

EFFECTS OF POYNTING-ROBERTSON (P-R) DRAG, RADIATION, AND OBLATENESS ON MOTION AROUND THE $L_{4,5}$ EQUILIBRIUM POINTS IN THE CR3BP

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ABSTRACT. The research paper under consideration investigates the motion in the vicinity of triangular equilibrium points of the circular restricted three-body problem of a passively gravitating dust particle in the gravitational field for the binaries system (Kruger 60 and Achird). The two bodies of the binary are both oblate radiating stars possessing P-R drag. The position of triangular equilibrium points of the particle is seen to be affected by oblateness, radiation and P-R drag. In our numerical exploration of the binary system Kruger 60 and Achird, we computed the radiation factors $q_i (i = 1, 2)$ and the velocity of light c_d which is a component of the P-R drag. The points are unstable due to the presence of a positive real part of the complex roots.

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1. INTRODUCTION

The Restricted Three-Body Problem (R3BP), where an infinitesimal mass (called the third body) moves around the gravitational field of the primaries in circular orbits about their common center of mass. It possesses five equilibrium points

$L_i (i = 1, 2, \dots, 5)$. The points L_1 , L_2 and L_3 are called collinear equilibrium points, while L_4 and L_5 are called triangular equilibrium points.

The classical R3BP considers the bodies involved to be strictly spherical; but in the solar (e.g. Sun, Earth, Jupiter and Saturn) and in the Stellar (e.g. Achernar, Alfa area, Kruger 60, Achird, Cen X-4) systems, some planets, stars and their satellites (Moon, Charon) are sufficiently oblate. The oblateness of a celestial body can produce perturbation from the two-body motion. It's well known example in the solar system in the orbit of the fifth satellite of Jupiter and Amatheia. This justifies the inclusion of oblateness in the study of motion of celestial bodies. In the context of stars, their fast rotation produces an equatorial bulge due to centrifugal force. Neutron stars and black dwarf may be considered oblate because of their rapid rotation. The millisecond pulsar PSRB1937+21 and the pulsar PSRJ1748-422ad are also assumed to be oblate owing to their swiftest spinning.

Radzievsky [1,2] was the first to study the photogravitational restricted three-body problem (PR3BP). He found besides the five libration points (classical problem), additional points L_6 and L_7 called out-of-plane equilibrium points. Chernikov [3] later introduced the Poynting-Robertson (P-R) effect.

In discussing light radiation force, the above studies of PR3BPs with or without oblateness have taken into account, just one of the three components of the light pressure field, which is due to the central force; the gravitation and the radiation pressure. The remaining two components are arising from the Doppler Shift and the absorption and subsequent re-emission of the incident radiation. The Poynting-Robertson (P-R) drag constitutes these last two terms, which causes small particles of the solar system to spiral into the sun at a cosmically rapid rate. Poynting [4] pointed out this problem when he considered the effect of absorption and subsequent re-emission of sunlight by small isolated particles in the solar system. Later, Robertson [5] modified his theory by improving the expression of the total radiation force using precise relativistic treatments of the first order in the ratio of the rate of change in time of the particle to that of light. By law, the solar radiation pressure force F_p is exactly opposite to the gravitational attraction force F_g and change with the distance. It satisfies that the result of the action of this force will lead to reducing the effective mass of the particles.

The study P-R effect is now significant in the area of zodiacal cloud, orbital evolution of cometary meteor streams, asteroidal particles and dust rings around planets. Knowing the importance of the (P-R) drag, this motivated several researchers like Perezhugin [6], Schuerman [7], Simmons et al. [8], Murray [9], Ragos and Zafiropoulos [10], Kushvah [11] and Das et al. [12] study the existence of equilibrium points and their stability in the PR3BPs with various issues. They have considered more reasonable and understandable issues related to the motion of the radiating primaries. All these works have arrived at the conclusion that the P-R effect renders unstable those equilibrium positions, which are conditionally stable in the classical case.

Authors of the R3BPS (Hamilton and Burns [13]; Singh and Ishwar [14]; Kunitsyn [15]; Singh [16]; Singh and Leke [17]; Singh et al. [18]; Ershkov [19]; Abouelmagd [20]) have addressed the influences of oblateness, radiation pressure, small changes in the Coriolis and centrifugal forces, variation of the masses, Yarkovsky effects and the P-R drag. Some significant studies of the R3BP with one or both oblate primaries whose equatorial plane is coincident with the plane of motion are investigated (Danby [21]; Sharma and SubbaRao [22]; Sharma [23]; Markellos et al. [24]; AbdulRaheem and Singh [25]; Abouelmagd [26]; Singh and Leke [27]). Also, Singh and Amuda [28] studied the motion of a test particle around the triangular equilibrium points under the effects of radiation pressure of the second primary together with its P-R drag when the first primary is an oblate spheroid. They found that the positions of triangular points are influenced by the presence of these parameters, the linear stability of the problem is investigated and applied to the binary system RXJ 0450.1-5856. It is observed that the triangular points are unstable due to the presence of positive roots. In considering both primaries as luminous-oblate spheroids, one can think of Kruger 60 and Achird which are the brightest naked eye star in the constellation.

In this paper, our effort is geared towards investigating the effect produced by oblateness of the primaries and their radiation pressures together with the P-R drag on the existence of triangular points and their linear stability in the CR3BP. The paper organization is as follows: Section 2 derives the equations of motion. The existence of triangular equilibrium points is discussed in Section 3, while Section 4

shows numerical applications. Section 5 examines the stability of these triangular points. Finally, section 6 summarizes the conclusion of the paper.

2. EQUATIONS OF MOTION

The barycentric rotating co-ordinate system $0xyz$ relative to an inertial system with angular velocity n and common z -axis is considered with a line joining the primaries as the x -axis. Let m_1, m_2 be the masses of bigger and smaller primaries (Fig. 1), respectively. In this context, r_1 and r_2 are the distances of the third body from the primaries. Let (x, y, z) be the coordinates of an infinitesimal mass m , and the units such that the sum of the masses and the distance between primaries are unity. The unit of time i.e. time period of m_2 about m_1 consists of 2π units such that the Gaussian constant of gravitational $\gamma = 1$. Then the perturbed mean n motion of the primaries is given by $n^2 = 1 + \frac{3}{2}(A_1 + A_2)$, where $A_i = \frac{AE_i^2 - AP_i^2}{5R^2}$ ($i = 1, 2$) are the oblateness coefficients of m_i and are such that $0 < A_i < 1$ (McCuskey, [29]). AE_i, AP_i are the equatorial and polar radii of m_i , respectively and R the distance between the primaries. Let $\mu = \frac{m_2}{m_1 + m_2}$ be the mass parameter, then $1 - \mu = \frac{m_1}{m_1 + m_2}$ with $m_1 > m_2$.

