

HYPERSURFACE-HOMOGENEOUS COSMOLOGICAL MODELS WITH MATTER AND DARK ENERGY

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ABSTRACT. We have presented exact solutions of Einstein field equations for a hypersurface-homogeneous space time in the presence of a perfect fluid with constant EoS parameter. One solution corresponds to a cosmological model filled with stiff matter which may be useful for the description of the early stages of the universe. The other two solutions represent recent accelerating expanding models filled with dark energy and negative EoS parameter. The physical and kinematical properties of the models are discussed.

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1. INTRODUCTION

The cosmological problems within the framework of general relativity consist in finding a model of the physical universe which predicts the results of astronomical observations and which is determined by those physical laws describing the behavior of matter on scales upto those of clusters of galaxies. The spatially homogeneous and isotropic FRW perfect fluid models represent the simplest models of the expanding universe. These models, in some sense, are good global approximation of the present day universe. But it is unreasonable to assume that the regular expansion predicted by these models is also suitable for describing the early stages of evolution of the universe. At very early times in the evolution of the universe, most of the radiations and matter currently observed are believed to have been created during the inflation. Spatially homogeneous and anisotropic cosmological models

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of Bianchi type I-IX play a significant role in the description of large scale behavior of the universe and such models have been studied widely in general relativity in search of a realistic picture of the universe in its early stages.

The recent astronomical observations of the luminosity of type Ia supernovae (Bahcall et al.[1], Perlmutter et al.[2], Riess et al.[3], cosmic microwave background radiation (Spergel et al.[4]), the galaxy power spectrum (Tegmark et al. [5]) etc. have indicated that our universe is undergoing in a late-time accelerating expansion. This appears to be in strong disagreement with the standard picture of the matter dominated universe. Many cosmologists have suggested a number of ideas to explain the current accelerating universe. The source driving this acceleration is known as dark energy (DE), whose origin is still a mystery in modern cosmology. The acceleration is realized with negative pressure and positive energy density that violates the strong energy condition. This violation gives a reverse gravitational effect. Due to this effect, the universe gets a jerk and the transition from the earlier deceleration phase to current acceleration phase takes place (Caldwell et al.[6]). The cause of this sudden transition and the source of dark energy is still unknown. Dark energy has conventionally been characterized by EoS parameter mentioned by $\omega = \frac{p}{\rho}$, which is not necessarily constant, where p is the fluid pressure and ρ is the energy density (Carroll and Hoffman[7]). The value $\omega < -\frac{1}{3}$ is necessary for cosmic acceleration. The simplest dark energy candidate is the vacuum energy density ($\omega = -1$) which is mathematically equivalent to the cosmological constant (Λ), but it needs to be extremely fine-tuned to satisfy the current value of DE density. The cosmological term provides a repulsive force opposing the gravitational pull between galaxies. The other conventional alternatives to DE, which can be described by minimally coupled scalar fields are quintessence $\omega > -1$ (Steinhardt[8]), phantom energy ($\omega < -1$) (Caldwell[9,10]) and quintom (that can cross from phantom region to quintessence region as evolved) and have time-dependent EoS parameter.

Pradhan et al.[11] have studied DE models by considering the special law of variation of Hubble's parameter that yields a constant value of deceleration parameter. Some authors (Akarsu and Kilinc[12,13], Yadav et al.[14], Yadav and Yadav[15], Kumar and Yadav[16], Yadav et al.[17], Adhav et al.[18], Yadav and Saha[19], Saha and Yadav[20] have studied DE models with variable EoS parameter. Recently, Adhav et al.[21] have studied anisotropic and spatially homogeneous LRS Bianchi type I, Bianchi type-V, Bianchi type-III, Bianchi type VI_0 and Kantowski-Sachs space times with variable EoS parameter in general relativity, which correspond to early decelerating and late time accelerating anisotropic cosmological models with dynamical EoS parameter.

Motivated by above works, in this paper, we discuss Einstein's field equations for hypersurface-homogeneous space-time filled with perfect fluid. The metric and the field equations are presented with isotropic EoS parameter. We obtain exact solutions of field equations for three different cases. We also discuss the physical behaviors of the cosmological models briefly.

2. METRIC AND FIELD EQUATIONS

We consider the metric of the hypersurface-homogeneous space time of the form

$$(1) \quad ds^2 = dt^2 - A^2(t)dx^2 - B^2(t) (dy^2 + \Sigma^2(y, K)dz^2)$$

where A and B are cosmic scale functions and $\Sigma(y, K) = \sin y, y, \sinh y$ for $K = 1, 0, -1$ respectively. The metric (1) admits a group of motion G_4 in V_3 , the isotropy group being spatial rotations. Stewart and Eills[22] have presented some general solutions of Einstein's field equations for a perfect fluid distribution satisfying a barotropic equation of state. Hajj-Boutros[23] developed a method to generate new perfect fluid solutions of the field equations not satisfying the equation of barotropic state. Verma and Shri Ram[24] have obtained some hypersurface-homogeneous bulk viscous fluid cosmological models with varying cosmological term and cosmological constant. Singh and Beesham[25] have investigated some hypersurface-homogeneous cosmological models in the presence of a dynamically anisotropic EoS parameter and a dynamical energy density. Recently, Katore et al.[26] have investigated accelerating hypersurface-homogeneous cosmological models in Barber's second self-creation theory.

