

DYNAMIC RELIABILITY INDICES FOR MULTI-STATE SYSTEM

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ABSTRACT. The Multi-State System reliability is investigated through Dynamic Reliability Indices in this paper. These indices are obtained in this paper for the first time. They estimate influence upon the Multi-State System reliability by the capacity levels of a system component. The mathematical tools of Multiple-Valued Logic are used for calculation of Dynamic Reliability Indices.

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Notation and Acronyms

MSS: Multi-State System

MVL: Multiple-Valued Logic

DRI: Dynamic Reliability Indices

DDRI: Dynamic Deterministic Reliability Indices

DIRI: Dynamic Integrated Reliability Indices

CDRI: Component Dynamic Reliability Indices

n : number of components in the system

m : number of states of the system and its components,
state $m - 1$: perfect-functioning, state 0: complete-failure

$\phi(x)$: system-state structure function: $\phi(x) \in \{0, 1, \dots, m - 1\}$

x_i : the i -th system component, $i = \overline{1, n}$

t_i : state of component i ; $t_i \in \{0, 1, \dots, m-1\}$, $i = \overline{1, n}$

$\mathbf{X}$: truth table vector of MVL function (vector of structure function $\phi(x)$)

$x^{(h)}$: h -th element of the vector of the truth table vector $\mathbf{X}$

$\partial\phi(j \rightarrow k)/\partial x_i(a \rightarrow b)$: Direct Partial Logic Derivative of structure function $\phi(x)$ by variable x_i

$\partial\mathbf{X}(j \rightarrow k)/\partial x_i(a \rightarrow b)$: truth table vector of Direct Partial Logic Derivative of structure function $\phi(x)$ by variable x_i

$\rho(i)_{a \rightarrow b}^{j \rightarrow k}$: number of system states when a change of i -th component state from a to b forces a modification of system reliability from j to k

$P(i)_{a \rightarrow b}^{j \rightarrow k}$: probability of change of MSS reliability from j to k if the component state change from a to b

$\{G\}$: set of DDRI

$\{G|x_i\}$: subset of system states when a change state of i -th component from a to b forces a change the system reliability from j to k

$P_{a \rightarrow b}^{j \rightarrow k}$: DIRI (is probability of a modification of system reliability from j to k if a component state changes from a to b)

$Q_f(i)$ or $Q_r(i)$: CDRI (is probabilities of MSS failure or repairing at a modification of a state of i -th system component)

1. INTRODUCTION

In this time the reliability analysis consists of two parts. The one part is research of the binary-state system reliability. This system has two states only: complete failures and perfect functioning. But the reliability analysis is focused on systems that have possibility to perform the task by different levels of exactitude or quality. This system is named *Multi-State System* (MSS). In addition, every system component has different levels to work too. The reliability analysis of MSS is more complicated and is insufficiently investigated. The investigation of MSS is the second part of the reliability analysis.

There are some approaches for MSS reliability analysis [1]: (a) the structure function approach, (b) the stochastic process approach; (c) universal generating function approach, (d) Monte-Carlo simulation technique. MSS reliability is investigated by the structure function in this paper. The structure function approach is first historically that is developed and is applied for the MSS reliability evaluation [2, 3]. Two directions for analysis MSS by structure function are known.

Generalization of some results of the binary-state system to the multi-state case is the first direction. For example, Boolean models use for representation structure function of MSS in papers [4, 5, 6]. Some investigators use the *Multiple-Valued Logic* (MVL) for MSS reliability analysis according to the second direction [7, 8]. In this case a structure function is interpreted as a MVL function.

Note, the structure function approach has two disadvantages [1]: (a) it is large dimension of the structure function; (b) it is inability to investigate dynamic behavior of

MSS. Different tools of decomposition solve the first problem [9, 10]. And the Logical Differential Calculus permits to investigate dynamic proprieties of MSS [11, 12, 13].

