

ALGEBRAIC NUCLEAR SURFACES IN STRING SPACE

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ABSTRACT. We show by algebraic methods that the requirement that a nucleon be a representation of the space in which we live, i.e. an irreducible representation (IR) of Lorentz space, is already sufficient to guarantee a dipolar or quadrupolar shape. Then when odd- A nuclei are modeled by the tensor product, it is found that these representations lie in a Calabi-Yau or string space. In this way a nucleus is bound by strings of electric flux lines generated by the spinning protons which dispenses with the 'strong' force. The mesons are the quanta of the string field. Additional achievements of the model are CP - invariance, canonical state labeling and a fibration that not only introduces a Yang-Mills field but also leads to the well-known angular momentum operators for a coupled system of P protons and N neutrons which together with their dual operators constitute the six generators of $O(4)$. Also the tensor product dictates a system of entangled nucleons in a spin lattice.

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1. INTRODUCTION

Nearly a hundred years ago Hudson [13] described, very elegantly, how the equation of the Kummer surface (Ibid. p 81) may be derived by insisting on Lorentz invariance alone or in other words by finding representations (IRs) of the centre D of the quaternion or Dirac ring. Now the notion of associating a particle with such a representation goes back to Dirac, Eddington[8] Biedenharn and Wigner ([3], Ch. 4). In fact Kahan [14] states in his preface that 'Particles may be said to be representations of the Lorentz group, just as the electron may be said to be the equation of Dirac'. A truncated form of the Kummer surface is the quadrupole depicted in Fig. 1 which characterizes many nuclei and this contribution is intended to show how a particle-like IR of the Dirac ring, namely (1) must yield this shape as well as the allied dipolar and octupolar varieties. Moreover an IR of the centre lies in a Kaehler manifold or string space if the atomic number A is odd. Such a representation has a Hodge decomposition (when the tensor product of (1) with itself is constructed) that can be associated with a quadrupolar section of a quantic algebraic surface ([10], [17]).

For example the quadrupole of Fig. 1 will be shown to have the same singularities

as ${}^9\text{Li}$, ${}^{11}\text{C}$ and their mirror nuclei. In the case of ${}^9\text{Li}$, three of the 'pouches', which are 3-branches, could house deuterons and the fourth an unstable dineutron that loses an electron to become a fourth deuteron. Another neutron indicated by a dot in the centre (to be explained in 3) gives us ${}^9\text{Be}$ which is the stable dipole in Fig. 2. Here we find geodesics on a Calabi-Yau manifold, calculated in 3, which are strings of electric flux lines generated by the spinning protons that hold the nucleus together. The figure shows how two pairs of deuterons coalesce to form two Helium nuclei. The 'circles' are average positions behind strings that pulsate and the whole nucleus also rotates.

Returning to the irreducible self-representation, or one-form

$$(1) \quad \frac{1}{4}\Psi = (iE_4\Psi_1 + E_{23}\Psi_2 + E_{14}\Psi_3 + E_{05}\Psi_4)e$$

of the centre of the Dirac ring [6]; we follow Eddington [8] and associate the commuting operators E_{23}, E_{14} and E_{05} respectively with independent rotations in 3-space, 4-space and isospace that correspond to the spin σ , parity π and charge T_3 carried by a single nucleon. E_4 is the unit 4×4 matrix while Ψ_2, Ψ_3, Ψ_4 are half-angles of rotation or 'turns' [3] and e is a primitive idempotent. To see how E_{14} is related to parity we notice that a rotation through π about t will send x to $-x$ without inverting the time axis t but instead changing to a left-handed coordinate system. Eddington's E-numbers are mapped into Dirac's matrices by

$$\gamma_\nu = iE_{0\nu}, \quad E_{\mu\nu} = E_{\rho\mu}E_{\rho\nu} = -E_{\nu\mu}, \quad E_{\mu\nu}^2 = -1, \quad \mu < \nu = 1, \dots, 5$$

and the commuting operators E_{23}, E_{14}, E_{05} obey the multiplication table

$$(2) \quad \begin{array}{c|ccc} & E_{23} & E_{14} & E_{05} \\ \hline E_{23} & i^2 & iE_{05} & iE_{14} \\ E_{14} & iE_{05} & i^2 & iE_{23} \\ E_{05} & iE_{14} & iE_{23} & i^2 \end{array}$$