Let F_g and F_p denote the gravitational and radiation forces acting on a particle. Then the resultant force on the particle is given by $F = F_g - F_p = F_g \left(1 - \frac{F_p}{F_g}\right) = qF_g$, where q is the factor characterizing radiation effects.

If the solar radiation flood fluctuations and a shadow effect of the planet are neglected, then q is assumed to be constant. Depending upon the value of q , the reduced particle mass is positive, negative or zero. In this paper, we consider the case when the gravitation prevails, i.e. $q > 0$. We denote the radiation factors as q_i ($i = 1, 2$) for the bigger and smaller primaries respectively, and they are given by $q_i = 1 - \left(\frac{F_{pi}}{F_{gi}}\right) = 1 - \delta_i$ ($i = 1, 2$) such that $0 < 1 - q_i < 1$ ($i = 1, 2$), where δ_i are the ratios of the radiation pressure force to the gravitational force of the primaries. The relativistic treatment of this problem was formulated by Robertson [5] and Chernikov [3], who showed that to first order in $\frac{v}{c}$ the radiation pressure force is given by.

$$(1) \quad \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

where,

$$\vec{F}_1 = F_p \frac{\vec{R}}{R}, \vec{F}_2 = -F_p \frac{\vec{v} \cdot \vec{R}}{cR} \frac{\vec{R}}{R}, \vec{F}_3 = -F_p \frac{\vec{v}}{c}$$

Here $F_p = \frac{3Lm}{16\pi R^2 \rho s c}$ denotes the measure of the radiation pressure force, $\vec{R}$ the position vector of a particle p with respect to radiation source, $\vec{v}$ the corresponding velocity vector and c the velocity of light. In the expression of F_p , L is the luminosity of the radiating body, while m , ρ and s are the mass, density and cross section of the particle respectively.

The first term in equation (1) expresses the radiation pressure, the second one represents the Doppler shift of the incident radiation and the third one is due to the absorption and subsequent re-emission of the incident radiation. These last two terms taken together are called the Poynting-Robertson drag. We denote the P-R drag of bigger and smaller primaries by the parameters $W_i (i = 1, 2)$ respectively.

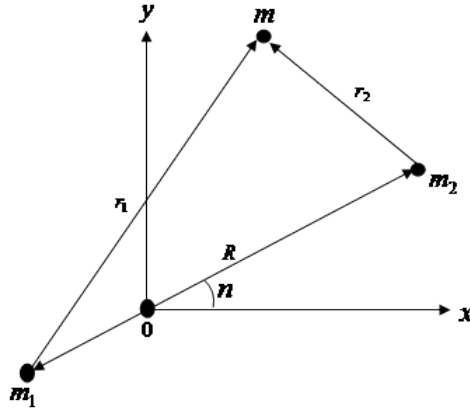


Figure 1: The primaries m_1 and m_2 are moving in circular orbits about their common center of mass unperturbed by the infinitesimal mass m in the CR3BP.

Following McCuskey [29], Ragos and Zafriopoulos [10] and Singh and Amuda [28], the equations of motion of a passively gravitating test body of infinitesimal mass in the CR3BP under the effects of oblateness of the primaries and their radiations together with P-R drag can be written as:

$$\vec{a} + 2\vec{\omega} \times \vec{v} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) = -\frac{(1-\mu)q_1\vec{r}_1}{r_1^3} - \frac{3(1-\mu)A_1q_1\vec{r}_1}{2r_1^5} -$$

$$(2) \quad \frac{(1-\mu)(1-q_1)}{r_1^2} \left[\frac{\vec{r}_1}{r_1} - \left(\frac{(\vec{r}_1 + w \times \vec{r}_1)\vec{r}_1}{c_d r_1} \right) \frac{\vec{r}_1}{r_1} - \frac{(\vec{r}_1 + w \times \vec{r}_1)}{c_d} \right] - \frac{\mu q_2 \vec{r}_2}{r_2^3} \\ - \frac{3\mu A_2 q_2 \vec{r}_2}{2r_2^5} - \frac{\mu(1-q_2)}{r_2^2} \left[\frac{\vec{r}_2}{r_2} - \left(\frac{(\vec{r}_2 + w \times \vec{r}_2)\vec{r}_2}{c_d r_2} \right) \frac{\vec{r}_2}{r_2} - \frac{(\vec{r}_2 + w \times \vec{r}_2)}{c_d} \right]$$

where

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}, \quad \vec{v} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}, \quad \vec{a} = \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k}, \quad \vec{w} = n\vec{k}, \\ \vec{r}_1 = (x + \mu)\vec{i} + y\vec{j} + z\vec{k}, \quad \vec{r}_2 = (x + \mu - 1)\vec{i} + y\vec{j} + z\vec{k}, \\ r_1^2 = (x + \mu)^2 + y^2 + z^2, \quad r_2^2 = (x + \mu - 1)^2 + y^2 + z^2, \\ \vec{w} \times \vec{v} = -n(\dot{y}\vec{i} - \dot{x}\vec{j}), \quad \vec{w} \times (\vec{w} \times \vec{r}) = -n^2(x\vec{i} + y\vec{j}), \\ \vec{r}_2 + \vec{w} \times \vec{r}_2 = (\dot{x} - ny)\vec{i} + [\dot{y} + n(x + \mu - 1)]\vec{j} + \dot{z}\vec{k}, \\ (\dot{\vec{r}}_2 + \vec{w} \times \vec{r}_2)\vec{r}_2 = [(x + \mu - 1)\dot{x} + y\dot{y} + z\dot{z}],$$

where the dots denote differentiations with respect to time t .

Equation (2), on simplifying and equating the coefficients of $\vec{i}, \vec{j}$ and $\vec{k}$ on both sides, is reduced to the system of equations:

$$\ddot{x} - 2n\dot{y} = n^2x - \frac{(1-\mu)(x+\mu)q_1}{r_1^3} - \frac{3(1-\mu)A_1(x+\mu)q_1}{2r_1^5} - \frac{\mu q_2(x+\mu-1)}{r_2^3} \\ - \frac{3\mu(x+\mu-1)A_2q_2}{2r_2^5} - \frac{W_1}{r_1^2} \left[\frac{(x+\mu)}{r_1^2} \{ (x+\mu)\dot{x} + y\dot{y} + z\dot{z} \} + \dot{x} - ny \right] \\ - \frac{W_2}{r_2^2} \left[\frac{(x+\mu-1)}{r_2^2} \{ (x+\mu-1)\dot{x} + y\dot{y} + z\dot{z} \} + \dot{x} - ny \right], \\ \ddot{y} + 2n\dot{x} = n^2y - \frac{(1-\mu)yq_1}{r_1^3} - \frac{3(1-\mu)yq_1A_1}{2r_1^5} - \frac{\mu q_2y}{r_2^3} - \frac{3\mu yA_2q_2}{2r_2^5} \\ - \frac{W_1}{r_1^2} \left[\frac{y}{r_1^2} \{ (x+\mu)\dot{x} + y\dot{y} + z\dot{z} \} + \dot{y} + n(x+\mu) \right] \\ - \frac{W_2}{r_2^2} \left[\frac{y}{r_2^2} \{ (x+\mu-1)\dot{x} + y\dot{y} + z\dot{z} \} + \dot{y} + n(x+\mu-1) \right], \\ \ddot{z} = -\frac{(1-\mu)zq_1}{r_1^3} - \frac{3(1-\mu)zq_1A_1}{2r_1^5} - \frac{\mu zq_2}{r_2^3} - \frac{3\mu zA_2q_2}{2r_2^5} \\ - \frac{W_1}{r_1^2} \left[\frac{z}{r_1^2} \{ (x+\mu)\dot{x} + y\dot{y} + z\dot{z} \} + \dot{z} \right] - \frac{W_2}{r_2^2} \left[\frac{z}{r_2^2} \{ (x+\mu-1)\dot{x} + y\dot{y} + z\dot{z} \} + \dot{z} \right],$$

with

$$W_1 = \frac{(1-\mu)(1-q_1)}{c_d}, \quad W_2 = \frac{\mu(1-q_2)}{c_d}.$$

Hence, the motion in the xy -orbital plane is described by the equations

$$(3) \quad \ddot{x} - 2n\dot{y} = U_x, \quad \ddot{y} + 2n\dot{x} = U_y$$

where

$$\begin{aligned} U_x = & n^2 x - \frac{(1-\mu)(x+\mu)q_1}{r_1^3} - \frac{3(1-\mu)A_1(x+\mu)q_1}{2r_1^5} - \frac{\mu q_2(x+\mu-1)}{r_2^3} \\ & - \frac{3\mu(x+\mu-1)A_2q_2}{2r_2^5} - \frac{W_1}{r_1^2} \left[\frac{(x+\mu)}{r_1^2} \{ (x+\mu)\dot{x} + y\dot{y} + z\dot{z} \} + \dot{x} - ny \right] \\ & - \frac{W_2}{r_2^2} \left[\frac{(x+\mu-1)}{r_2^2} \{ (x+\mu-1)\dot{x} + y\dot{y} + z\dot{z} \} + \dot{x} - ny \right], \\ U_y = & n^2 y - \frac{(1-\mu)yq_1}{r_1^3} - \frac{3(1-\mu)yA_1q_1}{2r_1^5} - \frac{\mu q_2 y}{r_2^3} - \frac{3\mu y A_2 q_2}{2r_2^5} - \\ & - \frac{W_1}{r_1^2} \left[\frac{y}{r_1^2} \{ (x+\mu)\dot{x} + y\dot{y} + z\dot{z} \} + \dot{y} + n(x+\mu) \right] \\ & - \frac{W_2}{r_2^2} \left[\frac{y}{r_2^2} \{ (x+\mu-1)\dot{x} + y\dot{y} + z\dot{z} \} + \dot{y} + n(x+\mu-1) \right], \end{aligned}$$

$$(4) \quad r_1^2 = (x+\mu)^2 + y^2, \quad r_2^2 = (x+\mu-1)^2 + y^2$$

$$(5) \quad n^2 = 1 + \frac{3}{2}A_1 + \frac{3}{2}A_2$$

These equations are affected by the oblateness of the primaries and their radiation pressures P-R drags. Next, we discuss the positions of triangular equilibrium points of the third body.

3. POSITION OF TRIANGULAR EQUILIBRIUM POINTS

The coordinates of triangular equilibrium points are the solutions of the equations $U_x = 0$, $U_y = 0$, $y \neq 0$ with $\dot{x} = \dot{y} = \ddot{x} = \ddot{y} = 0$, that is, they are the solutions of the equations

$$\begin{aligned} (6) \quad & n^2 x - \frac{(1-\mu)(x+\mu)q_1}{r_1^3} - \frac{3(1-\mu)A_1(x+\mu)q_1}{2r_1^5} - \\ & - \frac{\mu q_2(x+\mu-1)}{r_2^3} - \frac{3\mu(x+\mu-1)A_2q_2}{2r_2^5} + \frac{W_1 ny}{r_1^2} + \frac{W_2 ny}{r_2^2} = 0 \end{aligned}$$

and

$$\left[n^2 - \frac{(1-\mu)q_1}{r_1^3} - \frac{3(1-\mu)q_1 A_1}{2r_1^5} - \frac{\mu q_2}{r_2^3} - \frac{3\mu A_2 q_2}{2r_2^5} \right] y$$

$$(7) \quad -\frac{W_1 n(x + \mu)}{r_1^2} - \frac{W_2 n(x + \mu - 1)}{r_2^5} = 0$$

Now, when the P-R drag effect and oblateness factors are ignored, equations (6) and (7) become:

$$x \left[1 - \frac{(1 - \mu)q_1}{r_1^3} - \frac{\mu q_2}{r_2^3} \right] - (1 - \mu)\mu \left[\frac{q_1}{r_1^3} - \frac{q_2}{r_2^3} \right] = 0,$$

$$1 - \frac{(1 - \mu)q_1}{r_1^3} - \frac{\mu q_2}{r_2^3} = 0.$$

When the above equations are solved, we get.

$$(8) \quad r_1 = q_1^{\frac{1}{3}}, \quad r_2 = q_2^{\frac{1}{3}}$$

Knowing that equations (8) are the solutions when the primaries are radiating only. We can with their help, assume the solutions when P-R drag and oblateness of the primaries are present (i.e. $A \neq 0$, $W_2 \neq 0$) to be

$$(9) \quad r_1 = q_1^{\frac{1}{3}} + \epsilon_1, \quad r_2 = q_2^{\frac{1}{3}} + \epsilon_2; \quad |\epsilon_1| \ll 1, |\epsilon_2| \ll 1$$

Now, the exact x -coordinate of the triangular point is found by subtracting the second equation of (4) from the first to get

$$(10) \quad x = \frac{1}{2} - \mu + \frac{r_1^2 - r_2^2}{2}$$

substituting equations (9) in (10), we get

$$(11) \quad x = x_0 + q_1^{\frac{1}{3}} \epsilon_1 - q_2^{\frac{1}{3}} \epsilon_2$$

where $x_0 = \frac{1}{2} - \mu + \frac{q_1^{\frac{2}{3}} - q_2^{\frac{2}{3}}}{2}$.