The results to present in [11, 12, 13, 14] extend in this paper. So, R.Boedigheimer and K.Kapur in [11] proposed the customer's structure function methodology where the relates the components states to the system states by specifying when a change in the component states forces a change in the system states. Authors had examined the dynamic proprieties of MSS by the Logical Differential Calculus in [12, 13, 14]. In the paper [12] they had substantiated basic conceptions to use the Direct Partial Logic Derivatives (it is part of Logical Differential Calculus) for the reliability analysis of MSS and had determined the new class of reliability indices. This class had named *Dynamic Reliability Indices* (DRI). The dynamic character of DRI consists to determine states of the system failure or, in other case, the repair system when change state of one of a component. In other words, they define the boundary states of MSS. In paper [13] two groups of DRI were obtained. These indices allow to analyze system reliability if a capacity level of one component of MSS is modified. The group of deterministic and the group probabilistic indices were received. Authors had offered to calculate these groups of DRI by Direct Partial Logic Derivative and by well known GD-algorithm, which were developed in [15, 16]. The new group of DRI was investigated by Direct Partial Logic Derivative in [14]. Indices of this group examine the influence of change of capacity level of MSS component to the reliability system.

However algorithms for evaluation of reliability indices which were obtained in papers [12, 13, 14], have high computing complexity. In this paper this algorithms for calculation DRI is improved.

2. BASIC CONCEPTIONS

2.1. Mathematical model of MSS. The MSS of n components is examined in this paper. Let x_i ($i = \overline{1, n}$) is its component. Both the system and its components have m reliability states (level to perform a task). The numerical characteristic of the reliability state of i -th component is: $x_i = t_i$, where $t_i \in \{0, 1, \dots, m-1\}$; $x_i=0$ corresponds complete failure and $x_i = m-1$ is perfect functioning of i -th component.

The system reliability depends of its components states and is defined by the structure function [2, 3]:

$$(1) \quad \phi(x_1, x_2, \dots, x_n) = \phi(X) : \{0, 1, \dots, m-1\}^n \longrightarrow \{0, 1, \dots, m-1\},$$

where $\phi(X) = 0$ is complete failure of system and $\phi(X) = m-1$ is perfect functioning of system. The structure function (1) is interpreted as MVL function [8, 17]. The values and arguments of MVL function belong to set $\{0, 1, \dots, m-1\}$.

Example 1. The structure function $\phi(X)$ for three reliability levels ($m = 3$) of a well-known system "two of three" is shown in Table 1. This MSS consists of three components ($n = 3$) and functions at a refusal no more than one component.

Different indices are obtained over the structure function [2-8]. As a rule the set of minimal cut and the set of minimal path are calculated and are analyzed for these indices of MSS. They are important for the reliability of the system topology. But these indices do not show dependence of the MSS reliability from a modification of the component reliability. Papers [11, 12, 13] are need to remark here. Authors proposed to examine MSS when a change in the state of any one of the component forces a change in the system state and obtained the new class of indices to use for reliability analysis of MSS. These are DRI.

The DRI was defined in [12, 13] firstly. In these papers authors showed the relationships of DRI and Logic Deferential Calculation. So, DRI are calculated through Direct Partial Logic Derivatives that are of the Logic Deferential Calculation [15, 16].

TABLE 1. The structure function of MSS ($m = 3, n = 3$)

$x_1 x_2 x_3$	$\phi(X)$	$x_1 x_2 x_3$	$\phi(X)$	$x_1 x_2 x_3$	$\phi(X)$
000	0	100	0	200	0
001	0	101	1	201	1
002	0	102	1	202	2
010	0	110	1	210	1
011	1	111	2	211	2
012	1	112	2	212	2
020	0	120	1	220	2
021	1	121	2	221	2
022	2	122	2	222	2

2.2. Direct Partial Logic Derivatives for Calculation MSS Reliability. Any MVL function may be represented as the truth table vector $X = [x^{(0)} x^{(1)} \dots x^{(m^n-1)}]^T$ (X is the column of a truth table of the MVL function). The h -th element $x^{(h)}$ of the vector X is the value of MVL function when the value of the vector X corresponds to m -th representation of the parameter $h : X = (h_1 h_2 \dots h_n)$. The example of truth table vector X for MVL function ($m = 3$) of 2 variables ($n = 2$) is shown in Table 2.

