

DYNAMICAL ANISOTROPIC DARK ENERGY COSMOLOGICAL MODEL IN GENERAL RELATIVITY

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ABSTRACT. We have investigated an anisotropic dark energy cosmological model in the presence of anisotropic dark energy fluid coupled with mass less scalar field in Kantowski-Sachs space-time in general theory of relativity. A physically viable dark energy cosmological model is presented using (i) a hybrid expansion law proposed by Akarsu et al. (JCAP. **01**, 022: 2014) and (ii) a relation between the metric potential. The dark energy density, the equation of state (EoS) parameter and the skewness parameters of the model are determined and their physical significance is pointed out. The deceleration parameter and the jerk parameter of the model show that there is a smooth transition of the universe from early deceleration to late time acceleration which confirms the observations modern cosmology.

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1. INTRODUCTION

Dark energy which is considered to be responsible for the recent scenario of early inflation and late time acceleration [1]–[2] still remains a mystery. However several dark energy models [3]–[7] and modified theories of gravitation [8]–[10] have been proposed from time to time to explain and explore this mysterious concept in modern cosmology. It is believed

that our universe consists of cosmic fluid made up of 70% of dark energy and 30% of dark matter most of it being non-baryonic. In recent years, construction of dark energy models in general relativity and in modified theories of gravitation in the presence of anisotropic dark energy fluid coupled with scalar fields is inspiring several researchers due to the fact that scalar fields play an important role in the discussion of cosmology. Here we are interested in the investigation of anisotropic dark energy models in Einstein's theory of gravitation in the presence of zero mass scalar meson fields.

The study of interacting fields (one of the fields being scalar meson field) is basically an attempt to look into the yet unsolved problem of the unification of the gravitational and quantum theories. In fact, there are two types of scalar meson fields, namely, zero rest mass scalar fields and attractive massive scalar fields. The zero rest mass scalar fields represent long range interactions while massive scalar fields describe short range interactions. The investigation of Einstein field equations in the presence of scalar meson field has attracted the attention of many workers [11]-[13]. Chatterjee and Roy [14] have discussed the plane symmetric perfect fluid in the presence of mass less scalar fields. Subsequently, the investigation of interacting scalar fields has gained importance because of the fact that the observations reveal that spatially homogeneous and anisotropic cosmological models provide a richer structure of our early universe, both geometrically and physically, than the FRW models of the present day universe. Mohanty and Pradhan [15] have discussed some non-static solutions of Einstein field equations in the presence of viscous fluid and attractive massive scalar field. Singh and Shri Ram [16] have obtained plane symmetric cosmological models with viscous fluid and massive scalar field as the source. Rahaman et al. [17] obtained Kantowski-Sachs cosmological model in the presence of mass less scalar field in Lyra Manifold [18]. Singh et al. [19] investigated Bianchi type-III cosmological models in Lyra geometry in the presence of massive scalar field. Spatially homogeneous and anisotropic Bianchi type-V DE models in the presence of mass less scalar meson fields have not been studied recently by Reddy [20]. It is well known that exact solutions of general relativity for homogenous and anisotropic space times belong to either Bianchi type or Kantowski-Sachs. It may be mentioned here that Kantowski-Sachs models in scalar-tensor theories have been investigated by many authors [21]-[22]. Adhav et al. [23] have obtained exact solution for Kantowski-Sachs model in general relativity. Panigrahi and Sahu [24] studied micro and macro cosmological models in the presence of mass less scalar fields interacting with perfect fluid while Tiwari and Dwivedi [25] have studied cosmological

models coupled with perfect fluid and mass less scalar fields in general relativity. Rao et al. [26], Rao and Prasanthi [28] and Santhi et al. [29] have discussed Kantowski-Sachs anisotropic dark energy models in modified theories of gravitation. However, anisotropic Kantowski-Sachs dark energy models in the presence of scalar meson fields have not been, so far, investigated.

The main objective of this paper is to find Kantowski-Sachs cosmological model with anisotropic dark energy fluid coupled with mass less scalar meson fields in Einstein's theory of gravitation. The next section is devoted to the derivation of basic field equations corresponding to our problem. Section 3 determines the model by solving the field equations. The dynamical and geometrical parameters of the model are determined and their physical importance is pointed out in section 4. Summary and conclusions are presented in the last section.

