

## **ELECTRIC AND MAGNETIC PARTS OF THE WEYL TENSOR AND SPIN COEFFICIENTS**

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**ABSTRACT.** It is known that electric and magnetic parts of the Weyl tensor in General Relativity are analogous to that occur in the theory of electro-magnetism. On the other hand, Newman-Penrose formalism provides a new dimension to the solutions and interpretations for the theory of relativity. Its significance in understanding spacetimes with some particular Petrov types, solutions with different material contents is well known. This paper provides spin coefficient form of electric and magnetic parts of the Weyl tensor, which leads to a new and efficient way for the computation of electric and magnetic parts of the Weyl tensor. It is observed that the computational time is greatly reduced when done using computer algebra system in comparison to that is done manually. This technique is elaborated in the example of Pure Radiation metric.

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## 1. INTRODUCTION

Electric and magnetic parts of the Weyl tensor in General Relativity are useful in understanding nature of gravitational field which is analogous to electric field and magnetic field corresponding to Maxwell's field tensor. So far these tensors have not attracted relativists because in some situations expressions involving in these tensors or intermediate tensors are very lengthy and complicated. Thus, it becomes quite difficult to make meaningful conclusion. We have obtained expressions of these tensors in terms of spin coefficients in Newman-Penrose formalism (or NP formalism). Since NP formalism is a well developed geometric tool, it helps in efficient computation on one hand and also it will give good analytical insight of results on the other hand.

However, so far its use is limited to some Petrov classes, in general situation no much work is done. McIntosh *et al* [19] have used NP formalism to show that Petrov type  $D$  Weyl tensors are purely electric (magnetic) if and only if, in the canonical frame,  $\Psi_2$  is real (imaginary); also, they have proved that there are no vacuum spacetime metrics with purely magnetic petrov type  $D$  Weyl tensor. Ahsan [1] has shown that Petrov type  $III$  and  $N$  spacetimes are neither purely electric nor purely magnetic. Further, it is shown by Barnes and Rowlingson [3] that, there is an alignment between the fluid 4-velocity and a canonical null tetrad determined by the Weyl tensor, for the case of all perfect fluid spacetimes with a purely electric Weyl tensor; this alignment condition was generalized by Lozanovski and Carminati [17] and it was applied to show that purely magnetic Petrov type  $D$ , shear-free perfect fluids are locally rotationally symmetric (LRS). Cyganowski and Carminati [5] have investigated shear-free, perfect fluid solutions of Einstein's field equations in which they have shown that if the electric part of the Weyl tensor vanishes and the spacetime is not conformally flat then the fluid volume expansion is zero but the vorticity is necessarily non-zero. In 2003, Lozanovski and Carminati [18] have shown that all purely magnetic, LRS spacetimes belong to either LRS class  $I$  or  $III$ . Ferrando and Sáez [9] have proved results on the non-existence of purely magnetic solutions.

Computer algebra systems have been used for computations in general relativity by many researchers [6–8, 10–16, 20] using classical tensor approach. It is worth noting that the computations in general relativity involve up to second order derivatives of metric tensor and their non-linear combinations. Campbell and Wainwright [4]

gave a method for computing Weyl scalar, Ricci scalar etc. in NP formalism without computing corresponding tensors. In this paper, we have presented spin coefficient form of electric and magnetic parts of the Weyl tensor. In order to compute electric and magnetic parts of Weyl tensor using this approach, one needs to compute Christoffel symbols (to use in computations of covariant derivatives), spin coefficients and their derivatives. The simplicity in computation is revealed when the classical approach and spin coefficient approach are compared by a computer algebra system. This comparison is shown in Table 1 in section 4. We have come across some CAS packages developed for computations in GR in which classical tensor approach is used and it is observed that if spin coefficient approach is used then time consumed for computations will be less.