TABLE 2. The example of truth table vector X of MVL function MAX

Parameter h	Variables $x_1 x_2$	The truth table vector X	The truth table vector X of function MAX
0	0 0	$x^{(0)}$	0
1	0 1	$x^{(1)}$	1
2	0 2	$x^{(2)}$	2
3	1 0	$x^{(3)}$	1
4	1 1	$x^{(4)}$	1
5	1 2	$x^{(5)}$	2
6	2 0	$x^{(6)}$	2
7	2 1	$x^{(7)}$	2
8	2 2	$x^{(8)}$	2

The dynamic properties of MVL function are revealed through Logic Derivational Calculation and Direct Partial Logic Derivative in particular. A Direct Partial Logic Derivative $\partial\phi(j \rightarrow k)/\partial x_i(a \rightarrow b)$ of a MVL function $\phi(X)$ of n variables with respect to variable x_i reflects the fact of changing of function from j to k when the value of variable x_i is changing from a to b [15]:

$$(2) \quad \partial\phi(j \rightarrow k)/\partial x_i(a \rightarrow b) = \phi(x_1, \dots, x_{i-1}, a, x_{i+1}, \dots, x_n) \bullet \phi(x_1, \dots, x_{i-1}, b, x_{i+1}, \dots, x_n),$$

where $\bullet$ is the symbol of a comparison operation:

$$\partial\phi(j \rightarrow k)/\partial x_i(a \rightarrow b) = m - 1,$$

if $\phi(x_1, \dots, x_{i-1}, a, x_{i+1}, \dots, x_n) = j$ and $\phi(x_1, \dots, x_{i-1}, b, x_{i+1}, \dots, x_n) = k$

$\partial\phi(j \rightarrow k)/\partial x_i(a \rightarrow b) = 0$, in the other case.

Example 2. Compute the Direct Partial Logic Derivative $\partial\phi(0 \rightarrow 1)/\partial x_1(1 \rightarrow 2)$ of the MVL function $\phi(\mathbf{X})$ ($m = 3, n = 2$) with the truth table vector $\mathbf{X} = [111200210]^T$.

According to (1) for this function it is necessary to analyze values of the function when $x_1 = 1$ and $x_2 = 2$: $\partial\phi(0 \rightarrow 1)/\partial x_1(1 \rightarrow 2) = \phi(1, x_2) \bullet \phi(2, x_2)$. Therefore, the elements the truth table vector of the Direct Partial Logic Derivative $\partial\phi(0 \rightarrow 1)/\partial x_1(1 \rightarrow 2)$ are equal two if $\phi(1, x_2) = 0$ and $\phi(2, x_2) = 1$ only. This condition is fulfilled for $x_1 = 1$ and $x_2 = 1$ ($\phi(1, 1) = 0$ and $\phi(2, 1) = 1$). The truth table vector of Direct Partial Logic Derivative of the MVL function $\phi(\mathbf{X})$ is $\partial\phi(0 \rightarrow 1)/\partial x_1(1 \rightarrow 2) = [000020000]^T$.

The matrix algorithm of Direct Partial Logic Derivatives an evaluation is shown in [15].

3. THE DYNAMIC RELIABILITY INDICES

Three groups of DRI are introduced. The first group is the Dynamic Deterministic Reliability Indices (DDRI) [12, 13]. These indices are sets of the boundary system states. There are two types of these sets. One type is sets of system failure states. So, the decrease of system component capacity level determines in MSS failure. Another type is sets of system repair states. The system repair states are states that the increase of system component capacity level causes to repairing MSS. The second group of DRI is Dynamic Integrated Reliability Indices (DIRI) [13]. DIRI determine probability of a system failure (or system repairing) with a modification of capacity level of a system component. These are probabilistic analogs of DDRI. The third group of indices is named Component Dynamic Reliability Indices (CDRI) [14]. CDRI characterizes probability of a system failure (repairing) at a modification of capacity levels of each of components MSS. The DRI calculation methodology is considered below. Let's remind definitions of three groups of DRI according to [12, 13, 14].