The Einstein's field equations (with gravitational units $8\pi G = c = 1$) read as

$$(2) \quad R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij}$$

where T_{ij} is the energy momentum tensor for a perfect fluid is given by

$$(3) \quad T_{ij} = (\rho + p)v_iv_j - pg_{ij}.$$

Here ρ is the matter energy density, p is the pressure and v^i is the 4-velocity vector satisfying $v^iv_i = 1$. We assume that the matter energy density ρ and pressure p satisfies the equation of state

$$(4) \quad p = \omega\rho$$

where ω is not necessarily constant.

In comoving coordinates, the field equations (2),(3) and (4), for the metric (1) lead to the following set of equations

$$(5) \quad \frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{K}{B^2} = -\omega\rho,$$

$$(6) \quad \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -\omega\rho,$$

$$(7) \quad \frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{K}{B^2} = \rho.$$

Here a dot denotes ordinary derivative with respect to cosmic time t.

The physical parameter that are of cosmological interest are:

$$(8) \quad \text{Spatial volume : } V = AB^2,$$

$$(9) \quad \text{Expansion scalar : } \theta = \frac{\dot{A}}{A} + \frac{2\dot{B}}{B},$$

$$(10) \quad \text{Shear scalar : } \sigma^2 = \frac{1}{3} \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right)^2.$$

The generalized mean Hubble parameter H is defined by

$$(11) \quad H = \frac{1}{3}(H_1 + H_2 + H_3)$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = H_3 = \frac{\dot{B}}{B}$ are directional Hubble parameters. An important observational quantity is the deceleration parameter q defined by

$$(12) \quad q = -\frac{V\ddot{V}}{\dot{V}^2}.$$

The sign of q indicates whether the model inflates or not. The positive sign corresponds to the standard decelerating model whereas the negative sign indicates inflation.

3. SOLUTIONS OF FIELD EQUATIONS

We now have a set of three highly non-linear equations in four unknowns A , B , ρ and ω . In order to solve them completely, we need an extra condition either on physical ground or for mathematical convenience. We assume that

$$(13) \quad B = A^n$$

where n is a constant. This is a physically plausible condition as discussed by Thorne[27], Bali and Jain[28], Pradhan and Singh[29] etc.

Combining Eqs. (8). and (13), we obtain

$$(14) \quad A = V^{\frac{1}{2n+1}}, B = V^{\frac{n}{2n+1}}.$$

From Eqs. (5) and (6), we get

$$(15) \quad \frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}^2}{B^2} - \frac{K}{B^2} = 0$$

Inserting A and B from Eq. (14) into Eq.(15), we find equation for defining V as

$$(16) \quad \ddot{V} = \frac{K(2n+1)}{1-n} V^{\frac{1}{2n+1}}$$

Eq. (16) has the general solution in quadrature form as

$$(17) \quad \int \frac{dV}{\sqrt{V^{\frac{2(n+1)}{2n+1}} + C}} = \left(\sqrt{\frac{K}{1-n^2}} \right) (2n+1)t$$

where C is an integration constant. Eq. (17) imposes some restriction on the choice of n . Thus, we see that V plays central role here in defining all physical quantities. In what follows we find V from Eq. (17) for different values of K and $C = 0$.

One can not solve Eq. (17) in general. So, in order to solve the problem completely, we have to choose either C or n in such a manner that Eq.(17) be integrable. Obviously the easiest way is to choose $C = 0$ in (17). We now consider

the following cases of physically interest and obtain corresponding solutions.

Case 1. When $K = 0$ and $C = 0$, Eq. (17) gives $\ddot{V} = 0$, which has the solution

$$(18) \quad V = c_1 t + c_2$$

where c_1 and c_2 are integration constants. From Eqs.(14) and (18), we obtain

$$(19) \quad A = (c_1 t + c_2)^{\frac{1}{2n+1}},$$

$$(20) \quad B = (c_1 t + c_2)^{\frac{n}{2n+1}}.$$

The metric of the solutions can be written in the form

$$(21) \quad ds^2 = dT^2 - T^{\frac{2}{2n+1}} dx^2 - T^{\frac{2n}{2n+1}} (dy^2 + y^2 dz^2)$$

where $T = c_1 t + c_2$. The energy density and EoS parameter are given by

$$(22) \quad \rho = \frac{n(n+2)}{(2n+1)^2} \frac{1}{T^2},$$

$$(23) \quad \omega = 1.$$

Thus, the metric (21) represents a hypersurface-homogeneous space time in the presence of stiff matter ($\rho = p$). From theoretical perspective the present model is important in relativistic cosmology for the description of the very early stages of the universe (Zel'dovich[30]).