2. BASIC FIELD EQUATIONS

Spatially homogeneous and anisotropic Kantowski-Sachs space-time is given by

$$(1) \quad ds^2 = dt^2 - A^2 dr^2 - B^2(d\theta^2 + \sin^2\theta d\phi^2)$$

where A and B are functions of cosmic time t only.

Kantowski-Sachs space-times represent homogeneous but anisotropic expanding (contracting) cosmologies. They also provide models where the effects of anisotropy can be estimated and compared with FRW cosmologies (Thorne [30]). Also, these space-times play an important role in understanding the realistic picture of the universe immediately after the big bang.

The relativistic field equations for coupled anisotropic dark energy fluid and mass less scalar fields can be written as

$$(2) \quad R_{ij} - \frac{1}{2}g_{ij}R = -(T_{ij}^{(m)} + T_{ij}^{(s)})$$

where $T_{ij}^{(m)}$ and $T_{ij}^{(s)}$ are the energy momentum tensors, respectively, of the anisotropic DE fluid and zero mass scalar field given by

$$(3) \quad T_{ij}^{(m)} = (\rho_\Lambda + p_\Lambda)u_i u_j - g_{ij}p_\Lambda$$

$$(4) \quad T_{ij}^{(s)} = V_{,i}V_{,j} - \frac{1}{2}g_{ij}(V_{,k}V^{,k})$$

Here ρ_Λ and p_Λ are the energy density and pressure, respectively, of the DE fluid, V is the real scalar field satisfying the Klein –Gordon equation

$$(5) \quad g^{ij}V_{;ij} = 0$$

where a semi colon denotes covariant derivative and a comma indicates ordinary derivative, respectively.

The energy- momentum tensor of anisotropic DE fluid given by Eq.(3) can be parameterized as

$$(6) \quad T_{ij} = \text{diag}[1, \omega_\Lambda, -(\omega_\Lambda + \gamma), -(\omega_\Lambda + \delta)]\rho_\Lambda$$

where

$$(7) \quad \omega_\Lambda = \frac{p_\Lambda}{\rho_\Lambda}$$

is the EoS parameter of DE. Here the skewness parameters γ and δ are the deviations from ω_Λ along ϑ and φ directions respectively.

Using comoving coordinates and Eqs. (2)-(7) the explicit field equations for the metric(1) can be written as

$$(8) \quad 2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} + \frac{\dot{V}^2}{2} = -\omega_\Lambda\rho_\Lambda$$

$$(9) \quad \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{V}^2}{2} = -(\omega_\Lambda + \gamma)\rho_\Lambda$$

$$(10) \quad \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{V}^2}{2} = -(\omega_\Lambda + \delta)\rho_\Lambda$$

$$(11) \quad 2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} - \frac{\dot{V}^2}{2} = \rho_\Lambda$$

$$(12) \quad \ddot{V} + \dot{V}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) = 0$$

Also the energy conservation equation $T^{ij}_{;j} = 0$ leads us to the following equation

$$(13) \quad \dot{\rho}_\Lambda + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)(1 + \omega)\rho_\Lambda + (\delta + \gamma)\rho_\Lambda\frac{\dot{B}}{B} = 0$$

which is a consequence of the field equations(8)-(11). Here an over head dot denotes differentiation with respect to cosmic time.

We, now, define the following parameters which will be useful in solving the above field equations.

The average scale factor $a(t)$ and spatial volume V are given by

$$(14) \quad a(t) = (AB^2)^{\frac{1}{3}}$$

$$(15) \quad V = a^3 = AB^2$$

The average Hubble's parameter is

$$(16) \quad H = \frac{1}{3} \left(\frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} \right)$$

The scalar expansion θ and shear scalar σ^2 are given by

$$(17) \quad \theta = \frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B}$$

$$(18) \quad \sigma^2 = \frac{1}{2} \sigma^{ij} \sigma_{ij} = \frac{1}{3} \left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} - 2 \frac{\dot{A}}{A} \frac{\dot{B}}{B} \right)$$

Mean anisotropy parameter is

$$(19) \quad A_h = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2$$

3. SOLUTION AND THE MODEL

From the equations (9) and (10) we, immediately, obtain

$$(20) \quad \delta = \gamma$$

Now the field equations(8) –(13) constitute a system of four independent equations in six unknowns $A, B, V, \rho_\Lambda, \omega_\Lambda$ and δ . Hence we need two more conditions to obtain a determinate solution. Here we use the following physically viable conditions:

- (1) the hybrid expansion law for the average scale factor $a(t)$, proposed by Akarsu et al. [31]

$$(21) \quad a(t) = a_1 t^{\alpha_1} e^{\alpha_2 t}$$

here α_1, α_2 and a_1 are non –negative constants. Eq. (21) gives the exponential law when $\alpha_1 = 0$ and the power law when $\alpha_2 = 0$. Recently, Santhi et al. [32] and Reddy [33] have investigated some dark energy cosmological models using this hybrid expansion law for scale factor.