3.1. Dynamic Deterministic Reliability Indices (DDRI).

Definition 1. DDRI are the set $\{G\}$ of MSS states $t_1 t_2 \dots t_n$ such that the modifications of the state of one of the system components from a to b reduce to change of MSS reliability from j to k .

So, the set $\{G\}$ is union of subsets $\{G|x_i\}$:

$$(3) \quad \{G\} = \{G|x_1\} \cup \{G|x_2\} \cup \dots \cup \{G|x_i\},$$

where subset elements are system states when the change state of i -th component from a to b forces a change the system state from j to k and subsets elements $\{G|x_i\}$ correspond the values $t_1 \dots t_{i-1} a t_{i+1} \dots t_n$ of variables $x_1 \dots x_i \dots x_n$ for which the values of Direct Partial Logic Derivative $\partial\phi(j \rightarrow k)/\partial x_i(a \rightarrow b)$ with respect to i -th variable are not equal 0:

$$(4) \quad \partial\phi(j \rightarrow k)/\partial x_i(a \rightarrow b) \neq 0.$$

Two types of DDRI investigate in this paper. The first type is the set $\{G_f\}$ of MSS states when the failure of one of the system component causes the failure of MSS. The set $\{G_r\}$ of the system failure states, which are eliminated by the replacement of a failure component. This indices are calculated by of these Direct Partial Logic Derivatives: $\partial\phi(1 \rightarrow 0)/\partial x_i(1 \rightarrow 0)$ and $\partial\phi(0 \rightarrow \bar{k})/\partial x_i(1 \rightarrow m - 1)$, where $\bar{k} \in \{1, 2, \dots, m - 1\}$.

Example 3. Define DDRI $\{G_f\}$ and $\{G_r\}$ for MSS which is given by the structure function from Table 1. So, the DDRI $\{G_f\}$ are calculated for every system component x_i ($i = \bar{1}, 3$) through the Direct Partial Logic Derivative $\partial\phi(1 \rightarrow 0)/\partial x_i(1 \rightarrow 0)$ according to the Equation (3) and the Equation (4). The result is shown in Table 3.

Thus, the set $\{G_f\}$ according to the Table 1 is

TABLE 3. DDRI $\{G_f\}$

$\partial\phi(1 \rightarrow 0)/\partial x_1(1 \rightarrow 0)$				$\partial\phi(1 \rightarrow 0)/\partial x_2(1 \rightarrow 0)$				$\partial\phi(1 \rightarrow 0)/\partial x_3(1 \rightarrow 0)$			
$x_2 \backslash x_3$	0	1	2	$x_1 \backslash x_3$	0	1	2	$x_1 \backslash x_2$	0	1	2
0	0	2	2	0	0	2	2	0	0	2	2
1	2	0	0	1	2	0	0	1	2	0	0
2	2	0	0	2	2	0	0	2	2	0	0
$\{G_f x_1\} = \{101, 102, 110, 120\}$				$\{G_f x_1\} = \{011, 012, 110, 210\}$				$\{G_f x_1\} = \{011, 021, 101, 201\}$			

$$(5) \quad \{G_f\} = \{G_f|x_1\} \cup \{G_f|x_2\} \cup \{G_f|x_3\} = \{011, 012, 021, 101, 102, 110, 120, 201, 210\}.$$

The DDRI $\{G_r\}$ are calculated for every system component x_i ($i = \overline{1, 3}$) too. But in this case use the Direct Partial Logic Derivative $\partial\phi(0 \rightarrow 1)/\partial x_i(0 \rightarrow 2)$ and $\partial\phi(0 \rightarrow 2)/\partial x_i(0 \rightarrow 2)$. The Table 4 illustrates the result of evaluations.