We now follow Barth and Nieto [2] and Hudson [13] and look at each of the above operators as specifying a pair of 'fix-lines' given by its ± 1 eigenspace (or by the $\pm i$ eigenspaces). These are polar lines (eg. 1, +i, -i) defined by the fact that their two-dimensional vector spaces are orthogonal to each other, and the six fix-lines are the edges of an invariant tetrahedron which may be inscribed in a cube as illustrated by Fig. 3. In this way equation (1) characterizes such a tetrahedron and if it is to be invariant under reflections the most general quadric equation for Kummer's surface unchanged by the Dirac operators is

$$(3) \quad (x^2y^2 + z^2t^2) + (y^2z^2 + x^2t^2) + (x^2z^2 + y^2t^2) + kxyz = 0$$

if we also require four singular points at the vertices of the tetrahedron. This surface is plotted in Fig. 1 from a programme by Morris for $k=10$.

Thus we have already seen how the choice of (1) for a representation of a nucleon in the space where we live automatically accounts for its shape. Physically the four arms of Fig. 1 could accommodate positive and negative spin and isospin but a single proton or neutron would be a dipole. When we consider the many nucleon case in 2, equation (11) shows that they are independent with the same fix-lines which ensures that the IR of space-time is not affected. Instead we find 'entangled' states that are independent and yet belong to the same representation. In practice the nucleons are held together by mesons which are the quanta of a string field.

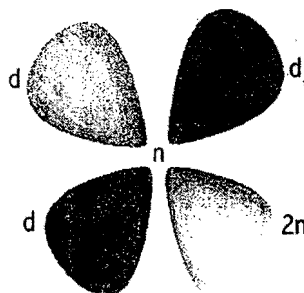


FIGURE 1. Algebraic Quadrupole Surface.

The tensor product representations exhibit other well-known nuclear characteristics such as CP-invariance (cf. Eqns. (18,19) ; (20,21)) and state labeling by a partition $A = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$ of the nuclear ensemble where

- $\lambda_1 =$ number of neutrons N with positive spin s and negative parity p ,
- $\lambda_2 =$ number of neutrons with negative spin and positive parity,
- $\lambda_3 =$ number of protons P with negative spin and negative parity,
- $\lambda_4 =$ number of protons with positive spin and positive parity. (4)

In this way a row of a representation is labeled by $[\lambda] = [\lambda_1 \lambda_2 \lambda_3 \lambda_4]$ and the spin and parity of a state are

$$(5) \quad s = \frac{1}{2}(A - 2(\lambda_2 + \lambda_3)), \quad p = \frac{1}{2}(2(\lambda_2 + \lambda_4) - A)$$

which may be substituted directly into the Hodge decompositions (18,19);(20,21) if we write $2is = \sigma_0, 2ip = \pi_0$ to obtain eigenvalues without any appeal to a matrix representation given by (6). The fact that there is perfect agreement justifies the canonical labeling system (4). There is a unique relationship between the state $[\lambda]_k$ and its eigenvalue γ_k .

As an example of entanglement consider the process

$$[\lambda_1 \lambda_2 \lambda_3 \lambda_4] \rightarrow [(\lambda_1 + 1)(\lambda_2 - 1)(\lambda_3 + 1)(\lambda_4 - 1)]$$

where one of the λ_2 neutrons changes spin from negative to positive thereby increasing the number of positive neutron spins λ_1 by unity. At the same time one of the positive proton spins λ_4 must decrease by unity to preserve the spin balance and thus the number λ_3 of negative spins increases by one. However the number of nucleons $(\lambda_2 + \lambda_4)$ with positive parity decreases by two. There is absolutely

nothing to require that these nucleons be nearest neighbors !