(2) The shear scalar σ^2 is proportional to scalar expansion θ so that we can take [34]

$$(22) \quad A = B^m$$

where $m \neq 1$ is a positive constant and preserves the anisotropy of the space time.

Now using equations (14), (21) and (22), we obtain the expressions for metric potentials (choosing $a_1 = 1$) as

$$(23) \quad \begin{aligned} A &= (t^{\alpha_1} e^{\alpha_2 t})^{\frac{3m}{(m+2)}} \\ B &= (t^{\alpha_1} e^{\alpha_2 t})^{\frac{3}{(m+2)}} \end{aligned}$$

Also from equations (12) and (23) the mass less scalar field in the model is given by

$$(24) \quad V = \int (V_0 t^{\alpha_1} e^{\alpha_2 t})^{-3} dt + \psi_0$$

where V_0 and φ_0 are constants of integration and the integration constant ψ_0 can be set equal to zero.

Now through a proper choice of coordinates and constants the metric (1) with the help of Eq. (23) can be written as

$$(25) \quad ds^2 = -dt^2 + \left(t^{\alpha_1} e^{\alpha_2 t}\right)^{\frac{6m}{(m+2)}} dr^2 + \left(t^{\alpha_1} e^{\alpha_2 t}\right)^{\frac{6}{(m+2)}} (d\theta^2 + \sin^2 \theta d\phi^2)$$

and the zero mass scalar field in the model is given by

$$(26) \quad V = \int (V_0 t^{\alpha_1} e^{\alpha_2 t})^{-3} dt$$

which describes the Kantowski-Sachs model in the presence of scalar-meson field with following physical and geometrical parameters.

4. GEOMETRICAL AND PHYSICAL PARAMETERS OF THE MODEL

Spatial volume v , scalar expansion θ , shear scalar σ^2 , skewness parameter δ and anisotropy parameter A_h are the geometrical parameters and the average Hubble parameter H , dark energy density ρ_Λ , the scalar field V and the EoS parameter ω_Λ are physical parameters which are all found to be

$$(27) \quad v = (t^{\alpha_1} e^{\alpha_2 t})^3$$

$$(28) \quad H = \frac{\dot{a}}{a} = \frac{\alpha_1}{t} + \alpha_2$$

$$(29) \quad \theta = 3H = 3\left(\frac{\alpha_1}{t} + \alpha_2\right)$$

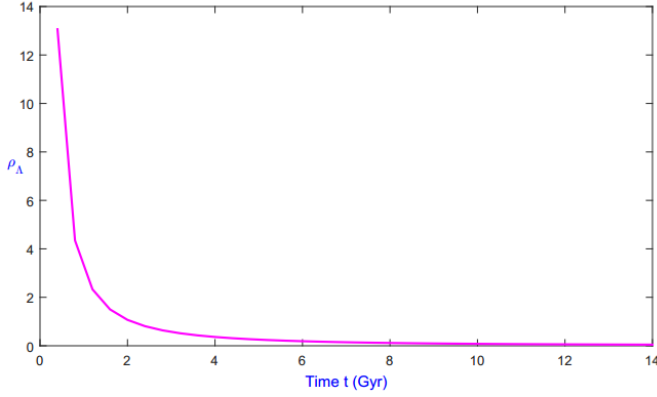


FIGURE 1. Plot of energy density of dark energy versus cosmic time for $\alpha_1 = 0.6, \alpha_2 = 0.06, V_0 = 0.01$ and $m = 0.98$.

$$(30) \quad \sigma^2 = \frac{9(m-1)^2 t^2}{2(m+2)^2 (\alpha_1 + \alpha_2 t)^2}$$

$$(31) \quad A_h = \frac{2(m-1)^2}{(m+2)^2}$$

The anisotropic dark energy density is determined from Eqs.(11),(25) and (26) as

$$(32) \quad \rho_\Lambda = \frac{9(\alpha_1 + \alpha_2 t)^2 (2m+1) + 2t^2 (m+2)^2 (t^{\alpha_1} e^{\alpha_2 t})^{\frac{-6}{m+2}} - V_0^2 t^2 (m+2)^2 (t^{\alpha_1} e^{\alpha_2 t})^{-6}}{t^2 (m+2)^2}$$