Now from $y^2 = r_1^2 - (x + \mu)^2$,

we get

$$(12) \quad y = \pm y_0 \left[1 + \frac{1}{2y_0^2} \left(q_1^{\frac{1}{3}} \epsilon_1 + q_2^{\frac{1}{3}} \epsilon_2 \right) - \frac{1}{2y_0^2} \left(q_1^{\frac{2}{3}} - q_2^{\frac{2}{3}} \right) \left(q_1^{\frac{1}{3}} \epsilon_1 - q_2^{\frac{1}{3}} \epsilon_2 \right) \right]$$

where

$$y_0 = \pm \frac{1}{2} \left[-1 + 2 \left(q_1^{\frac{1}{3}} + q_2^{\frac{1}{3}} \right) + \left(q_1^{\frac{2}{3}} - q_2^{\frac{2}{3}} \right) \right]^{\frac{1}{2}}$$

Expressing $q_i = 1 - \delta_i$ ($i = 1, 2$) where δ_i are very small and substituting them in x_0 and y_0 , we have

$$(13) \quad x_0 = \frac{1}{2} - \mu - \frac{1}{3}\delta_1 + \frac{1}{3}\delta_2 \quad \text{and} \quad y_0 = \pm \frac{\sqrt{3}}{2} \left(1 - \frac{2}{9}\delta_1 - \frac{2}{9}\delta_2 \right)$$

Now, substituting equations (5), (9), (11) and (12) into equations (6) and (7), neglecting higher order terms of small quantities, we get the respective equations:

$$(14) \quad a_1 \epsilon_1 + b_1 \epsilon_2 = c_1$$

$$(15) \quad a_2 \epsilon_1 + b_2 \epsilon_2 = c_2$$

where

$$a_1 = 3x_0 - 3\mu x_0 + 3\mu - 3\mu^2$$

$$b_1 = -3\mu + 3\mu^2 + 3\mu x_0$$

$$c_1 = -\frac{3x_0}{2}(A_1 + A_2) + \frac{3x_0}{2}A_1 - \frac{3\mu x_0}{2}A_1 + \frac{3\mu}{2}A_1 - \frac{3\mu^2}{2}A_1 + \frac{3\mu x_0}{2}A_2 + \frac{3\mu^2}{2}A_2 - \frac{3\mu}{2}A_2 - y_0 W_1 - y_0 W_2$$

$$a_2 = 3y_0 - 3\mu y_0$$

$$b_2 = 3\mu y_0$$

$$c_2 = -\frac{3y_0}{2}(A_1 + A_2) + \frac{3y_0}{2}A_1 - \frac{3\mu y_0}{2}A_1 + \frac{3\mu y_0}{2}A_2 + x_0 W_1 + \mu W_1 + x_0 W_2 + \mu W_2 - W_2.$$

The values of ϵ_1 and ϵ_2 can be found using the relations:

$$\epsilon_1 = \frac{b_2 c_1 - b_1 c_2}{a_1 b_2 - a_2 b_1}, \quad \epsilon_2 = \frac{a_1 c_2 - a_2 c_1}{a_1 b_2 - a_2 b_1}$$

and they are

$$\epsilon_1 = -\frac{A_2}{2} - \frac{W_1}{3\sqrt{3}(1-\mu)} - \frac{2W_2}{3\sqrt{3}(1-\mu)}$$

$$(16)$$

$$\epsilon_2 = -\frac{A_1}{2} + \frac{2W_1}{3\mu\sqrt{3}} + \frac{W_2}{3\mu\sqrt{3}}$$

Putting the values of ϵ_1 and ϵ_2 above and equation (13) into equations (11) and (12), we at once have

$$x_4 = \frac{1}{2} - \mu - \frac{1}{3}\delta_1 + \frac{1}{3}\delta_2 + \frac{A_1}{2} - \frac{A_2}{2} - \frac{W_1(2-\mu)}{3\mu\sqrt{3}(1-\mu)} - \frac{W_2(1+\mu)}{3\mu\sqrt{3}(1-\mu)}$$

$$(17)$$

$$y_4 = \pm \left[\frac{\sqrt{3}}{2} \left(1 - \frac{2}{9}\delta_1 - \frac{2}{9}\delta_2 - \frac{A_1}{3} - \frac{A_2}{3} \right) + \frac{W_1(2-3\mu)}{9\mu(1-\mu)} + \frac{W_2(1-3\mu)}{9\mu(1-\mu)} \right]$$

Equations (17) give the coordinates of the triangular points of the system under investigation. Since $r_1 \neq r_2$, the two points defined by (17) form scalene triangles with the primaries. These points are denoted by $L_{4,5}(x_4, \pm y_4)$ and are called the triangular equilibrium points by virtue of the two triangles they form with lines

joining the primaries. The positions of these points depend on the mass ratio, oblateness of the primaries and their radiation pressures and P-R drag effects.

4. NUMERICAL APPLICATION

In order to show the effects of parameters involved in the position and stability of triangular equilibrium points, we consider the binaries system Kruger 60 and Achird. We first compute the dimensionless velocity of light $c_d = \frac{c}{\sqrt{\gamma(M_1+M_2)}}$ (Ragos, O., et al. [30]), where ' c ' is the velocity of light, ' γ ' is the gravitational constant, ' a ' the binary separation, and M_1 and M_2 are the masses of primaries. Then, we obtain the ratios δ_i and consequently radiation factors q_i by means of relation $q = 1 - \frac{AkL}{a\rho M}$ (Xuetang and Lizhong, [31]) and $\delta_2 = \delta_1 \frac{L_2 M_1}{L_1 M_2}$ (Das, M.K., et al. [12]), taking $k = 1$ on the basis of Stefan- Boltzmann's law. L_i, M_i refer to luminosity and masses of the primaries, and a and ρ are the radius and density of a moving body; κ is the radiation pressure efficiency factor of a star; $A = \frac{3}{16\pi CG}$ is a constant. In C.G.S system, $A = 2.9838 \times 10^{-5}$. We suppose that the dust grain has a radius and density $a = 2 \times 10^{-2} cm$ and $\rho = 1.4 gcm^{-3}$, respectively. All these necessary quantities are listed in the table 1.

Binaries	Luminosity (L_0)		Mass (M_0)	
	L_1	L_2	M_1	M_2
Kruger 60	0.01	0.0034	0.271	0.176
Achird	1.29	0.06	0.95	0.62

Table 1a: Numerical data for Kruger 60 and Achird binary system.

Binaries	Radiation Pressure (q_i)		Binary separation (AU) a	Dimensionless velocity of light c_d
	q_1	q_2		
Kruger 60	0.99992	0.99996	9.5	46393.84
Achird	0.9971	0.9997	71	67675.52

Table 1b: Numerical data for Kruger 60 and Achird binary system.

