

GENERALIZED SYNCHRONIZATION OF NUCLEAR SPIN GENERATOR SYSTEM AND THE APPLICATION IN SECURE COMMUNICATION

MOHAMMAD ALI KHAN¹ AND SWARUP PORIA²

¹ DEPARTMENT OF MATHEMATICS, GARHBETA RAMSUNDAR VIDYABHABAN,
PASCHIM MEDINIPUR, WEST BENGAL, INDIA

² DEPARTMENT OF APPLIED MATHEMATICS, UNIVERSITY OF CALCUTTA,
KOLKATA, WEST BENGAL, INDIA
E-MAILS: MDMATHS@GMAIL.COM
SWARUP.P@YAHOO.COM

(Received: 25 March 2012, Accepted: 23 May 2012)

ABSTRACT. This paper investigates the synchronization of Nuclear spin generator system. Sufficient theorem for the synchronization of Nuclear spin generator system are deduced. A scheme of secure communication based on the synchronization of Nuclear spin generator system is presented. Numerical simulation shows its feasibility.

AMS Classification: 34D06.

Keywords: Generalized synchronization(GS); Nuclear spin generator system; Secure communication.

1. INTRODUCTION

Experimental observations have pointed out that chaotic systems are common in nature. It is found that in Chemistry (Belouov-Zhabotinski reaction), in Nonlinear Optics (lasers), in Electronics (Chua-Matsumoto circuits), in Fluid Dynamics

(Rayleigh-Benard convection), etc chaotic systems exist. Chaos is found in meteorology, solar system, heart and brain of living organisms and so on. Synchronization and control of interacting chaotic oscillators is one of the fundamental phenomena of nonlinear dynamics and chaos. Experimental realization of chaos Synchronization and control have been achieved with a magnetoelastic ribbon, a heart, a thermal convection loop, a diode oscillator, an optimal multimode chaotic solid-state laser, a Belousov-Zhabotinski reaction diffusion chemical system, and many other experiments.

One of the most striking discoveries in the study of chaos is that chaotic systems can be made to synchronize with each other. Synchronization of chaos is a phenomenon that may occur when two or more chaotic dynamical systems are coupled. This was discovered by Pecorra and carroll[1]. They produced the idea for chaos synchronization and verified by the electric current later. Over the last decade, chaos synchronization has received increasing attention due to its potential application in many areas such as secure communication, information processing, biological systems and chemical reactions. Since Pecora and Carroll's [9] work many effective methods namely, OGY method [7], adaptive control [1], differential geometric method [2], inverse optimal control [12], lag synchronization [8], projective synchronization [6] etc for chaos control and synchronization have been proposed. Usually two dynamical systems are called synchronized if the distance between their corresponding states converges to zero as time goes to infinity. This type of synchronization is known as identical synchronization [9]. A generalization of the concept for unidirectionally coupled dynamical systems was proposed by Rulkov et al.[10], where two systems are called synchronized if a static functional relationship exists between the states of the systems. They called this kind of synchronization a generalized synchronization (GS). Kocarev and Parlitz [5] formulated a condition for the occurrence of GS between two coupled continuous dynamical systems. Yang and Chua [13] proposed GS of continuous dynamical systems via linear transformations. Khan and Mondal [4] introduce synchronization between two generalized coupled Rossler systems via linear transformations.

2. GENERALIZED SYNCHRONIZATION BETWEEN TWO NUCLEAR SPIN GENERATOR(NSG) SYSTEMS

The nuclear spin generator problem was studied by Sachdev and Sarthy [11] and Hegazi et.al [3]. They showed that the system displays rich and typical bifurcation and chaotic phenomena for some values of the control parameters. The system consists a suitable simple of matter containing proper nuclei in a relatively strong magnetic field defining Z-direction, an exciting coil with axis in the X-direction, perpendicular to Z, a pick-up coil with axis in the Y-direction, perpendicular to both X and Z, and a high-gain amplifier feeding the voltage induced in the pick-up coil back to the exciting coil. We first reformulate its equation in the following form

$$\begin{aligned}
 \dot{x} &= -\beta x + y \\
 \dot{y} &= -x - \beta y(1 - kz) \\
 \dot{z} &= \beta(\alpha(1 - z) - ky^2)
 \end{aligned}
 \tag{1}$$

such that α , β and k are parameters, where $\beta\alpha \geq 0$ and $\beta \geq 0$ are linear damping terms, the non-linearity parameters βk is proportional to the amplifier gain in the voltage feedback. Physical consideration limit the parameters α to the range $0 < \alpha \leq 1$.