In particular it was shown by de Wet[6] that if we choose a fibration consisting of all those states that have the same quantum numbers of spin, parity and charge then the bundle space has a beautiful de Rham decomposition into isobaric multiplets each of which is characterised by the matrix representations

$$(6) \quad \sigma_i = E_N \otimes^P \Gamma_i + {}^N \Gamma_i \otimes E_P, \quad \pi_i = E_N \otimes^P \Gamma_i - {}^N \Gamma_i \otimes E_P, \quad i = 1, 2, 3$$

where ${}^P \Gamma_i, {}^N \Gamma_i$ are $(P+1)-$, $(N+1)-$ dimensional Lie operators of $so(3)$ and E_P, E_N are unit matrices. The operator σ_i is the well-known angular momentum operator for a coupled system of P protons and N neutrons and the dual π_i is a parity operator. In fact σ_i, π_i are the six generators of $O(4)$ and the fibration introduces a Yang-Mills field with connections on a fibre bundle ([19], Introduction). In §3 these bundles will be seen to be sections of a sheaf which not only determines a shape but also may be provided with a metric. Each section is a state labelled by $[\lambda]_k$ which is characterized by the eigenvalue γ_k .

Moreover Lawson and Michelsohn [16], Ch.1, have shown that (6) follows directly from the Z_2 grading of the Dirac algebra without any appeal to group representation theory. Furthermore because we are in a spin space the Kaehler manifold is Calabi-Yau. Thus (1) is confirmed yet again.

In the remaining sections we will briefly introduce the tensor product and indicate how it can be reduced to a Hodge decomposition of the enveloping algebra. The eigenvalues γ_k of the decomposition are the basis of an exponential map into the sheaf (cf.Eqn.(24)) . The singularities of the metric associated with a preferred section labeled by γ_k determine the covering algebraic variety. Details appear in Ref. [7]. The dipole of Fig. 2 is a parametric plot or geodesic of complementary sheaf wave functions and is defined by the quadric

$$(7) \quad x^2 + y^2 - z^2 = 0$$

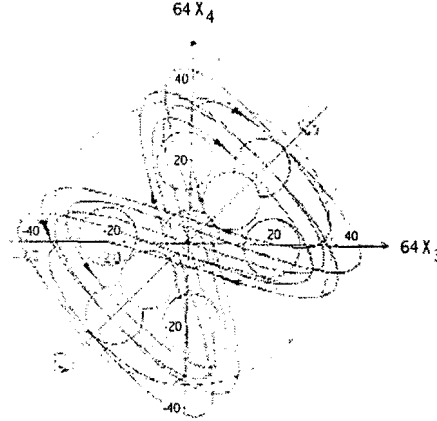
which is also Dirac invariant. In fact a quadric surface parametrizes all the lines meeting two fix-lines generated by E_{23}, E_{14} or E_{05} and is a conic [2]. Finally Cynk and Szmberg [4] show that one can find a double covering (i.e. an octupole) that remains a Calabi-Yau threefold. This can be seen graphically in Fig. 3.

The four singular points of (3) at the origin correspond to the four vertices $(x,y,z,t) = (1,0,0,0) ; (0,1,0,0) ; (0,0,1,0) ; (0,0,0,1)$ of an invariant tetrahedron rotating into itself, but at the same time Fig. 3 shows that this involves the rotation of a second tetrahedron. The eight singularities at the corners of the cube are the singular points of an octahedron composed of two quadrahedra. After a metric has been introduced in §3 ; Fig. 4 depicts the eight singularities of ${}^{15}N$ which clearly divide into two sets of four.

The representations considered in this contribution are all for odd-A nuclei because when A is even IRs of the tensor product do not lie in a Calabi-Yau manifold or string space. However it is still possible to construct representations by the methods of the next section and even to get a Calabi-Yau space if we assume that a deuteron occupies the centre of Fig. 1. In this way the even A nuclei can also exhibit a quadrupole.