## 2. NEWMAN-PENROSE FORMALISM

Newman-Penrose formalism consists of four linearly independent null vectors (also called null tetrad)  $l^i, n^i, m^i$  and  $\bar{m}^i$ . In which null vectors  $l^i$  and  $n^i$  are real and  $m^i$  and  $\bar{m}^i$  are complex conjugates of each other, satisfying orthogonality conditions

$$(1) \quad l^i m_i = l^i \bar{m}_i = n^i m_i = n^i \bar{m}_i = 0,$$

along with

$$(2) \quad l^i l_i = n^i n_i = m^i m_i = \bar{m}^i \bar{m}_i = 0.$$

Also, they have the following orthonormal properties

$$(3) \quad l^i n_i = 1, m^i \bar{m}_i = -1.$$

Relations between metric tensor and null tetrad are

$$(4) \quad g^{ij} = l^i n^j + n^i l^j - m^i \bar{m}^j - \bar{m}^i m^j$$

and

$$(5) \quad g_{ij} = l_i n_j + n_i l_j - m_i \bar{m}_j - \bar{m}_i m_j.$$

The spin coefficients in NP formalism are given by

$$\begin{aligned}
 \kappa &= m^i D l_i, & \tau &= m^i \Delta l_i, \\
 \sigma &= m^i \delta l_i, & \rho &= m^i \bar{\delta} l_i, \\
 \pi &= -\bar{m}^i D n_i, & \nu &= -\bar{m}^i \Delta n_i, \\
 \mu &= -\bar{m}^i \delta n_i, & \lambda &= -\bar{m}^i \bar{\delta} n_i, \\
 \epsilon &= \frac{1}{2}(n^i D l_i - \bar{m}^i D m_i), \\
 \gamma &= \frac{1}{2}(n^i \bar{\delta} l_i - \bar{m}^i \bar{\delta} m_i), \\
 \beta &= \frac{1}{2}(n^i \delta l_i - \bar{m}^i \delta m_i), \\
 (6) \quad \alpha &= \frac{1}{2}(n^i \Delta l_i - \bar{m}^i \Delta m_i),
 \end{aligned}$$

with

$$(7) \quad D\phi = l^i \phi_{;i}, \quad \Delta = n^i \phi_{;i}, \quad \delta = m^i \phi_{;i}, \quad \bar{\delta} = \bar{m}^i \phi_{;i},$$

where  $\phi$  is a scalar, vector or tensor quantity and semicolon denotes covariant derivative.

In this formalism, ten independent components of the Weyl tensor are represented by five complex scalars

$$\begin{aligned}
 \Psi_0 &= -C_{ijkl} l^i m^j l^k m^l, \\
 \Psi_1 &= -C_{ijkl} l^i n^j l^k m^l, \\
 \Psi_2 &= -C_{ijkl} l^i m^j \bar{m}^k n^l, \\
 \Psi_3 &= -C_{ijkl} l^i n^j \bar{m}^k n^l, \\
 (8) \quad \Psi_4 &= -C_{ijkl} n^i \bar{m}^j n^k \bar{m}^l
 \end{aligned}$$

and ten components of Ricci tensor are represented by four real and three complex scalars as follows

$$\begin{aligned}
 \Phi_{00} &= -\frac{1}{2}R_{ij}l^i l^j, & \Phi_{22} &= -\frac{1}{2}R_{ij}n^i n^j, \\
 \Phi_{01} &= -\frac{1}{2}R_{ij}l^i m^j, & \Phi_{10} &= -\frac{1}{2}R_{ij}l^i \bar{m}^j, \\
 \Phi_{02} &= -\frac{1}{2}R_{ij}m^i m^j, & \Phi_{20} &= -\frac{1}{2}R_{ij}\bar{m}^i \bar{m}^j, \\
 \Phi_{12} &= -\frac{1}{2}R_{ij}m^i n^j, & \Phi_{21} &= -\frac{1}{2}R_{ij}\bar{m}^i n^j, \\
 \Phi_{11} &= -\frac{1}{4}(R_{ij}l^i n^j + R_{ij}m^i \bar{m}^j), \\
 (9) \quad \Lambda &= \frac{1}{24}R = \frac{1}{12}(R_{ij}l^i n^j - R_{ij}m^i \bar{m}^j).
 \end{aligned}$$