Using the software Mathematica, Table 1(a,b) and equation (17), we locate numerically the positions of triangular points given in tables 2(a,b)-5(a,b). Figure 2 and

3 are the graph of table 2(a,b) and 4(a,b) for the triangular points of the binaries system Kruger 60 and Achird.

q_1	q_2	A_1	A_2
1	1	0	0
0.99992	0.99996	0.00001	0.00002
0.99992	0.99996	0.0001	0.0002
0.99992	0.99996	0.001	0.002
0.99992	0.99996	0.01	0.02
0.99992	0.99996	0.1	0.2

Table 2a. Effect of oblateness on $L_{4,5}$ for $\mu = 0.3937$, $c_d = 46393.84$, $0 < 1 - q_i < 1$ and $0 \leq A_i \leq 0.2$

W_1	W_2	x_4	$\pm y_4$
0	0	0.106300	0.866025
1.04548×10^{-9}	3.39442×10^{-10}	0.106282	0.86594
1.04548×10^{-9}	3.39442×10^{-10}	0.106237	0.865916
1.04548×10^{-9}	3.39442×10^{-10}	0.105787	0.865136
1.04548×10^{-9}	3.39442×10^{-10}	0.101287	0.857342
1.04548×10^{-9}	3.39442×10^{-10}	0.056287	0.779400

Table 2b. Effect of oblateness on $L_{4,5}$ for $\mu = 0.3937$, $c_d = 46393.84$, $0 < 1 - q_i < 1$ and $0 \leq A_i \leq 0.2$

q_1	q_2	A_1	A_2
0.99	0.99996	0.001	0.002
0.9	0.99996	0.001	0.002
0.8	0.99996	0.001	0.002
0.7	0.99996	0.001	0.002
0.6	0.99996	0.001	0.002
0.5	0.99996	0.001	0.002
0.99992	0.95	0.001	0.002
0.99992	0.85	0.001	0.002
0.99992	0.75	0.001	0.002
0.99992	0.65	0.001	0.002
0.99992	0.55	0.001	0.002
0.99992	0.50	0.001	0.002

Table 3a. Effect of radiation pressure on $L_{4,5}$ for $\mu = 0.3937$, $c_d = 46393.84$, $0 < 1 - q_i < 1$ and $0 \leq A_i \leq 0.2$

W_1	W_2	x_4	$\pm y_4$
1.30685×10^{-7}	3.39442×10^{-10}	0.102480	0.863227
1.30685×10^{-6}	3.39442×10^{-10}	0.072748	0.845907
2.61371×10^{-6}	3.39442×10^{-10}	0.039143	0.826663
3.92056×10^{-6}	3.39442×10^{-10}	0.005808	0.807418
5.22742×10^{-6}	3.39442×10^{-10}	-0.027527	0.788174
6.53427×10^{-10}	3.39442×10^{-10}	-0.060862	0.768929
1.04548×10^{-9}	4.24302×10^{-7}	0.122440	0.855521
1.04548×10^{-9}	1.27291×10^{-6}	0.155772	0.836276
1.04548×10^{-9}	2.12151×10^{-6}	0.189104	0.817031
1.04548×10^{-9}	2.97011×10^{-6}	0.222437	0.797786
1.04548×10^{-9}	3.81872×10^{-6}	0.255769	0.778541
1.04548×10^{-9}	4.24302×10^{-6}	0.272435	0.768919

Table 3b. Effect of radiation pressure on $L_{4,5}$ for $\mu = 0.3937$, $c_d = 46393.84$, $0 < 1 - q_i < 1$ and $0 \leq A_i \leq 0.2$

q_1	q_2	A_1	A_2
1	1	0	0
0.9971	0.9997	0.00001	0.00002
0.9971	0.9997	0.0001	0.0002
0.9971	0.9997	0.001	0.002
0.9971	0.9997	0.01	0.02
0.9971	0.9997	0.1	0.2

Table 4a. Effect of oblateness on $L_{4,5}$ for $\mu = 0.3949$, $c_d = 67675.52$, $0 < 1 - q_i < 1$ and $0 \leq A_i \leq 0.2$

W_1	W_2	x_4	$\pm y_4$
0	0	0.104233	0.865410
2.59295×10^{-8}	1.75056×10^{-9}	0.104228	0.865401
2.59295×10^{-8}	1.75056×10^{-9}	0.104183	0.865323
2.59295×10^{-8}	1.75056×10^{-9}	0.103733	0.864544
2.59295×10^{-8}	1.75056×10^{-9}	0.099233	0.856749
2.59295×10^{-8}	1.75056×10^{-9}	0.054233	0.778807

Table 4b. Effect of oblateness on $L_{4,5}$ for $\mu = 0.3949$, $c_d = 67675.52$, $0 < 1 - q_i < 1$ and $0 \leq A_i \leq 0.2$

q_1	q_2	A_1	A_2
0.99	0.9997	0.001	0.002
0.9	0.9997	0.001	0.002
0.8	0.9997	0.001	0.002
0.7	0.9997	0.001	0.002
0.6	0.9997	0.001	0.002
0.5	0.9997	0.001	0.002
0.9971	0.95	0.001	0.002
0.9971	0.85	0.001	0.002
0.9971	0.75	0.001	0.002
0.9971	0.65	0.001	0.002
0.9971	0.55	0.001	0.002
0.9971	0.50	0.001	0.002

Table 5a. Effect of radiation pressure on $L_{4,5}$ for $\mu = 0.3949$, $c_d = 67675.52$, $0 < 1 - q_i < 1$ and $0 \leq A_i \leq 0.2$

W_1	W_2	x_4	$\pm y_4$
8.94119×10^{-8}	1.75056×10^{-9}	0.101367	0.863177
8.94119×10^{-7}	1.75056×10^{-9}	0.071366	0.845857
1.78824×10^{-6}	1.75056×10^{-9}	0.038031	0.826612
2.68236×10^{-6}	1.75056×10^{-9}	0.004697	0.807368
3.57648×10^{-6}	1.75056×10^{-9}	-0.028638	0.788123
4.47060×10^{-6}	1.75056×10^{-9}	-0.061972	0.768878
2.59295×10^{-8}	2.91760×10^{-7}	0.120300	0.854979
2.59295×10^{-8}	8.75280×10^{-7}	0.153632	0.835734
2.59295×10^{-8}	1.45880×10^{-6}	0.186965	0.816489
2.59295×10^{-8}	2.04232×10^{-6}	0.220298	0.797244
2.59295×10^{-8}	2.62584×10^{-6}	0.253630	0.777999
2.59295×10^{-8}	2.9176×10^{-6}	0.270297	0.768376

Table 5b. Effect of radiation pressure on $L_{4,5}$ for $\mu = 0.3949$, $c_d = 67675.52$, $0 < 1 - q_i < 1$ and $0 \leq A_i \leq 0.2$

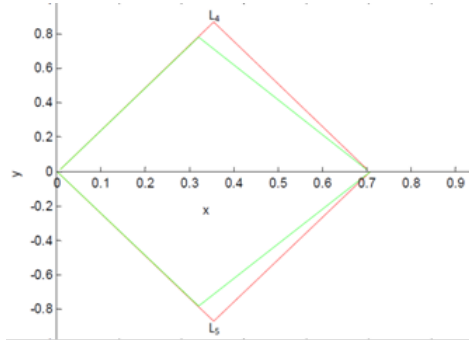


Figure 2: Showing the effect of oblateness on $L_{4,5}$ for Kruger 60.