2. THE TENSOR PRODUCT

A many particle representation is found in the enveloping algebra $A(\gamma)$ of the Dirac ring by constructing the tensor product of (1) with itself. The basis elements are

FIGURE 2. Geodesics or Strings on ${}^9\text{Be}$.

the $A^4 \times A^4$ matrices

$$(8) \quad E_{\mu\nu}^l = E_4 \otimes \cdots \otimes E_4 \otimes E_{\mu\nu} \otimes E_4 \otimes \cdots \otimes E_4$$

with $E_{\mu\nu}$ in the l -th position. The elements $E_{\mu\nu}^1, E_{\mu\nu}^{l+1}$ commute but $E_{23}^l, E_{14}^l, E_{05}^l$ obey the multiplication table (2). Then $A(\gamma)$ has the generators, or de Broglie operators

$$(9) \quad \Gamma_\nu^{(A)} = \frac{1}{2}(E_{0\nu}^1 + E_{0\nu}^2 + \cdots + E_{0\nu}^A), \quad \nu = 1, \dots, 5$$

$$(10) \quad \sigma_{\mu\nu}^{(A)} = [\Gamma_\mu^{(A)}, \Gamma_\nu^{(A)}] = \frac{1}{2}(E_{\mu\nu}^1 + \cdots + E_{\mu\nu}^A), \quad \mu < \nu = 1, \dots, 5$$

$$(11) \quad \eta_\nu^{(A)} = E_{0\nu}^1 \otimes \cdots \otimes E_{0\nu}^A = E_{0\nu}^1 E_{0\nu}^2 \cdots E_{0\nu}^A = \eta_\mu^{(A)} \eta_\nu^{(A)} = E_{\mu\nu}^1 E_{\mu\nu}^2 \cdots E_{\mu\nu}^A$$

In particular we shall find it convenient to employ the spin s , parity p and charge operator T_3

$$(12) \quad \sigma_0 = (E_{23}^1 + \cdots + E_{23}^A) = 2is, \quad \pi_0 = (E_{14}^1 + \cdots + E_{23}^A) = 2ip$$

$$T_0 = (E_{05}^1 + \cdots + E_{05}^A) = i(Z - N) = 2T_3$$

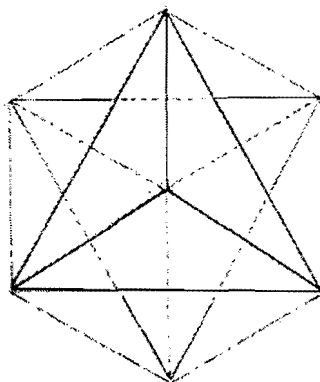


FIGURE 3. Tetrahedra Inscribed in a Cube.

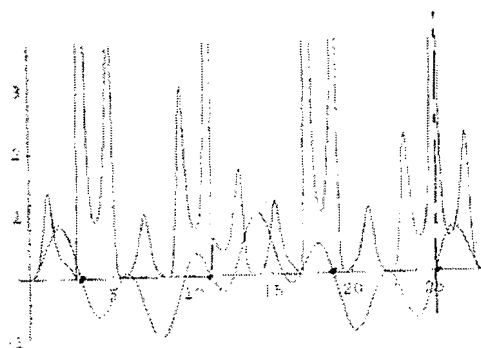


FIGURE 4. Metric of ^{15}N .

The IRs or minimum left ideals of $A(\gamma)$ are (de Wet [5])

$$(13) \quad \Psi^{(A)} = \sum_{\lambda} C_{[\lambda]} P_{[\lambda]}$$

where $P_{[\lambda]}$ is a projection operator

$$(14) \quad P_{[\lambda]}^2 = P_{[\lambda]} \psi_1^{\lambda_1} \psi_2^{\lambda_2} \psi_3^{\lambda_3} \psi_4^{\lambda_4}$$

with precisely the same structure as (1) but in the configuration space $A = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$ embodying the idempotent $\epsilon = e \otimes e \cdots \otimes e$ (A times). The essential difference between (14) and (1) lies in the fact that the operators E_{23}, E_{14} or E_{05} are replaced by their tensor products η_{23}^A, η_{14}^A or η_{05}^A but this will not affect the 'fix-lines' which determine the edges of the invariant tetrahedron, and hence (3), because by (8) the $E_{\mu\nu}^i$ in (11) commute. Therefore each nucleon in the ensemble of A nucleons will specify the same tetrahedron! Furthermore this implies that there is no interaction between nucleons which however are held together by meson exchange introduced by the string field.