TABLE 4. DDRI $\{G_r\}$

$\partial\phi(0 \rightarrow 1)/\partial x_1(0 \rightarrow 2)$				$\partial\phi(0 \rightarrow 1)/\partial x_2(0 \rightarrow 2)$				$\partial\phi(0 \rightarrow 1)/\partial x_3(0 \rightarrow 2)$			
$x_2 \backslash x_3$	0	1	2	$x_1 \backslash x_3$	0	1	2	$x_1 \backslash x_2$	0	1	2
0	0	2	0	0	0	2	0	0	0	2	0
1	2	0	0	1	2	0	0	1	2	0	0
2	0	0	0	2	0	0	0	2	0	0	0
$\partial\phi(0 \rightarrow 2)/\partial x_1(0 \rightarrow 2)$				$\partial\phi(0 \rightarrow 2)/\partial x_2(0 \rightarrow 2)$				$\partial\phi(0 \rightarrow 2)/\partial x_3(0 \rightarrow 2)$			
$x_2 \backslash x_3$	0	1	2	$x_1 \backslash x_3$	0	1	2	$x_1 \backslash x_2$	0	1	2
0	0	0	2	0	0	0	2	0	0	0	2
1	0	0	0	1	0	0	0	1	0	0	0
2	2	0	0	2	2	0	0	2	2	0	0
$\{G_r x_1\} = \{001, 002, 010, 020\}$				$\{G_r x_2\} = \{001, 002, 100, 200\}$				$\{G_r x_3\} = \{010, 020, 100, 200\}$			

The DDRI $\{G_r\}$ is

$$(6) \quad \{G_r\} = \{G_r|x_1\} \cup \{G_r|x_2\} \cup \{G_r|x_3\} = \{001, 002, 010, 020, 100, 200\}.$$

Thus, the Equation (5) determines functioning system boundary states for the special attention, because failure of one of components causes MSS failure. So, the system state $\phi(011) = 1$ (the first element of set $\{G_f\}$) shows, that the system is failure, if these components x_2 or x_3 are failure.

Elements of the set $\{G_r\}$ are states of failure MSS for which the replacement of one of components allows to restore the system. In this example the system comes back to serviceability from states $\phi(001) = 0$ or $\phi(002) = 0$ when the first component and the second component are failure by the replacement of one of them.

These Boundary States are represented in Table 5 for MSS which are investigated in this example.

TABLE 5. Boundary States for MSS

	The Boundary States											
	of Failure System								of Repair System			
x_1	0	0	0	1	1	1	1	2	2	0	0	0
x_2	1	1	2	0	0	1	2	0	1	0	0	1
x_3	1	2	1	1	2	0	0	1	0	1	2	0
	The system will be failure if the state of any component is change from $x_i = 1$ to $x_i = 0$ ($i=1,3$) for this component states								The system will be restoring if the failure component is replaced			

There is a possibility to receive other types of DDRI. For example, indices to assessment that relate the component states to the system states when reductions in a capacity level of any one of the system components are investigate in [12, 17]. These indices calculate by the Direct Partial Logic Derivative: $\partial\phi(1 \rightarrow 0)/\partial x_i(s \rightarrow s-1)$.

3.2. Dynamic Integrated Reliability Indices (DIRI).

Definition 2. DIRI are probability of a modification of system reliability from j to k if a component state changes from a to b :

$$(7) \quad P_{a \rightarrow b}^{j \rightarrow k} = \sum_{i=1}^n P(i)_{a \rightarrow b}^{j \rightarrow k} \times \prod_{q=1, q \neq i}^n 1 - P(i)_{a \rightarrow b}^{j \rightarrow k},$$

where $P(i)_{a \rightarrow b}^{j \rightarrow k}$ is probability of a modification of system reliability from j to k if i -th component reliability changes from a to b :

$$(8) \quad P(i)_{a \rightarrow b}^{j \rightarrow k} = \frac{\rho(i)_{a \rightarrow b}^{j \rightarrow k}}{m^n},$$

$\rho(i)_{a \rightarrow b}^{j \rightarrow k}$ is number of system states when a change the reliability of i -th component from a to b forces a modification of system reliability from j to k and the $\rho(i)_{a \rightarrow b}^{j \rightarrow k}$ is number of values of Direct Partial Logic Derivative $\partial\phi(j \rightarrow k)/\partial x_i(a \rightarrow b)$ with respect to i -th variable Equation (4) which are not equal 0. In other words $\rho(i)_{a \rightarrow b}^{j \rightarrow k}$ is a elements count of set $\{G|x_i\}$ in Equation (3).