The NP field equations [21] are given by

$$\begin{aligned}
 (10) \quad D\rho - \bar{\delta}\kappa &= (\rho^2 + \sigma\bar{\sigma}) + (\epsilon + \bar{\epsilon})\rho - \bar{\kappa}\tau - \kappa(3\alpha + \bar{\beta} - \pi) + \Phi_{00}, \\
 (11) \quad D\alpha - \bar{\delta}\epsilon &= (\rho + \bar{\epsilon} - 2\epsilon)\alpha + \beta\bar{\sigma} - \bar{\beta}\epsilon - \kappa\lambda - \bar{\kappa}\gamma + (\epsilon + \rho)\pi + \Phi_{10}, \\
 (12) \quad D\lambda - \bar{\delta}\pi &= (\rho - 3\epsilon + \bar{\epsilon})\lambda + \bar{\sigma}\mu + (\pi + \alpha - \bar{\beta})\pi - \nu\bar{\kappa} + \Phi_{20}, \\
 (13) \quad \Delta\mu - \delta\nu &= -(\mu + \gamma + \bar{\gamma})\mu - \lambda\bar{\lambda} + \bar{\nu}\pi + (\bar{\alpha} + 3\beta - \tau)\nu - \Phi_{22}, \\
 (14) \quad \Delta\beta - \delta\gamma &= (\bar{\alpha} + \beta - \tau)\gamma - \mu\tau + \sigma\nu + \epsilon\bar{\nu} + (\gamma - \bar{\gamma} - \mu)\beta - \alpha\bar{\lambda} - \Phi_{12}, \\
 (15) \quad \Delta\sigma - \delta\tau &= -(\mu - 3\gamma + \bar{\gamma})\sigma - \bar{\lambda}\rho - (\tau + \beta - \bar{\alpha})\tau + \kappa\bar{\nu} - \Phi_{02}, \\
 (16) \quad D\sigma - \delta\kappa &= (\rho + \bar{\rho} + 3\epsilon - \bar{\epsilon})\sigma - (\tau - \bar{\pi} + \bar{\alpha} + 3\beta)\kappa + \Psi_0, \\
 (17) \quad D\beta - \delta\epsilon &= (\alpha + \pi)\sigma + (\bar{\rho} - \bar{\epsilon})\beta - (\mu + \gamma)\kappa - (\bar{\alpha} - \bar{\pi})\epsilon + \Psi_1, \\
 (18) \quad \Delta\alpha - \bar{\delta}\gamma &= (\rho + \epsilon)\nu - (\tau + \beta)\lambda + (\bar{\gamma} - \bar{\mu})\alpha + (\bar{\beta} - \bar{\tau})\gamma - \Psi_3, \\
 (19) \quad \Delta\lambda - \bar{\delta}\nu &= -(\mu + \bar{\mu} + 3\gamma - \bar{\gamma})\lambda + (3\alpha + \bar{\beta} + \pi - \bar{\tau})\nu - \Psi_4, \\
 (20) \quad D\tau - \Delta\kappa &= (\tau + \bar{\pi})\rho + (\bar{\tau} + \pi)\sigma + (\epsilon - \bar{\epsilon})\tau - (3\gamma + \bar{\gamma})\kappa + \Psi_1 + \Phi_{01}, \\
 (21) \quad \delta\lambda - \bar{\delta}\mu &= (\rho - \bar{\rho})\nu + (\mu - \bar{\mu})\pi + (\alpha + \bar{\beta})\mu + (\bar{\alpha} - 3\beta)\lambda - \Psi_3 + \Phi_{21}, \\
 (22) \quad D\nu - \Delta\pi &= (\pi + \bar{\tau})\mu + (\bar{\pi} + \tau)\lambda + (\gamma - \bar{\gamma})\pi - (3\epsilon + \bar{\epsilon})\nu + \Psi_3 + \Phi_{21}, \\
 (23) \quad \delta\rho - \bar{\delta}\sigma &= (\bar{\alpha} + \beta)\rho - (3\alpha - \bar{\beta})\sigma + (\rho - \bar{\rho})\tau + (\mu - \bar{\mu})\kappa - \Psi_1 + \Phi_{01},
 \end{aligned}$$