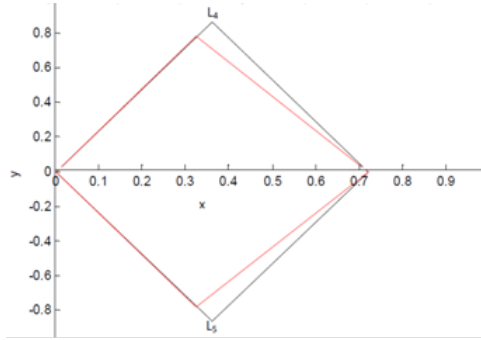


Figure 3: Showing the effect of oblateness on $L_{4,5}$ for Achird.

In the next section, we investigate the stability of triangular points.

5. STABILITY OF TRIANGULAR EQUILIBRIUM POINTS

We now examine the stability of an equilibrium configuration, that is, its ability to restrain the body motion in its vicinity. To do so, we displace the third body a little from an equilibrium point with a small velocity. If its motion is a rapid departure from the vicinity of the point, we call such a position an unstable one. However, if the body merely oscillates about the point, it is said to be a stable position. Let the location of an equilibrium point be denoted by (a_0, b_0) and consider a small displacement (ξ, η) from the point such that $x = a_0 + \xi$ and $y = b_0 + \eta$, substituting these values in (3), we obtain the variational equations

$$\ddot{\xi} - 2n\dot{\eta} = (U_{x\dot{x}})^0 \dot{\xi} + (U_{x\dot{y}})^0 \dot{\eta} + (U_{xx})^0 \xi + (U_{xy})^0 \eta$$

(18)

$$\ddot{\eta} + 2n\dot{\xi} = (U_{y\dot{x}})^0 \dot{\xi} + (U_{y\dot{y}})^0 \dot{\eta} + (U_{yx})^0 \xi + (U_{yy})^0 \eta$$

Here, only linear terms in ξ and η have been taken. The second order partial derivatives of U are denoted by subscripts. The superscript o indicates that the derivatives are to be evaluated at the equilibrium point a_0, b_0 .

The characteristic equation corresponding to equations (18) is

$$\begin{bmatrix} \lambda^2 - \lambda U_{x\dot{x}}^0 - U_{xx}^0 & -2n\lambda - \lambda U_{x\dot{y}}^0 - U_{xy}^0 \\ 2n\lambda - \lambda U_{y\dot{x}}^0 - U_{yx}^0 & \lambda^2 - \lambda U_{y\dot{y}}^0 - U_{yy}^0 \end{bmatrix} = 0$$

That is,

$$(19) \quad \lambda^4 + a\lambda^3 + b\lambda^2 + c\lambda + d = 0$$

where

$$\begin{aligned} a &= -(U_{y\dot{y}}^0 + U_{x\dot{x}}^0) \\ b &= 4n^2 + U_{x\dot{x}}^0 U_{y\dot{y}}^0 - U_{xx}^0 - U_{yy}^0 - (U_{x\dot{y}}^0)^2 \\ c &= U_{x\dot{x}}^0 U_{y\dot{y}}^0 + U_{xx}^0 U_{yy}^0 + 2nU_{xy}^0 - 2nU_{yx}^0 - U_{y\dot{x}}^0 U_{xy}^0 - U_{yx}^0 U_{x\dot{y}}^0 \\ d &= U_{xx}^0 U_{yy}^0 - U_{xy}^0 U_{yx}^0. \end{aligned}$$

Evaluating the second order partial derivatives at the equilibrium points, we obtain

$$\begin{aligned} U_{xx}^0 &= -(x_0 + \mu)^2 W_1 - W_1 - (x_0 + \mu - 1)^2 W_2 - W_2 \\ U_{x\dot{y}}^0 &= -(x_0 + \mu)y_0 W_1 - (x_0 + \mu - 1)y_0 W_2 = U_{yx}^0 \text{ say} \\ U_{y\dot{y}}^0 &= -(y_0^2 + 1)W_1 - (y_0^2 + 1)W_2 \end{aligned}$$

$$\begin{aligned} U_{xx}^0 &= 3x_0^2 - 3\mu^2 + 3\mu + (2x_0^2 + 4\mu x_0 + 2\mu^2 - 2\mu^2 x_0 - 4\mu^2 x_0 - 2\mu^3)\delta_1 + (2\mu x_0^2 \\ &+ 2\mu^3 + 4\mu^2 x_0 - 4\mu x_0 - 4\mu^2 + 2\mu)\delta_2 + (\mu + 5x_0^2 + 10\mu x_0 + 5\mu^2 - 5\mu x_0^2 - 10\mu^2 x_0 \\ &- 5\mu^3)\frac{3}{2}A_1 + (1 + 4\mu + 5\mu x_0^2 + 5\mu^3 + 10\mu^2 x_0 - 10\mu x_0 - 10\mu^2)\frac{3}{2}A_2 \\ &+ (3 - 3\mu - 15\mu^2 + 15\mu^3 + 6x_0 - 15x_0^2 - 30\mu x_0 + 15\mu x_0^2 + 30\mu^2 x_0)\epsilon_1 \\ &+ (-12\mu + 30\mu^2 - 15\mu^3 - 6x_0 - 15\mu x_0^2 + 30\mu x_0 - 30\mu^2 x_0)\epsilon_2 \\ &- (2x_0 y_0 + 2\mu y_0)W_1 - (2x_0 y_0 + 2\mu y_0 - 2y_0)W_2 \\ U_{xy}^0 &= 3x_0 y_0 + (2x_0 y_0 - 2\mu y_0 - 2\mu x_0 y_0 - 2\mu^2 y_0)\delta_1 + (2\mu x_0 y_0 + 2\mu^2 y_0 - 2\mu y_0)\delta_2 \\ &+ (x_0 y_0 + \mu y_0 - \mu x_0 y_0 - \mu^2 y_0)\frac{15}{2}A_1 + (\mu x_0 y_0 + \mu^2 y_0 - \mu y_0)\frac{15}{2}A_2 \\ &+ \left(3y_0 + \frac{3x_0}{2y_0} - 15x_0 y_0 - 15\mu y_0 + 15\mu x_0 y_0 + 15\mu^2 y_0\right)\epsilon_1 \\ &+ \left(-3y_0 + \frac{3x_0}{2y_0} + 15\mu y_0 - 15\mu x_0 y_0 - 15\mu^2 y_0\right)\epsilon_2 + (1 - 2y_0^2)W_1 + (1 - 2y_0^2)W_2 \\ U_{yx}^0 &= 3x_0 y_0 + (2x_0 y_0 - 2\mu y_0 - 2\mu x_0 y_0 - 2\mu^2 y_0)\delta_1 + (2\mu x_0 y_0 + 2\mu^2 y_0 - 2\mu y_0)\delta_2 \\ &+ (x_0 y_0 + \mu y_0 - \mu x_0 y_0 - \mu^2 y_0)\frac{15}{2}A_1 + (\mu x_0 y_0 + \mu^2 y_0 - \mu y_0)\frac{15}{2}A_2 \\ &+ \left(3y_0 + \frac{3x_0}{2y_0} - 15x_0 y_0 - 15\mu y_0 + 15\mu x_0 y_0 + 15\mu^2 y_0\right)\epsilon_1 \\ &+ \left(-3y_0 + \frac{3x_0}{2y_0} + 15\mu y_0 - 15\mu x_0 y_0 - 15\mu^2 y_0\right)\epsilon_2 + \end{aligned}$$