The EoS parameter can be found, from Eqs. (8), (25), (26) and (32), as

$$(33) \quad \omega_\Lambda = - \left[\frac{54(\alpha_1 + \alpha_2 t)^2 - 12\alpha_1(m+2) + 2t^2(m+2)^2 (t^{\alpha_1} e^{\alpha_2 t})^{\frac{-6}{m+2}} - V_0^2 t^2 (m+2)^2 (t^{\alpha_1} e^{\alpha_2 t})^{-6}}{9(\alpha_1 + \alpha_2 t)^2 (2m+1) + 2t^2(m+2)^2 (t^{\alpha_1} e^{\alpha_2 t})^{\frac{-6}{m+2}} - V_0^2 t^2 (m+2)^2 (t^{\alpha_1} e^{\alpha_2 t})^{-6}} \right]$$

The skewness parameters can be found from Eqs.(8),(10),(25) and (32) as

$$(34) \quad \delta = \gamma = \left[\frac{3\alpha_1(m-1)(m+2) - 9(\alpha_1 + \alpha_2 t)^2 (m+2)^2 (m-1) + (m+2)^2 t^2 (t^{\alpha_1} e^{\alpha_2 t})^{\frac{-6}{m+2}}}{9(\alpha_1 + \alpha_2 t)^2 (2m+1) + 2t^2(m+2)^2 (t^{\alpha_1} e^{\alpha_2 t})^{\frac{-6}{m+2}} - V_0^2 t^2 (m+2)^2 (t^{\alpha_1} e^{\alpha_2 t})^{-6}} \right]$$

The deceleration parameter of the model is defined and is given as

$$(35) \quad q = -1 + \frac{d}{dt} \left(\frac{1}{H} \right) = -1 + \frac{\alpha_1}{(\alpha_1 + \alpha_2 t)^2}$$

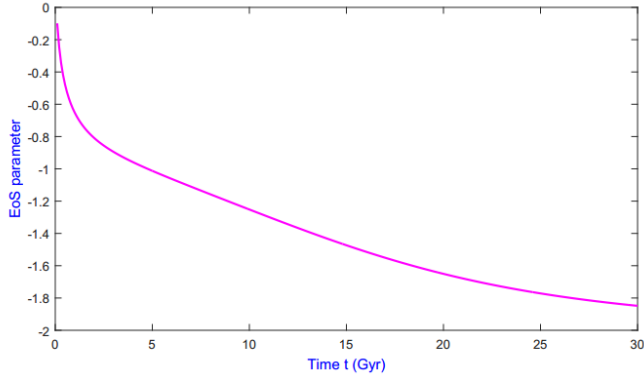


FIGURE 2. Plot of EoS parameter of dark energy versus cosmic time for $\alpha_1 = 0.6, \alpha_2 = 0.06, V_0 = 0.01$ and $m = 0.98$.

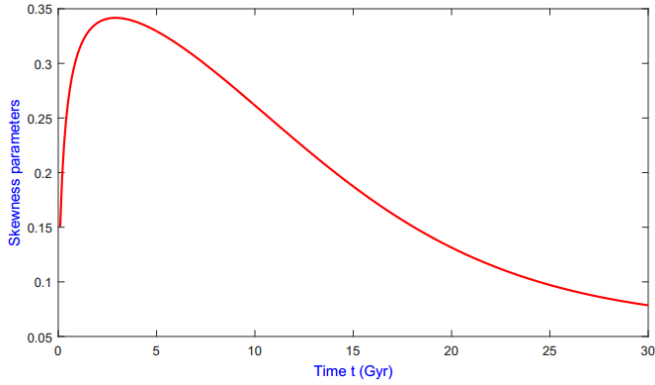


FIGURE 3. Plot of skewness parameter versus cosmic time for $\alpha_1 = 0.6, \alpha_2 = 0.06, V_0 = 0.01$ and $m = 0.98$.