$$(24) \quad \begin{aligned} D\gamma - \Delta\epsilon = & (\tau + \bar{\pi})\alpha + (\bar{\tau} + \pi)\beta - (\epsilon + \bar{\epsilon})\gamma - (\gamma + \bar{\gamma})\epsilon + \tau\pi - \nu\kappa \\ & + \Psi_2 - \Lambda + \Phi_{11}, \end{aligned}$$

$$(25) \quad \begin{aligned} \delta\alpha - \bar{\delta}\beta = & \mu\rho - \lambda\sigma + \alpha\bar{\alpha} + \beta\bar{\beta} - 2\alpha\beta + (\rho - \bar{\rho})\gamma + (\mu - \bar{\mu})\epsilon \\ & - \Psi_2 + \Lambda + \Phi_{11}, \end{aligned}$$

$$(26) \quad D\mu - \delta\pi = (\bar{\rho} - \epsilon - \bar{\epsilon})\mu + \sigma\lambda + (\bar{\pi} - \bar{\alpha} - \beta)\pi - \nu\kappa + \Psi_2 + 2\Lambda,$$

$$(27) \quad \Delta\rho - \bar{\delta}\tau = (\gamma + \bar{\gamma} - \bar{\mu})\rho - \sigma\lambda + (\bar{\beta} - \alpha - \bar{\tau})\tau + \nu\kappa - \Psi_2 - 2\Lambda.$$

NP field equations establish relations between spin coefficients, Weyl scalars and Ricci Scalars; and hence by solving the system we get these quantities. The solution may appear non-unique when looked in terms of spin coefficients. However, the final form of these quantities will agree when values of spin coefficients are substituted.

### 3. ELECTRIC AND MAGNETIC PARTS OF THE WEYL TENSOR AND SPIN COEFFICIENTS

For an observer with a unit time-like velocity vector  $u^i$ , electric and magnetic parts of the Weyl tensor are defined by [2]

$$(28) \quad \begin{aligned} E_{ik} &= C_{ijkl}u^ju^l, \\ H_{ik} &= {}^*C_{ijkl}u^ju^l, \end{aligned}$$

where  ${}^*C_{ijkl}$  represents dual of the Weyl tensor

$$(29) \quad {}^*C_{ijkl} = \frac{1}{2}\eta_{ijmn}C^{mn}_{kl},$$

in which  $\eta_{ijkl}$  is Levi-Civita permutation tensor.

$E_{ik}$  and  $H_{ik}$  have the following properties

$$(30) \quad \begin{aligned} E_{ik} &= E_{ki}, E_{ik}u^k = 0, E_{ik}g^{ik} = E^i_i = 0, \\ H_{ik} &= H_{ki}, H_{ik}u^k = 0, H_{ik}g^{ik} = H^i_i = 0. \end{aligned}$$

From electric and magnetic parts of Weyl tensor, a new complex quantity  $Q_{ik}$  is defined as follows (see [22])

$$(31) \quad 2Q_{ik} = E_{ik} - iH_{ik} = 2C^+_{ijkl}u^ju^l,$$

where

$$(32) \quad 2C^+_{ijkl} = C_{ijkl} - iC^*_{ijkl}.$$

Looking to the properties of electric and magnetic parts of the Weyl tensor,  $Q_{ik}$  satisfies

$$Q_{ik} = Q_{ki}, \quad Q_{ik}u^k = 0, \quad Q^i{}_i = 0.$$

Using  $Q_{ik}$ , two invariants  $I$  and  $J$  of the Weyl tensor are formed as

$$(33) \quad I = Q^i{}_j Q^j{}_i, \quad J = Q^i{}_j Q^j{}_k Q^k{}_i,$$

which can be expressed into electric and magnetic parts of the Weyl tensor as follows

$$(34) \quad I = (E^i{}_j E^j{}_i - H^i{}_j H^j{}_i) + 2i E^i{}_j H^j{}_i,$$

$$(35) \quad J = E^i{}_j E^j{}_k E^k{}_i - 3E^i{}_j H^j{}_k H^k{}_i - i(H^i{}_j H^j{}_k H^k{}_i - 3E^i{}_j E^j{}_k H^k{}_i).$$