Let the system (1) be the drive system and y be the drive variable, then the response system can be written as follows:

$$\begin{aligned}
 \dot{u} &= -\beta u + v \\
 \dot{v} &= -u - \beta v + \beta kyw \\
 \dot{w} &= \beta(\alpha(1 - w) - kv)
 \end{aligned}
 \tag{2}$$

Theorem: When $\beta > 0$ and $0 < \alpha \leq 1$, then the response system (2) can synchronize the drive system (1).

Proof. Let the state tracking error be $e = (e_1, e_2, e_3)^T = (u - x, v - y, w - z)^T$, then the error dynamics is

$$\begin{aligned}
 \dot{e}_1 &= -\beta e_1 + e_2 \\
 \dot{e}_2 &= -e_1 - \beta e_2 + \beta kye_3 \\
 \dot{e}_3 &= \beta(-\alpha e_3 - ke_2)
 \end{aligned}
 \tag{3}$$

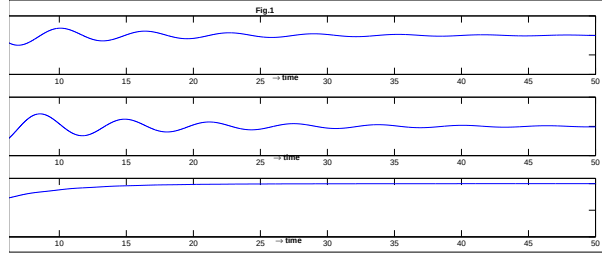


FIGURE 1. The tracking error for $\alpha = 0.75, \beta = 0.15$ and $k = 1$.

Now the synchronization of two identical Nuclear spin generator system is equivalent to the asymptotic stability of the zero solution for the error system (3). Construct a Lyapunov function of the form

$$(4) \quad E(e_1, e_2, e_3) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2).$$

Then its derivative along the solution of (3) is

$$(5) \quad \frac{dE}{dt} = -\beta e_1^2 - \beta e_2^2 - \alpha \beta e_3^2 < 0$$

when $\beta > 0$ and $0 < \alpha \leq 1$. Therefore $e \rightarrow 0$ as $t \rightarrow \infty$. Thus the drive system (1) and response system (2) synchronized globally asymptotically.

3. RESULTS AND DISCUSSIONS

Numerical simulations have been performed for demonstration of the synchronization method. Fourth-order Runge-Kutta method is used for solving coupled Nuclear spin generator system with step size .001. The parameter of Nuclear spin generator system is chosen as $\alpha = 0.75, \beta = 0.15$ and $k = 1$. The initial conditions for the driving and driven system are taken, respectively as $(x_1(0), x_2(0), x_3(0)) = (-1, -1, -1)$ and $(y_1(0), y_2(0), y_3(0)) = (1, 1, 1)$. Fig.1 shows the tracking error.

4. THE APPLICATION IN SECURE COMMUNICATION

Assume that $m(t)$ is the message signal, adding it to the right of the first equation for the transmitter (drive system), then we have

$$\begin{aligned} \dot{x} &= -\beta x + y + m(t) \\ \dot{y} &= -x - \beta y(1 - kz) \\ \dot{z} &= \beta(\alpha(1 - z) - ky^2) \end{aligned} \quad (6)$$

Select the output $y(t)$ of the system (5) as the transmitted signal, then construct the receiver as follows:

$$\begin{aligned} \dot{u} &= -\beta u + v + p(t) \\ \dot{v} &= -u - \beta v + \beta kyw \\ \dot{w} &= \beta(\alpha(1 - w) - kyv) \\ \dot{p} &= k(x - u) \end{aligned} \quad (7)$$

where k is a parameter.