Example 4. Evaluate DIRI for MSS with structure function from Table 1. The Equation (7) is used for estimation of these indices for failure $P_{1 \rightarrow 0}^{1 \rightarrow 0}$ and for repair $P_{0 \rightarrow 1}^{0 \rightarrow 1}$ and $P_{0 \rightarrow 2}^{0 \rightarrow 2}$ of system:

$$(9) \quad P_{1 \rightarrow 0}^{1 \rightarrow 0} = \sum_{i=1}^3 P(i)_{1 \rightarrow 0}^{1 \rightarrow 0} \times \prod_{q=1, q \neq i}^3 1 - P(i)_{1 \rightarrow 0}^{1 \rightarrow 0} = 0.412,$$

$$(10) \quad P_{0 \rightarrow 1}^{0 \rightarrow 1} = \sum_{i=1}^3 P(i)_{0 \rightarrow 1}^{0 \rightarrow 1} \times \prod_{q=1, q \neq i}^3 1 - P(i)_{0 \rightarrow 1}^{0 \rightarrow 1} = 0.134,$$

$$(11) \quad P_{0 \rightarrow 2}^{0 \rightarrow 2} = \sum_{i=1}^3 P(i)_{0 \rightarrow 2}^{0 \rightarrow 2} \times \prod_{q=1, q \neq i}^3 1 - P(i)_{0 \rightarrow 2}^{0 \rightarrow 2} = 0.134,$$

TABLE 6. Dynamic Integrated Reliability Indices

System components	The number of system states when the change the reliability of i -th component from a to b forces a modification of the system reliability from j to k			The probability of a modification of the system reliability from j to k if i -th component reliability changes from a to b		
	$\rho(i)_{1 \rightarrow 0}^{1 \rightarrow 0}$	$\rho(i)_{0 \rightarrow 2}^{0 \rightarrow 1}$	$\rho(i)_{0 \rightarrow 2}^{0 \rightarrow 2}$	$P(i)_{1 \rightarrow 0}^{1 \rightarrow 0}$	$P(i)_{0 \rightarrow 2}^{0 \rightarrow 1}$	$P(i)_{0 \rightarrow 2}^{0 \rightarrow 2}$
x_1	4	2	2	0,444	0,222	0,222
x_2	4	2	2	0,444	0,222	0,222
x_3	4	2	2	0,444	0,222	0,222

So, the probability of the system failure is $P_{1 \rightarrow 0}^{1 \rightarrow 0} = 0.412$ when one of the components is failure. The probability of the MSS repairing by replacement of the failure component is $P_{0 \rightarrow 2}^{0 \rightarrow 1} = 0.134$ for a case of restoring of the system reliability up to a level 1 and is $P_{0 \rightarrow 2}^{0 \rightarrow 2} = 0.134$ for a case when system reliability is restored up to a level 2. Generally, the probability of repairing MSS by a replacement of the component failure is

$$P_{0 \rightarrow 2}^{rep} = P_{0 \rightarrow 2}^{0 \rightarrow 1} \cdot (1 - P_{0 \rightarrow 2}^{0 \rightarrow 2}) + P_{0 \rightarrow 2}^{0 \rightarrow 2} \cdot (1 - P_{0 \rightarrow 2}^{0 \rightarrow 1}) = 0.232.$$

For calculation DIRI in Equation (9) – Equation (11) the probabilities of three types are used (Table 6):

- probability of the system failure if any one of the component is failure $P(i)_{1 \rightarrow 0}^{1 \rightarrow 0}$, $i = \overline{1, 3}$;
- probability of the system repairing up to a level 1 if any one of the component replaces $P(i)_{0 \rightarrow 2}^{0 \rightarrow 1}$, $i = \overline{1, 3}$;
- probability of the system repairing up to a level 2 if any one of the component replaces $P(i)_{0 \rightarrow 2}^{0 \rightarrow 2}$, $i = \overline{1, 3}$;

Probabilities in Table 6 are determined by Equation (8).