Further, using bivectors,  $C_{ijkl}^+$  can be written as follows

$$(36) \quad \begin{aligned} -C_{ijkl}^+ &= \Psi_0 U_{ij} U_{kl} + \Psi_1 (U_{ij} M_{kl} + U_{kl} M_{ij}) \\ &\quad + \Psi_2 (M_{ij} M_{kl} + V_{ij} U_{kl} + V_{kl} U_{ij}) \\ &\quad + \Psi_3 (V_{ij} M_{kl} + V_{kl} M_{ij}) + \Psi_4 V_{ij} V_{kl}, \end{aligned}$$

where

$$(37) \quad U_{ij} = n_j \bar{m}_i - n_i \bar{m}_j,$$

$$(38) \quad V_{ij} = l_i m_j - l_j m_i,$$

and

$$(39) \quad M_{ij} = l_j n_i - l_i n_j + m_i \bar{m}_j - m_j \bar{m}_i.$$

Now, we have obtained the expression of  $Q_{ik}$  by projection of (36) with choice of  $u^i$  as  $\frac{l^i + n^i}{\sqrt{2}}$ . Since this expression is in terms of complex quantities, the expressions of  $E_{ik}$  and  $H_{ik}$  can be obtained by finding complex conjugate of  $Q_{ik}$ . This gives

$$(40) \quad \begin{aligned} E_{ik} &= -Re[(\Psi_0 + \bar{\Psi}_4) \bar{m}_i \bar{m}_k \\ &\quad + (\Psi_1 - \bar{\Psi}_3) \{\bar{m}_i (n_k - l_k) + \bar{m}_k (n_i - l_i)\} \\ &\quad + \Psi_2 \{(n_i - l_i)(n_k - l_k) - m_i \bar{m}_k - m_k \bar{m}_i\}] \end{aligned}$$

and

$$\begin{aligned}
H_{ik} = & \operatorname{Im}[(\Psi_0 - \bar{\Psi}_4)\bar{m}_i\bar{m}_k \\
& + (\Psi_1 + \bar{\Psi}_3)\{\bar{m}_i(n_k - l_k) + \bar{m}_k(n_i - l_i)\} \\
& + \Psi_2\{(n_i - l_i)(n_k - l_k) - m_i\bar{m}_k - m_k\bar{m}_i\}].
\end{aligned}
\tag{41}$$

Earlier it was mentioned that, the Petrov type  $D$  Weyl tensors are purely electric (magnetic) if and only if  $\Psi_2$  is real (imaginary), the expressions (40) and (41) support this result.

In order to express electric and magnetic parts of the Weyl tensor in terms of spin coefficients, we have exploited NP field equations to replace Weyl scalars by spin coefficients in (40) and (41). The final expressions are as follows

$$\begin{aligned}
E_{ik} = & -\operatorname{Re}[\{D\sigma - \Delta\bar{\lambda} - \delta(\kappa - \bar{\nu}) - (\rho + \bar{\rho} + 3\epsilon - \bar{\epsilon})\sigma - (\mu + \bar{\mu} + 3\bar{\gamma} - \gamma)\bar{\lambda} \\
& + (\bar{\alpha} + 3\beta)\kappa + (3\bar{\alpha} + \beta)\bar{\nu} + (\tau - \bar{\pi})(\kappa - \bar{\nu})\}\bar{m}_i\bar{m}_k \\
& + \{D\beta + \Delta\bar{\alpha} - \delta(\epsilon + \bar{\gamma}) - (\alpha + \pi)\sigma - (\bar{\rho} - \bar{\epsilon})\beta + (\mu + \gamma)\kappa + (\bar{\alpha} - \bar{\pi})\epsilon \\
& - (\bar{\rho} + \bar{\epsilon})\bar{\nu} + (\bar{\tau} + \bar{\beta})\bar{\lambda} - (\gamma - \mu)\bar{\alpha} - (\beta - \tau)\bar{\gamma}\}\{\bar{m}_i(n_k - l_k) + \bar{m}_k(n_i - l_i)\} \\
& + \frac{1}{3}\{D(\mu + \gamma) - \Delta\epsilon - \delta(\alpha + \pi) + \bar{\delta}\beta + (\bar{\alpha} - \beta - \tau - \bar{\pi})(\alpha + \pi) \\
& - (\alpha - \bar{\beta} + \bar{\tau} + \pi)\beta + (\rho - \bar{\rho} + \epsilon + \bar{\epsilon})(\mu + \gamma) + (\gamma + \bar{\gamma} + \mu - \bar{\mu})\epsilon \\
& + 2\nu\kappa - 2\sigma\lambda\}\{(n_i - l_i)(n_k - l_k) - m_i\bar{m}_k - m_k\bar{m}_i\}]
\end{aligned}
\tag{42}$$

