

## A METRIC FOR 5D INTERIOR BLACK-HOLE SOLUTION

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**Dedicated to Professor S J Bhatt on his superannuation whose  
continuous encouragement motivated us for research.**

(Received: 29 June 2012, Accepted: 25 October 2012 )

ABSTRACT. Rosa et al [5], Khambholja [4] and also Hasmani and Khambholja [2] have discussed nature of metric for the interior black hole solution in 4D using the frame work of general relativity. It is non-static. In this paper we have extended the solution [2], [4] to 5D case in the framework of Kaluza Klein theory.

**AMS Classification:** 83C57, 83E15

**Keywords:** Interior Black Hole Region; Kaluza Klein type metric; 5D region.

## 1. INTRODUCTION

In general relativity the interior black hole spacetime, a remarkable change occurs in the nature of the spacetime namely the external spatial radial and temporal coordinates interchange their characters [2],[4], [5]. Hence the interior black hole metric becomes a non-static. Rosa et al [5], Khambholja [4] and also Hasmani and Khambholja [2] have discussed nature of metric for the interior black hole solution in 4D using the frame work of general relativity. The present paper deals with the 5D region represented by a metric that is spherical symmetric extension of the interior black hole solution in 4D. So, the metric presented here is of the Kaluza-Klein type metric as there are 4 space and 1 time coordinates.

## 2. THE INTERIOR BLACK HOLE SOLUTION IN 4D

The empty annular region of spacetime for a spherical star inside its Schwarzschild radius  $2m$  and outside its physical radius  $a$  ;  $a < 2m$ , that is for a black hole, the spacetime geometry is characterized by the spherically symmetric non static line element

$$(1) \quad ds^2 = B(z, t)dz^2 - A(z, t)dt^2 - t^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

where the coefficient functions can be obtained by solving Einstein's field equations  $R_{ij} = 0$ , representing empty sapcetime.

This solution takes the form [4], [5]

$$(2) \quad ds^2 = \left( \frac{2\xi}{t} - 1 \right) dz^2 - \left( \frac{2\xi}{t} - 1 \right)^{-1} dt^2 - t^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

Thus (2) represents metric for interior black hole [2] , [4], [5].

## 3. THE INTERIOR BLACK HOLE SOLUTION IN 5D

The Riemannian metric for spherically symmetric region in 5D can be obtained by extending (1) as [6],

$$(3) \quad ds^2 = B(z, t)dz^2 - A(z, t)dt^2 - t^2(d\theta_1^2 + \sin^2 \theta_1 d\theta_2^2 + \sin^2 \theta_1 \sin^2 \theta_2 d\theta_3^2)$$

The independent non-zero components of Ricci tensor for metric (3) using Mathematica [3] are

$$(4) \quad R_{11} = \frac{1}{4BA}(\dot{B}^2 - A'B') + \frac{1}{4tA^2}\{t(\dot{B}\dot{A} - B'^2) - \frac{1}{2tBA}(3\dot{B} + t\ddot{B} - tB'')\},$$

$$(5) \quad R_{15} = \frac{-3A'}{2tA},$$

$$(6) \quad R_{22} = \frac{1}{2} \left[ -4 - \frac{4 + \frac{t\dot{B}}{B}}{A} + \frac{t\dot{A}}{A^2} \right],$$

$$(7) \quad R_{33} = -\frac{1}{2BA^2} \left[ \sin^2 \theta_1 \{tA\dot{B} + B(4A + 4A^2 - t\dot{A})\} \right],$$

$$(8) \quad R_{44} = \sin^2 \theta_2 R_{33},$$

$$(9) \quad R_{55} = \frac{1}{4tBA} \{-6B\dot{A} + t(-\dot{B}\dot{A} + A'^2)\} + \frac{1}{4B^2} \{-\dot{B}^2 + A'B' + 2B(\ddot{B} - A'')\}$$

here and in what follows an overhead dot and prime denote the derivatives with respect to  $t$  and  $z$ , respectively.

For Kaluza-Klein type 5D empty region the field equations  $R_{ij} = 0$  give the following:  $A = A(t)$  and  $\frac{d}{dt}(AB) = 0$

Consequently,  $AB = \text{constant}$ , say  $\alpha$ .

This result is analogous to the case of 4D [2], we proceed on the line of that case.

Thus, without a significant loss of generality the integrating constant can be taken  $\alpha = 1$ .

So,  $B = A^{-1}$  and  $B = B(t)$ .

As  $B$  is related to proper time  $A, B > 0$ .

Following the notations of Rosa et al [5], the solution takes the form

$$(10) \quad B = A^{-1} = \left( \frac{2\xi}{t^2} - 1 \right)$$

where  $\xi = \text{constant}$  arises from integration.

Using (14) in (3) we get

$$(11) \quad ds^2 = \left( \frac{2\xi}{t^2} - 1 \right) dz^2 - \left( \frac{2\xi}{t^2} - 1 \right)^{-1} dt^2 - t^2 (d\theta_1^2 + \sin^2 \theta_1 d\theta_2^2 + \sin^2 \theta_1 \sin^2 \theta_2 d\theta_3^2)$$

Comparing this with (2) we interpret this as the metric for 5D region inside the *Schwarzschild radius* and outside the physical radius of a black hole in 5D.

#### 4. CONCLUSION

Here we have obtained a solution for the 5D interior black hole. The metric components  $g_{11}$  and  $g_{55}$  obtained in (10) are with  $t^2$  in the denominator. In the case of interior black hole solution obtained by us [2] in four dimensions, metric components  $g_{11}$  and  $g_{44}$  were with  $1/t$ . Further we have extended the computations for six and higher dimensions and it is observed that power of  $t$  in denominator further increases with each increment of dimension. The physical relevance of the solution is yet to be studied.

#### 5. ACKNOWLEDGEMENTS

The author thanks to UGC for UGC-SAP-DRS(F-510/5/DRS/2004 (SAP-II)) awarded to the department under which the work was carried out.

#### REFERENCES

- [1] Adler R., Bazin M., Schiffer , *Introduction to General Relativity*, Mc-Graw Hill Ltd-1975
- [2] AH Hasmani and VG Khambholja, *Interior Black-hole Solution with Anisotropic Fluid*, Journal of Dynamical Systems and Geometric Theories, VOL. 10, Number 1 (2012) 47-52
- [3] VG Khambholja and AH Hasmani, *Algebraic Computation of Riemann Curvature Tensor for a 5D Space Using Mathematica*, Published Online
- [4] VG Khambholja, *M.Phil. dissertation* (Unpublished), Sardar Patel University
- [5] Rosa Doran, Francisco S. N. Lobo and Paulo Crawford, *Interior of a Schwarzschild Black Hole Revisited*, Found. Phys. 38 (2008), 160-187
- [6] Wang Yong-Jiu and Huang Xiu-Ju, *Higher Dimensional Cosmological Models with Density-Parameter-Dependent Cosmological Constant*, Chin.Phys.Lett., Vol.21, No.8(2004)1670