By virtue of the isomorphism between Clifford multiplication and the exterior product

$$(15) \quad C_{[\lambda]} = i^{\lambda_1} \sum (E_{23}^1 \cdots E_{23}^{\lambda_2} E_{14}^{\lambda_2+1} \cdots E_{14}^{\lambda_2+\lambda_3} E_{05}^{\lambda_2+\lambda_3+1} \cdots E_{05}^{A-1})$$

is a three form in the centre D . Here the summation is over all the $N_{[\lambda]} = A! / (\lambda_1! \lambda_2! \lambda_3! \lambda_4!)$ combinations of the basis elements and by (14) a matrix representation of many nucleons will have the rows of $C_{[\lambda]}$ labeled by the states $[\lambda] = [\lambda_1 \lambda_2 \lambda_3 \lambda_4]$. However a Hodge decomposition of the fundamental relation (15) may be obtained without any appeal to matrices and also the eigenvalues γ_k that determine the metric may be found up to sign by (5). The fact that there is a perfect agreement (apart from a possible spin mutation in ^{13}C) justifies the canonical labeling. To obtain the decomposition of (15) we write it

$$(16) \quad C_{[\Lambda]} = i^{\Lambda_1} \sigma_0^{\Lambda_2} \pi_0^{\Lambda_3} T_0^{\Lambda_4} - \sum_{\lambda} i^{\lambda_1} \sigma_0^{\lambda_2} \pi_0^{\lambda_3} T_0^{\lambda_4}$$

where $[\Lambda] = [\Lambda_1 \Lambda_2 \Lambda_3 \Lambda_4]$ is the ground state which is unique in that $\Lambda_1 = \Lambda_4 \geq \Lambda_2, \Lambda_3$ otherwise there is no decomposition. Specifically this means that all the other states $[\lambda]$ are Zeeman sub-levels of the ground state and not so-called 'excited' states recorded by the decay products of other nuclei. The summation is over all those terms arising from repeated indices eg. $E_{23}^k E_{23}^k = i^2, E_{23}^k E_{14}^k = i E_{05}^k, E_{23}^k E_{05}^k = i E_{14}^k$ which yield a single term according to (2). An elementary example of (16) is given in [7]. In general we will find a decomposition into (p,q) forms whenever the three form $\sigma_0 \pi_0 T_0$ contains products with the same indices and there will be coupling constants which count the number of times a (p,q) form occurs. The calculation begins with

$$(17) \quad P(E_{23}^i E_{23}^j) = \sigma_{00} - i\pi_0$$

where the unknown P embodies a summation over the $A!/(A-n)!$ combinations in the bracket and n is the number of terms in parenthesis (in this case $n=2$). Then we 'add' one nucleon at a time by multiplying by σ_0, π_0, T_0 until $\Lambda_2 + \Lambda_3 + \Lambda_4 = A - \Lambda_1$ and in this way we find the $A=9$ two-forms (after setting $T_0 = i(Z - N)$)

$$(18) \quad {}^9Li : C_{[3303]} = i^3 P(E_{23}^i E_{23}^j E_{23}^k E_{05}^l E_{05}^m E_{05}^n) / (3!3!) \\ = \frac{1}{8} [34(\sigma_0 + \pi_0) + 9(\sigma_0 \pi_0^2 + \sigma_0^2 \pi_0) + (\sigma_0^3 + \pi_0^3)]$$

$$(20) \quad {}^9B : C_{[3123]} = -\frac{1}{2}[34(\sigma_0 + \pi_0) + (\sigma_0\pi_0^2 + \sigma_0^2\pi_0) + (\sigma_0^3 + \pi_0^3)]$$

$$(21) \quad {}^9Be : C_{[3213]} = -\frac{1}{2}[34(\sigma_0 - \pi_0) + (\sigma_0\pi_0^2 - \sigma_0^2\pi_0) + (\sigma_0^3 - \pi_0^3)]$$