and

$$\begin{aligned}
H_{ik} = & \operatorname{Im}[\{D\sigma + \Delta\bar{\lambda} - \delta(\kappa + \bar{\nu}) - (\rho + \bar{\rho} + 3\epsilon - \bar{\epsilon})\sigma + (\mu + \bar{\mu} + 3\bar{\gamma} - \gamma)\bar{\lambda} \\
& + (\bar{\alpha} + 3\beta)\kappa - (3\bar{\alpha} + \beta)\bar{\nu} + (\tau - \bar{\pi})(\kappa + \bar{\nu})\}\bar{m}_i\bar{m}_k \\
& + \{D\beta - \Delta\bar{\alpha} - \delta(\epsilon - \bar{\gamma}) - (\alpha + \pi)\sigma - (\bar{\rho} - \bar{\epsilon})\beta + (\mu + \gamma)\kappa + (\bar{\alpha} - \bar{\pi})\epsilon \\
& + (\bar{\rho} + \bar{\epsilon})\bar{\nu} - (\bar{\tau} + \bar{\beta})\bar{\lambda} + (\gamma - \mu)\bar{\alpha} + (\beta - \tau)\bar{\gamma}\}\{\bar{m}_i(n_k - l_k) + \bar{m}_k(n_i - l_i)\} \\
& + \frac{1}{3}\{D(\mu + \gamma) - \Delta\epsilon - \delta(\alpha + \pi) + \bar{\delta}\beta + (\bar{\alpha} - \beta - \tau - \bar{\pi})(\alpha + \pi) \\
& - (\alpha - \bar{\beta} + \bar{\tau} + \pi)\beta + (\rho - \bar{\rho} + \epsilon + \bar{\epsilon})(\mu + \gamma) + (\gamma + \bar{\gamma} + \mu - \bar{\mu})\epsilon \\
& + 2\nu\kappa - 2\sigma\lambda\}\{(n_i - l_i)(n_k - l_k) - m_i\bar{m}_k - m_k\bar{m}_i\}].
\end{aligned}
\tag{43}$$

From the expressions (42) and (43) of electric and magnetic parts of the Weyl tensor, it is clear that spin coefficients are sufficient for the computation of electric and magnetic parts, no need to go for Weyl tensor. Also, (40) and (41) (or (42)

and (43)) are useful in obtaining invariants (34) and (35) in terms of Weyl scalars (or spin coefficients).

#### 4. ALGEBRAIC COMPUTATIONS

We have done the computations of electric and magnetic parts of the Weyl tensor using classical tensorial approach as well as spin coefficient approach. We have used computer algebra system Mathematica for these computational comparison<sup>1</sup>; for this we have used an earlier program based on tensorial technique [16] and also we have developed a program for the spin coefficient technique presented here. The comparison is shown in Table 1.

The classical techniques of computing electric and magnetic parts of the Weyl tensor involves computations of Christoffel symbols from the metric tensor, then computations of Riemann tensor, Ricci tensor, Ricci scalar and Weyl tensor. Finally, Weyl tensor and its dual are projected on unit timelike velocity vector to get electric and magnetic parts of the Weyl tensor, respectively. It should be noted that for some spacetimes, these computations becomes lengthy and complicated.