The jerk parameter in cosmology is defined as the third derivative of the average scale factor of the universe and is given by [35]

$$(36) \quad j(t) = \frac{\ddot{a}}{aH^3} = q + 2q^2 - \frac{\dot{q}}{H}$$

For our model, using Eqs. (28), (35) and (36), we obtain

$$(37) \quad j(t) = 1 - \frac{3\alpha_1}{(\alpha_1 + \alpha_2 t)^2} + \frac{2\alpha_1}{(\alpha_1 + \alpha_2 t)^3}$$

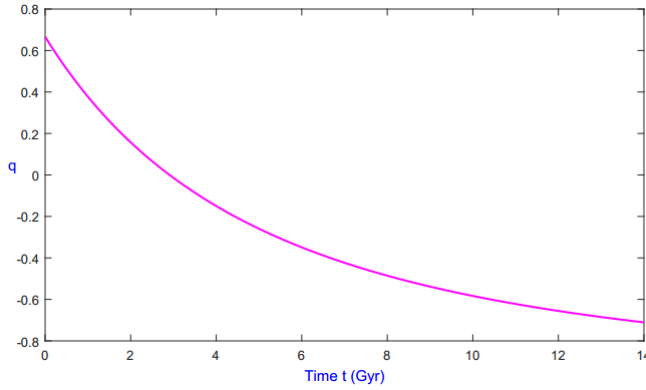


FIGURE 4. Plot of deceleration parameter versus cosmic time for $\alpha_1 = 0.6, \alpha_2 = 0.06$.

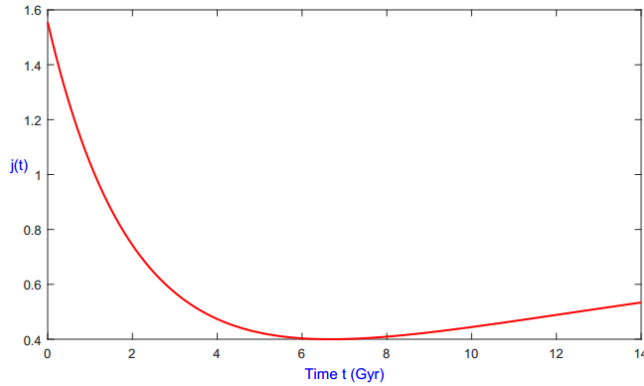


FIGURE 5. Plot of jerk parameter versus cosmic time for $\alpha_1 = 0.6, \alpha_2 = 0.06$.

In cosmology the jerk parameter is essential to describe models close to Λ CDM. Also, it is believed that our universe evolves smoothly from decelerated phase to present accelerated phase when $j(t)$ is positive and q is negative and attains Λ CDM when $j=1$.

Discussion: The above cosmological parameters are useful in the discussion of the behavior of our model given by Eqs. (25) and (26). It may be noted that the model is free from initial singularities and volume of the model increases with time. This shows the spatial expansion of our universe. The quantities H, θ, σ^2 and the mass less scalar field V all diverge at the initial epoch, $t=0$ and vanish as $t \rightarrow \infty$. It can be observed that anisotropy

parameter and shear scalar vanish at $m=1$ which implies that the universe attains isotropy and becomes shear free.

From Fig. 1 it can be seen that the dark energy density is positive throughout the evolution of the universe and vanishes for large values of time t . Fig. 2 depicts the evolution of EoS parameter versus cosmic time. We observe that the EoS parameter is always negative and varies in both quintessence ($\omega_\Lambda > -1$) and phantom ($\omega_\Lambda < -1$) regions. It is easy to see from the Fig. 3 that the skewness parameter is positive decreasing function of time and vanishes at late times. Fig. 4 shows variation deceleration parameter with respect to cosmic time t . This shows that the universe represents, initially, the deceleration phase and then enters into negative region indicating the accelerating phase and attains the present value $q_0=-0.703$ ultimately. Also from Fig. 3 we see that the jerk parameter is positive throughout the evolution of the universe and approaches to one at late times which implies that the universe exhibits a smooth transition of the universe from early deceleration to the present accelerated phase. Thus our model is confirms the current cosmological observations.

5. SUMMARY AND CONCLUSIONS

Dark energy models in the presence of scalar fields are an interesting subject of investigation in recent years in modern cosmology to explain the scenario of accelerated expansion of the universe. In this paper, we have obtained an anisotropic Kantowski-Sachs dark energy cosmological model coupled with a mass less scalar field. In order to obtain a deterministic model we have used a relation between the metric potentials and a hybrid expansion law. We have studied the behavior of the model by computing dark energy density, EoS parameter, deceleration parameter and jerk parameter. It is observed that the scalar field model varies in both quintessence and phantom, and attains a constant value in phantom region. The model undergoes a smooth transition from decelerated phase to an accelerated phase at late times. This behavior of the universe is in accordance with the observational data.

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