$$\begin{aligned}
& +2(x_0 + \mu)^2 W_1 - W_1 + 2(x_0 + \mu - 1)^2 W_2 - W_2 \\
U_{yy}^0 &= 3y_0^2 + (2y_0^2 - 2\mu y_0^2)\delta_1 + (2\mu y_0^2)\delta_2 + (\mu + 5y_0^2 - 5\mu y_0^2)\frac{3}{2}A_1 \\
& + (1 - \mu + 5\mu y_0^2)\frac{3}{2}A_2 + (6 - 3\mu - 15y_0^2 + 15\mu y_0^2)\epsilon_1 + (3 + 3\mu - 15\mu y_0^2)\epsilon_2 \\
& + (2x_0 y_0 + 2\mu y_0)W_1 + (2x_0 y_0 + 2\mu y_0 - 2y_0)W_2.
\end{aligned}$$

Substituting for $U_{xi}^0, U_{xj}^0, U_{yj}^0, U_{yx}^0, U_{xx}^0, U_{xy}^0, U_{yy}^0$ and Ω_{yx}^0 in the characteristic equation (19), we get

$$a = 3W_1 + 3W_2$$

$$b = 1 - \frac{3}{2}A_1(1 - 2\mu) + \frac{3}{2}A_2(-1 + 2\mu) + \frac{W_1}{\sqrt{3}} - \frac{W_2}{\sqrt{3}}$$

$$c = -\frac{W_1}{4}(12 + 9\mu) - \frac{W_2}{4}(21 - 9\mu)$$

$$\begin{aligned}
d &= \frac{27}{4}\mu(1 - \mu) + \frac{3}{2}\mu(1 - \mu)\delta_1 + \frac{3}{2}\mu(1 - \mu)\delta_2 + \frac{117}{4}\mu(1 - \mu)A_1 \\
&+ \frac{117}{4}\mu(1 - \mu)A_2 + \frac{27W_1}{4\sqrt{3}(1-\mu)}(-3\mu^2 + 5\mu - 2) + \frac{27W_2}{4\sqrt{3}(1-\mu)}(-3\mu^2 + 4\mu - 1)
\end{aligned}$$

and the roots of (19) are:

$$\lambda_1 \rightarrow -\frac{a}{4} - \frac{1}{2}\sqrt{L} - \frac{1}{2} \left[\frac{a^2}{4} - \frac{4b}{3} - \frac{G}{3} \left(\frac{2}{H} \right)^{\frac{1}{2}} - \left(\frac{H}{32} \right)^{\frac{1}{2}} - \frac{a^3 + 4ab - 8c}{4\sqrt{L}} \right]^{\frac{1}{2}}$$

$$\lambda_2 \rightarrow -\frac{a}{4} - \frac{1}{2}\sqrt{L} + \frac{1}{2} \left[\frac{a^2}{4} - \frac{4b}{3} - \frac{G}{3} \left(\frac{2}{H} \right)^{\frac{1}{2}} - \left(\frac{H}{32} \right)^{\frac{1}{2}} - \frac{a^3 + 4ab - 8c}{4\sqrt{L}} \right]^{\frac{1}{2}}$$

$$(20) \quad \lambda_3 \rightarrow -\frac{a}{4} + \frac{1}{2}\sqrt{L} + \frac{1}{2} \left[\frac{a^2}{4} - \frac{4b}{3} - \frac{G}{3} \left(\frac{2}{H} \right)^{\frac{1}{2}} - \left(\frac{H}{32} \right)^{\frac{1}{2}} - \frac{a^3 + 4ab - 8c}{4\sqrt{L}} \right]^{\frac{1}{2}}$$

$$\lambda_4 \rightarrow -\frac{a}{4} + \frac{1}{2}\sqrt{L} - \frac{1}{2} \left[\frac{a^2}{4} - \frac{4b}{3} - \frac{G}{3} \left(\frac{2}{H} \right)^{\frac{1}{2}} - \left(\frac{H}{32} \right)^{\frac{1}{2}} - \frac{a^3 + 4ab - 8c}{4\sqrt{L}} \right]^{\frac{1}{2}}$$

where,

$$L = \frac{a^2}{4} - \frac{2b}{3} + \frac{G}{3} \left(\frac{2}{H} \right)^{\frac{1}{2}} + \left(\frac{H}{32} \right)^{\frac{1}{2}}$$

$$F = 2b^3 - 9abc + 27(c^2 + a^2d) - 72bd$$

$$G = b^2 - 3ac + 12d$$

$$H = F + \sqrt{-4G^3 + F^2}$$

The roots $\lambda_i (i = 1, 2, 3, 4)$ of (20) are computed numerically for the motion of a test particle in the neighborhood of a binary star in table 6(a,b) and 7(a,b) below. In

particular, the binaries system, "Kruger 60 and Achird" are suitable model for our problem.