In contrast to the classical method, spin coefficient form of electric and magnetic parts of the Weyl tensor presented here is efficient as far as the time taken is reduced; and also the role of spin coefficients as more fundamental quantities is apparent.

TABLE 1. Comparison of Time Consumed

| Metric         | Time Taken in Seconds<br>(Tensor Method) | Time Taken in Seconds<br>(Spin Coefficient Method) |
|----------------|------------------------------------------|----------------------------------------------------|
| Gödel          | 0.45                                     | 0.29                                               |
| Vaidya         | 0.64                                     | 0.30                                               |
| Pure Radiation | 1.74                                     | 0.62                                               |
| Szekeres       | 7.50                                     | 6.26                                               |

Among the metrics given in table 1, the Pure Radiation metric is worked out in detail in the next section-5 and the details of other metrics are given in [16], for brevity they are not given here.

<sup>1</sup>These computations were performed on Windows 7 pc with 4 GB ram, i-3 processor having Mathematica 9.0 at the Department of Mathematics, Sardar Patel University, Vallabh Vidyanagar, Gujarat, India

## 5. PURE RADIATION METRIC - AN EXAMPLE

The line element in  $(r, x, y, u)$  coordinated is

$$(44) \quad ds^2 = 2dudr - \frac{4r}{x}dudx - \frac{1}{8\tilde{P}^2}(dx^2 + dy^2) + (-2xW^\circ - \frac{8r^2}{x^2}\tilde{P}^2)du^2,$$

where  $\tilde{P}$  is an arbitrary non-zero constant and  $W^\circ$  is an arbitrary function of  $u, x$  and  $y$ .

A suitable null tetrad for (44) is

$$(45) \quad \begin{aligned} l^i &= \delta_1^i, & n^i &= (xW^\circ - \frac{12r^2}{x^2}\tilde{P}^2)\delta_1^i - \frac{16r}{x}\tilde{P}^2\delta_2^i + \delta_4^i, \\ m^i &= -2\tilde{P}(\delta_2^i - i\delta_3^i), & \bar{m}^i &= -2\tilde{P}(\delta_2^i + i\delta_3^i). \end{aligned}$$

For above choice of tetrad (45), non-vanishing spin-coefficients are

$$(46) \quad \begin{aligned} 2\alpha &= 2\beta = \pi = \tau = \frac{2\tilde{P}}{x} \\ \gamma &= \frac{12\tilde{P}^2r}{x^2}, \quad \lambda = \mu = \frac{8\tilde{P}^2r}{x^2} \\ \nu &= 2\tilde{P} \left( ix \frac{\partial W^\circ}{\partial y} + x \frac{\partial W^\circ}{\partial x} + W^\circ + \frac{24\tilde{P}^2r^2}{x^3} \right) \end{aligned}$$

Using spin coefficients in relevant NP field equations (16)-(19), (24)-(27), we get

$$(47) \quad D\beta = 0$$

$$(48) \quad \Delta\alpha - \bar{\delta}\gamma = -(\tau + \beta)\lambda + (\bar{\gamma} - \bar{\mu})\alpha + (\bar{\beta} - \bar{\tau})\gamma$$

$$(49) \quad \Delta\lambda - \bar{\delta}\nu = -(\mu + \bar{\mu} + 3\gamma - \bar{\gamma})\lambda + (3\alpha + \bar{\beta})\nu - \Psi_4$$

$$(50) \quad D\gamma = (\tau + \bar{\pi})\alpha + (\bar{\tau} + \pi)\beta + \tau\pi$$

$$(51) \quad \delta\alpha - \bar{\delta}\beta = 0$$

$$(52) \quad D\mu - \delta\pi = \pi\bar{\pi}$$

$$(53) \quad \bar{\delta}\tau = \tau\bar{\tau}$$

which leads to only one non-vanishing component of the Weyl tensor

$$(54) \quad \Psi_4 = -4\tilde{P}^2x \left( \frac{\partial W^\circ}{\partial x^2} + 2i \frac{\partial W^\circ}{\partial x \partial y} - \frac{\partial W^\circ}{\partial y^2} \right)$$

Hence, the gravitational field is of Petrov type  $N$  with  $l^i$  as the propagation vector.