The expansion scalar (θ), shear scalar (σ) and Hubble parameter (H) are found to

$$(24) \quad \theta = \frac{1}{T},$$

$$(25) \quad \sigma^2 = \frac{1}{3} \left(\frac{1-n^2}{2n+1} \right)^2 \frac{1}{T^2},$$

$$(26) \quad H = \frac{3}{T}.$$

The deceleration parameter have the value $q = 0$, which shows the coasting cosmology i.e, inverse inflates with marginal inflation.

The model (21) has singularity at $T = 0$. At this epoch the energy density, expansion scalar, shear scalar and Hubble parameter all are infinite. These physical parameters are decreasing functions of time and ultimately tend to zero as $T \rightarrow \infty$. The model would essentially give an empty space time for large T .

Case 2. When $K = 1$ and $C = 0$, Eq.(17) can be integrated to give

$$(27) \quad V = \left[\frac{n(t+k)}{\sqrt{1-n^2}} \right]^{\frac{2n+1}{n}}, -1 < n < 1$$

where k is an integrating constant. Then from Eqs. (14) and (27), we find the solution for the scale factors of the form

$$(28) \quad A = \left[\frac{n(t+k)}{\sqrt{1-n^2}} \right]^{\frac{1}{n}},$$

$$(29) \quad B = \frac{n(t+k)}{\sqrt{1-n^2}}.$$

Hence, the metric of our solutions can be written in the form

$$(30) \quad ds^2 = dT^2 - \left(\frac{nT}{\sqrt{1-n^2}} \right)^{\frac{2}{n}} dx^2 - \left(\frac{nT}{\sqrt{1-n^2}} \right)^2 (dy^2 + \sin^2 y dz^2)$$

where again $T = t + k$.

The energy density, EoS parameter and deceleration parameter are given by

$$(31) \quad \rho = \frac{1+2n}{n^2 T^2},$$

$$(32) \quad \omega = -\frac{1}{2n+1},$$

$$(33) \quad q = -\frac{1+n}{2n+1}.$$

For the energy density to be positive and ω, q to be negative, we must have $-\frac{1}{2} < n < 1$. We restrict ourselves to $-\frac{1}{2} < n < 1$. The expansion scalar θ , shear scalar σ and Hubble parameter are found to be

$$(34) \quad \theta = \frac{1+2n}{nT},$$

$$(35) \quad \sigma^2 = \frac{1}{3} \left(\frac{1-n}{nT} \right)^2,$$

$$(36) \quad H = \frac{1+2n}{3nT}.$$

We observe that this model has a singularity at $T = 0$. Since $\frac{\sigma}{\theta}$ tends to a constant as $T \rightarrow \infty$, the anisotropy in the model is maintained through the passage of time.

Thus, the model (30) represents a hypersurface-homogeneous cosmological model in presence of dark energy. The negative EoS parameter may be attributed to the current accelerating expansion of the universe.

Case 3. When $K = -1$, $C = 0$, Eq.(17) can be integrated to give

$$(37) \quad V = \left[\frac{n(t+l)}{\sqrt{n^2-1}} \right]^{\frac{2n+1}{n}}$$

where l is constant of integration. For physically relativistic model we take $n > 1$. Combining Eqs. (14) and (37), we obtain

$$(38) \quad A = \left[\frac{n(t+l)}{\sqrt{n^2-1}} \right]^{\frac{1}{n}},$$

$$(39) \quad B = \frac{n(t+l)}{\sqrt{n^2-1}}.$$

The metric of the solutions can be written in the form

$$(40) \quad ds^2 = dT^2 - \left(\frac{nT}{\sqrt{n^2 - 1}} \right)^{\frac{2}{n}} dx^2 - \left(\frac{nT}{\sqrt{n^2 - 1}} \right)^2 (dy^2 + \sinh^2 y dz^2).$$

For this model the energy density, EoS parameter and deceleration parameter are obtained as

$$(41) \quad \rho = \frac{1 + 2n}{n^2 T^2},$$

$$(42) \quad \omega = -\frac{1}{2n + 1},$$

$$(43) \quad q = \frac{1 - n}{2n + 1}$$

If $n > 1$, ρ is positive and ω, q are negative. The expansion scalar, shear scalar and Hubble parameter has the values given by Eqs. (34) to (36) with $n > 1$. Since the deceleration parameter is negative, the present solution represents an accelerating expanding model filled with DE. We also observe that the anisotropy is maintained for all time.

4. CONCLUSION

In summary we have investigated hypersurface-homogeneous cosmological models filled with stiff fluid and DE under some choice of the problem parameters. The model filled with stiff matter is important in relativistic cosmology for the description of very early stages of the universe. For other two models the EoS parameter ω is negative constant with restrictions over n , which may be attributed to the current accelerated expansion of the universe. The anisotropy in the models are maintained throughout the evolution of the universe. The models obtained in this paper are of considerable interest and may be useful for describing the early evolution and studying the present accelerating model of the universe.

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