q_1	q_2	A_1	A_2
1	1	0	0
0.99992	1	0	0
1	0.99996	0	0
0.99992	0.99996	0	0
1	1	0.02	0
1	1	0	0.01
1	1	0.02	0.01
1	1	0.02	0.02
0.99992	0.99996	0.02	0.01
0.99992	0.99996	0.01	0.02
0.99992	1	0	0
1	0.99996	0	0
0.99992	0.99996	0	0
0.99992	0.99996	0.02	0
0.99992	0.99996	0	0.01
0.99992	0.99996	0.02	0.01
0.99992	0.99996	0.02	0.02

Table 6a. Roots of the characteristic equation (19) for Kruger 60 with varying parameters of radiation, oblateness and P-R drag for $\mu = 0.3937$, $c_d = 46393.84$, $0 \leq 1 - q_i \leq 1$ and $0 \leq A_i \leq 0.2$

W_1	W_2	$\lambda_{1,2}$	$\lambda_{3,4}$
0	0	$-0.620218 + 0.940569i$	$0.620218 + 0.940569i$
0	0	$-0.620223 + 0.940572i$	$0.620223 + 0.940572i$
0	0	$-0.62022 + 0.9405710i$	$0.62022 + 0.9405710i$
0	0	$-0.620225 + 0.940574i$	$0.620225 + 0.940574i$
0	0	$-0.636994 + 0.953141i$	$0.636994 + 0.953141i$
0	0	$-0.612934 + 0.939058i$	$0.612934 + 0.939058i$
0	0	$-0.630065 + 0.951756i$	$0.630065 + 0.951756i$
0	0	$-0.623019 + 0.950343i$	$0.623019 + 0.950343i$
0	0	$-0.630071 + 0.951761i$	$0.630071 + 0.951761i$
0	0	$-0.61442 + 0.9440100i$	$0.61442 + 0.9440100i$
1.04548×10^{-9}	0	$-0.620223 + 0.940572i$	$0.620223 + 0.940572i$
0	3.39442×10^{-9}	$-0.62022 + 0.9405710i$	$0.62022 + 0.9405710i$
1.04548×10^{-9}	3.39442×10^{-10}	$-0.620225 + 0.940574i$	$0.620225 + 0.940574i$
1.04548×10^{-9}	3.39442×10^{-10}	$-0.637 + 0.953145000i$	$0.637 + 0.953145000i$
1.04548×10^{-9}	3.39442×10^{-10}	$-0.612941 + 0.939062i$	$0.612941 + 0.939062i$
1.04548×10^{-9}	3.39442×10^{-10}	$-0.630071 + 0.951761i$	$0.630071 + 0.951761i$
1.04548×10^{-9}	3.39442×10^{-10}	$-0.623025 + 0.950348i$	$0.623025 + 0.950348i$

Table 6b. Roots of the characteristic equation (19) for Kruger 60 with varying parameters of radiation, oblateness and P-R drag for $\mu = 0.3937$, $c_d = 46393.84$, $0 \leq 1 - q_i \leq 1$ and $0 \leq A_i \leq 0.2$

q_1	q_2	A_1	A_2
1	1	0	0
0.9971	1	0	0
1	0.9997	0	0
0.9971	0.9997	0	0
1	1	0.02	0
1	1	0	0.01
1	1	0.02	0.01
1	1	0.02	0.02
0.9971	0.9997	0.02	0.01
0.9971	0.9997	0.01	0.02
0.9971	1	0	0
1	0.9997	0	0
0.9971	0.9997	0	0
0.9971	0.9997	0.02	0
0.9971	0.9997	0	0.01
0.9971	0.9997	0.02	0.01
0.9971	0.9997	0.02	0.02

Table 7a. Roots of the characteristic equation (19) for Achird with varying parameters of radiation, oblateness and P-R drag for $\mu = 0.3949$, $c_d = 6767.52$, $0 \leq q_i \leq 1$ and $0 \leq A_i \leq 0.2$

W_1	W_2	$\lambda_{1,2}$	$\lambda_{3,4}$
0	0	$-0.62049 + 0.9407490i$	$0.62049 + 0.9407490i$
0	0	$-0.620655 + 0.940857i$	$0.620655 + 0.940857i$
0	0	$-0.620507 + 0.940760i$	$0.620507 + 0.940760i$
0	0	$-0.620672 + 0.940869i$	$-0.620672 + 0.940869i$
0	0	$-0.637257 + 0.953345i$	$0.637257 + 0.953345i$
0	0	$-0.613173 + 0.939199i$	$0.613173 + 0.939199i$
0	0	$-0.630297 + 0.951924$	$0.630297 + 0.951924$
0	0	$-0.623219 + 0.950475i$	$0.623219 + 0.950475i$
0	0	$-0.630472 + 0.952040i$	$0.630472 + 0.952040i$
0	0	$-0.614799 + 0.944242i$	$0.614799 + 0.944242$
2.5929×10^{-8}	0	$-0.620655 + 0.940857i$	$0.620655 + 0.940857i$
0	1.75056×10^{-9}	$-0.620507 + 0.940760i$	$0.620507 + 0.940760i$
2.5929×10^{-8}	1.75056×10^{-9}	$-0.620672 + 0.940869i$	$0.620672 + 0.940869i$
2.5929×10^{-8}	1.75056×10^{-9}	$-0.637428 + 0.953459i$	$0.637428 + 0.953459i$
2.5929×10^{-8}	1.75056×10^{-9}	$-0.613358 + 0.939321i$	$0.613358 + 0.939321i$
2.5929×10^{-8}	1.75056×10^{-9}	$-0.630472 + 0.952040i$	$0.630472 + 0.952040i$
2.5929×10^{-8}	1.75056×10^{-9}	$-0.623397 + 0.950591i$	$0.623397 + 0.950591i$

Table 7b. Roots of the characteristic equation (19) for Achird with varying parameters of radiation, oblateness and P-R drag for $\mu = 0.3949$, $c_d = 6767.52$, $0 \leq q_i \leq 1$ and $0 \leq A_i \leq 0.2$

6. CONCLUSION

The circular R3BP under the effects of oblateness and radiation pressure of the primaries together with P-R drag is studied. In our model, the position of triangular points is seen to be influenced by the oblateness, radiation factors, mass ratio and the P-R drag. These points are different from those of the classical R3BP of Szebehely [32], and those obtained by AbdulRaheem and Singh [25], Singh and Umar [33] and Abouelmagd [20]. The triangular equilibrium points $L_{4,5}$ under the joint action of oblateness of the primaries and the radiational forces is seen to be unstable in the presence of the P-R drag, while in their absence, they are conditionally stable.

Our numerical investigation in the computations of these roots as shown above reveals the existence of at least one complex root with positive real part. Hence, we conclude that the triangular equilibrium points are unstable in the Lyapunov sense due to the presence of at least one complex root (19) having positive real part.

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