Thus, equations (42) and (43) reduce merely to

$$(55) \quad E_{ik} = \text{Re}[\{\Delta\bar{\lambda} - \delta\bar{\nu} + (\mu + \bar{\mu} + 3\bar{\gamma} - \gamma)\bar{\lambda} - (3\bar{\alpha} + \beta)\bar{\nu}\}\bar{m}_i\bar{m}_k]$$

$$(56) \quad H_{ik} = \text{Im}[\{\Delta\bar{\lambda} - \delta\bar{\nu} + (\mu + \bar{\mu} + 3\bar{\gamma} - \gamma)\bar{\lambda} - (3\bar{\alpha} + \beta)\bar{\nu}\}\bar{m}_i\bar{m}_k]$$

Non-vanishing independent components of electric part for the Pure Radiation metric are

$$(57) \quad \begin{aligned} E_{22} &= \frac{1}{4}x \left( \frac{\partial^2 W^\circ}{\partial x^2} - \frac{\partial^2 W^\circ}{\partial y^2} \right), & E_{23} &= \frac{1}{2}x \frac{\partial^2 W^\circ}{\partial x \partial y}, \\ E_{24} &= 4\tilde{P}^2 r \left( \frac{\partial^2 W^\circ}{\partial x^2} - \frac{\partial^2 W^\circ}{\partial y^2} \right), & E_{33} &= \frac{1}{4}x \left( \frac{\partial^2 W^\circ}{\partial y^2} - \frac{\partial^2 W^\circ}{\partial x^2} \right), \\ E_{34} &= 8\tilde{P}^2 r \frac{\partial^2 W^\circ}{\partial x \partial y}, & E_{44} &= \frac{64\tilde{P}^4 r^2}{x} \left( \frac{\partial^2 W^\circ}{\partial x^2} - \frac{\partial^2 W^\circ}{\partial y^2} \right) \end{aligned}$$

and non-vanishing independent components of magnetic part are

$$(58) \quad \begin{aligned} H_{22} &= -\frac{1}{2}x \frac{\partial^2 W^\circ}{\partial x \partial y}, & H_{23} &= \frac{1}{4}x \left( \frac{\partial^2 W^\circ}{\partial x^2} - \frac{\partial^2 W^\circ}{\partial y^2} \right), \\ H_{24} &= -8\tilde{P}^2 r \frac{\partial^2 W^\circ}{\partial x \partial y}, & H_{33} &= \frac{1}{2}x \frac{\partial^2 W^\circ}{\partial x \partial y}, \\ H_{34} &= 4\tilde{P}^2 r \left( \frac{\partial^2 W^\circ}{\partial x^2} - \frac{\partial^2 W^\circ}{\partial y^2} \right), & H_{44} &= -\frac{128\tilde{P}^4 r^2}{x} \frac{\partial^2 W^\circ}{\partial x \partial y}. \end{aligned}$$

From above expressions it is apparent that the Pure Radiation metric has non-zero electric part and non-zero magnetic part, thus the expressions support the result that the Petrov type  $N$  spacetimes are neither purely electric nor magnetic (see [1]).

## 6. CONCLUSION

This paper describes electric and magnetic parts of the Weyl tensor in terms of spin coefficients. According to these expressions to compute electric and magnetic parts of the Weyl tensor there is no need to compute the components of the Weyl tensor (in fact the computation of Christoffel symbols is sufficient) and hence this method is helpful in developing a time efficient algorithm for obtaining electric and magnetic parts of the Weyl tensor. It is felt that the proposed method will be used by computation using computer algebra systems as well as for doing manually. It is hoped that the expressions obtained here will become useful in further analysis of electric and magnetic parts of the Weyl tensor and relevant quantities. This method

is useful as far as computational time is concern for the invariants (34) and (35).

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