3.3. Component Dynamic Reliability Indices (CDRI).

Definition 3. CDRI are probabilities of MSS failure or repairing at a modification of a state of i -th system component:

$$(12) \quad Q_f(i) = \sum P(i)_{a \rightarrow b}^{1 \rightarrow 0}, \quad \text{for } \forall a, b \in \{0, 1, \dots, m-1\} \quad \text{and} \quad a > b,$$

$$(13) \quad Q_r(i) = \sum P(i)_{s \rightarrow t}^{0 \rightarrow 1}, \quad \text{for } \forall s, t \in \{0, 1, \dots, m-1\} \quad \text{and} \quad t > s,$$

where $P(i)_{a \rightarrow b}^{1 \rightarrow 0}$ is probability of the system failure if i -th component states changes from a to b and $P(i)_{s \rightarrow t}^{0 \rightarrow 1}$ is probability of the system repairing if reliability level of i -th component changes from s to t :

$$P(i)_{a \rightarrow b}^{1 \rightarrow 0} = \frac{\rho(i)_{a \rightarrow b}^{1 \rightarrow 0}}{m^n} \quad \text{and} \quad P(i)_{s \rightarrow t}^{0 \rightarrow 1} = \frac{\rho(i)_{s \rightarrow t}^{0 \rightarrow 1}}{m^n};$$

$\rho(i)_{a \rightarrow b}^{1 \rightarrow 0}$ is number of system states when a change i -th component state from a to b forces the system failure and $\rho(i)_{s \rightarrow t}^{0 \rightarrow 1}$ is number of system states when system repairing bring by a change i -th component state from s to t .

Note, numbers $\rho(i)_{a \rightarrow b}^{1 \rightarrow 0}$ and $\rho(i)_{s \rightarrow t}^{0 \rightarrow 1}$ are obtained as numbers of values of Direct Partial Logic Derivative $\partial \phi(1 \rightarrow 0) / \partial x_i(a \rightarrow b)$ and $\partial \phi(0 \rightarrow 1) / \partial x_i(s \rightarrow t)$ with respect to i -th variable Equation (8) which are not equal 0.

TABLE 7. Component Dynamic Reliability Indices

System components	$Q_f(i)$	$Q_r(i)$
x_1	0.444	0.444
x_2	0.444	0.444
x_3	0.444	0.444

Example 5. Calculate CDRI for MSS for MSS with structure function from Table 1. CDRI for this MSS are obtained by Equation (12) for system failure and Equation (13) by repairing of MSS:

$$Q_f(i) = \sum P(i)_{a \rightarrow b}^{1 \rightarrow 0} = P(i)_{1 \rightarrow 0}^{1 \rightarrow 0} \quad \text{and} \quad Q_r(i) = \sum P(i)_{s \rightarrow t}^{0 \rightarrow 1} = P(i)_{0 \rightarrow 1}^{0 \rightarrow 1} + P(i)_{0 \rightarrow 2}^{0 \rightarrow 2}.$$

In this example one type of boundary state for failure system is investigated only. So, CDRI for failure of the system $Q_f(i)$ coincide with the $P(i)_{1 \rightarrow 0}^{1 \rightarrow 0}$ and CDRI for system repairing $Q_r(i)$ include two parts $P(i)_{0 \rightarrow 1}^{0 \rightarrow 1}$ and $P(i)_{0 \rightarrow 2}^{0 \rightarrow 2}$ (Table 7).

These indices $Q_f(i)$ and $Q_r(i)$ are identical for this example. Generally, they are appreciable different.

4. CONCLUSION

In this paper Dynamic Reliability Indices (DRI) have investigated. DRI include three groups. The first group (DDRI) is represented as sets of boundary system states. In other words, if MSS has such state, a modification the system component state occurs MSS failure or it's repairing. But DDRI have a large calculation complexity and cause the certain difficulties. Probability analogies of these indices are often used in practice. These analogies are the second group (DIRI). DIRI characterize boundary system states as well as DDRI. The third group (CDRI) is presented more detailed in this paper for the first time. CDRI allow to estimate influence of each of a system component on a possibility of MSS failure or it's repairing.