Let the tracking error be $e = (e_1, e_2, e_3, e_4) = (x - u, y - v, z - w, m - p)$, then the error dynamics is

$$\begin{aligned} \dot{e}_1 &= -\beta e_1 + e_2 + e_4 \\ \dot{e}_2 &= -e_1 - \beta e_2 + \beta kye_3 \\ \dot{e}_3 &= \beta(-\alpha e_3 - kye_2) \\ \dot{e}_4 &= \frac{dm}{dt} - ke_1 \end{aligned} \quad (8)$$

where $k > 0$. Since the eigen frequency of the message signal $m(t)$ is much less than the oscillating frequency of the chaotic system in practice, then $\frac{dm}{dt} - ke_1(t) = -ke_1(t)$. Construct a Lyapunov function as follows:

$$E(e_1, e_2, e_3, e_4) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2). \quad (9)$$

If $k = 1$, then its derivative along the solution of (8) is

$$\frac{dE}{dt} = -\beta e_1^2 - \beta e_2^2 - \alpha \beta e_3^2 < 0 \quad (10)$$

Therefore $E \rightarrow 0$ as $t \rightarrow \infty$. That is $e_4 = m(t) - p(t) \rightarrow 0$ as $t \rightarrow \infty$. Thus the variable $p(t)$ can recover the message signal $m(t)$.

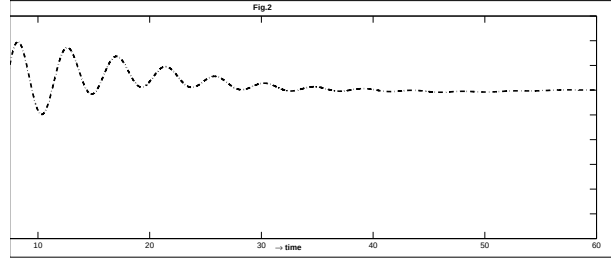


FIGURE 2. The tracking error $m(t) - p(t)$ for $\alpha = 0.75, \beta = 0.15$ and $k = 1$.

Taking $m(t) = 0.1\cos(0.1\pi t)$, $k = 1$, Fig.2 shows the error $m(t) - p(t)$. It is noticed that the chaos synchronization sensitively depends on the parameter values in above scheme for secure communication.

5. CONCLUSIONS

The chaos synchronization for two identical NSG systems is investigated by taking y as a drive variable and sufficient condition for chaos synchronization is gained. Furthermore, a scheme for secure communication is presented in theory, and its feasibility is verified by numerical simulations.

REFERENCES

- [1] S. Chen and J. Lu, Synchronization of an uncertain unified chaotic system via adaptive control, *Chaos, Soliton and Fractals* 14(4)(2002).
- [2] C.C. Fuh and P.C.Tung, Controlling chaos using differtial geometric method, *Phys.Rev.Lett.*75, 16(1995).
- [3] Hegazi, Agiza and Eledessoky, Adaptive control and synchronization of a modified Chua's circuit system, *Chaos,solitons and Fractals* 12, 40(2001).
- [4] M.A. Khan and A.K.Mandal, Generalized chaos synchronization of coupled Rossler systems. *Bull. Cal. Math. Soc.* 101,2(2009).
- [5] L. Kocarev and U. Parlitz, Generalized synchronization, predictability, an equivalence of uni-directionally coupled dynamical systems, *Phys.Rev.Lett.* 76(1996).
- [6] Ronnie Mainieri and Jan Rehacek, Projective synchronization in three dimensional chaotic systems, *Phys.Rev.Lett.*82,(1999).
- [7] E. Ott, C. Grebogi, and J.A. York, Controlling chaos *Physical Review Letter.* 64(1990).

- [8] D. Pazo, M.A. Zaks and J. Kurths, Role of unstable periodic orbit in phase and lag synchronization between coupled chaotic oscillators, *Chaos*, 13,1(2003).
- [9] L.M. Pecorra and T.L. Carrol, Synchronization in chaotic systems. *Phys.Rev.Lett.* 64, 8(1990).
- [10] N.F. Rulkov, M.M. Sushchik, L.S. Tsimring and H.D.I. Abarbanel, Generalized synchronization of chaos in directionally coupled chaotic systems, *Phys. Rev.E* 51(1995).
- [11] Sachdev and Sarthy, Periodic and chaotic solutions for a non-linear system arising from a nuclear spin generator, *Chaos, solitons and Fractals* 4, 11 (1994).
- [12] E.N. Sanchez, J.P. Perez, M. Martinez and G. Chen, Chaos stabilization: an inverse optimal control approach, *Latin Am Appl Res: Int J* 32(2002).
- [13] T. Yang and L.O. Chua, Generalized synchronization of chaos via linear transformations, *International Journal of Bifurcation and Chaos*, 9,1(1999).