REFERENCES

- [1] I. Ushakov, G. Levitin, A. Lisnianski, Multi-State System Reliability: from Theory to Practice, *Proc 3rd Int Conf on Mathematical Methods in Reliability: methodology and practice*, Trondheim, Norway, June 2002, pp. 563-566.
- [2] R.E. Barlow, A.S. Wu, Coherent Systems with Multi-State Components, *Mathematics of Operations Research*, 1978, 3(11), pp. 275-281.
- [3] E. El-Newehi, F. Proschan, J. Seturaman, Multistate Coherent Systems, *J. Applied Probability*, 1978, 15(12), pp. 675-688.
- [4] M. Veeraraghavan, K.S. Trivedi, A Combinatorial Algorithm for Performance and Reliability Analysis Using Multistate Models, *IEEE Transactions on Computers*, 1994, 43, pp. 226-229.
- [5] X. Zang, H. Sun, K. Trivedi, A BDD-based Algorithm for Reliability Analysis of Phased-Mission Systems, *IEEE Transactions on Reliability*, 1999, R-48(1), pp. 50-60.
- [6] J. Hudson, K. Kapur, Reliability Bounds for Multistate Systems with Multistate Components, *Operations Research*, 1985, 33, pp. 735-744.
- [7] S. Garriba, E. Guagnini, P. Mussio, Multiple-Valued Logic Trees: Meaning and Prime Implicants, *IEEE Transactions on Reliability*, 1985, R-34(5), pp. 463-470.
- [8] S. Garriba, P. Mussio, F. Naldi, Multivalued logic in the representation of engineering systems, *Synthesis and Analysis Methods for Safety and Reliability Studies*, Eds., G. Apostolakis, S. Garriba, G. Volta, Plenum Press, 1980, pp 183-197.
- [9] H. Block, T. Savits, Decomposition for Multistate Monotone Systems, *Reliability Theory and Models: Stochastic Failure Models, Optimal Maintenance Policies, Life Testing and*

- Structures*, Eds., M. Abdel-Hameed, E. Cinlar, J. Quinn, Academic Press Inc, Orlando, 1984, pp. 231-241.
- [10] S. Popowicz, A Disjunctive-Conjunctive and Conjunctive-Disjunctive Decomposition for Multistate Monotone Systems, *Postepy Cybernetyki*, 1988, 11(4), pp. 43-52.
 - [11] R. Boedigheimer, K. Kapur, Customer-Driven Reliability Models for Multistate Coherent Systems, *IEEE Transactions on Reliability*, 1994, 43(1), pp. 46-50.
 - [12] E. Zaitseva, V. Levashenko, New Reliability Indices for Multi-State System. *Proc. 15th IEEE European Conference on Circuit Theory and Design*, Espoo, Finland, September 2001, pp. 345-349.
 - [13] E. Zaitseva, V. Levashenko, Design of Dynamic Reliability Indices, *Proc. IEEE 32th Int. Symp. on Multiple-Valued Logic*, Boston, USA, May 2002, pp. 144-148.
 - [14] E. Zaitseva, V. Levashenko New Dynamic Reliability Indices for Multi-State System, *Proc. 3rd Int. Conf. on Mathematical Methods in Reliability: methodology and practice*, Trondheim, Norway, June 2002, pp. 687-690.
 - [15] V. Shmerko, V. Levashenko, S. Yanushkevich, Technique of Computing Logic Derivatives for MVL-Functions, *Proc. IEEE 26th Int. Symp. on Multiple-Valued Logic*, Santiago de Compostela, Spain, May 1996, pp. 267-272.
 - [16] S. Yanushkevich, V. Shmerko, V. Levashenko, Test Pattern Generation for Combinational Multi-Valued Networks Based on Generalized D-Algorithm, *Proc. IEEE 27th Int. Symp. on Multiple-Valued Logic*, Nova Scotia, Canada, 1997, pp. 139-144.
 - [17] E. Zaitseva, V. Levashenko, Dynamic Deterministic Reliability Indices for Multi-State System, *Proc. 6th Int. Conf. on Pattern Recognition and Information Processing*, Minsk, Belarus, May 2001, pp. 147-154.

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