which are manifestly CP-invariant and may be written in matrix form by appealing to (6). The representations of mirror nuclei are identical up to an equivalence transformation of rows and columns. Equations (18,19) ; (20,21) are in terms of the harmonics

$$(22) \quad H^{1,0} + H^{0,1} = (\sigma_0 + \pi_0); H^{2,1} + H^{1,2} = (\sigma_0^2\pi_0 + \sigma_0\pi_0^2), H^{3,0} + H^{0,3} = (\sigma_0^3 + \pi_0^3)$$

which is the required Hodge decomposition of the Calabi-Yau manifold K consisting of classes that are closed but not exact and thus have no even terms. Apart from the contribution of Griffiths [10], Green, Schwarz and Witten [9] show in §16.3.2 that these harmonics characterize a quintic hypersurface illustrated by the section of Fig. 1. However bearing the Introduction in mind it should be clear that there is no unique mapping between Hodge classes and algebraic surfaces after harmonics of order three. Nevertheless a Hodge decomposition does prove a unique IR of the centre D .

3. THE METRIC

We will need to exponentiate an irreducible subspace μ of the matrix $C[\lambda]$ which is a complex projective space P^3 characterized by

$$(23) \quad \mu = \begin{bmatrix} 0 & X \\ -X^T & 0 \end{bmatrix}$$

where X is a real symmetric p matrix with coordinates $k = \gamma_k t$ and there is an isomorphism between the eigenvalues γ_k and the state $[\lambda]_k = [\lambda_1 \lambda_2 \lambda_3 \lambda_4]_k$. The exponential formula [12] states that

$$(24) \quad e^{\mu t} = \mu \sum_{k=0,1}^n \frac{F_k(\mu) \cos \gamma_k t}{i \gamma_k F(i \gamma_k)} + i \sum_{k=1,2}^n \frac{F_k(\mu) \sin \gamma_k t}{F_k(i \gamma_k)}$$

where the coupling constants are

$$(25) \quad F(\mu) = \mu(\mu^2 + 1)(\mu^2 + \gamma_2^2) \dots (\mu + \gamma_n^2) = 0$$

$$F_0(\mu) = F(\mu)/\mu, \quad F_k(\mu) = F(\mu)/(\mu^2 + \gamma_k^2), \quad F_j(\mu)F_k(\mu) = 0$$

and

$$(26) \quad K_k(\mu) = \frac{i \gamma_k F_k(\mu)}{\mu F_k(i \gamma_k)}$$

is idempotent. Then (24) follows by differentiating at $t = 0$ because $\sum K_k(\mu)$ is a partition of unity. In practice we evaluate (24) at a specific element $[\mu^n]_{ij}$ of μ^n . In this way $e^{\mu t}$ can be expressed in terms of rows and columns of wave functions but one can select complementary wave functions at the centre of the matrix. An example is (33) below. Equation (24) is a sheaf of sections ([11], Chs.1,2) and each section k is a Zeeman sublevel. In particular an algebraic variety or surface is a collection of sections and after we have found a metric we shall see how the singularities of certain preferred sections of 9Li and ${}^{11}C$ imply the quadrupole

equation (3). Because $e^{\mu t}$ is orthogonal and unimodular (24) is an IR of $SO(4)$ that lies in the six-dimensional Calabi-Yau space K . In other words we are looking at representations of $SO(10) = SO(4) \times K$. Then a metric may be found by writing (24):

$$(27) \quad e^{\mu t} = Z_0(cost) + Z_1(sint) = \begin{bmatrix} Z_0 & Z_1 \\ -Z_1 & Z_0 \end{bmatrix}$$

and using the Fubini-Study form suggested by Wong [17] namely

$$(28) \quad ds^2 = Tr \frac{dT}{(1 + T\overline{T}^T)} \frac{d\overline{T}^T}{(1 + T\overline{T}^T)}$$

where

$$(29) \quad T = Z_1 Z_0^{-1} = -T^T = \mu \sum_{k=1,2}^n \frac{i(F_k(\mu)/\mu) \tan \gamma_k t}{F_k(i_k)}$$

$$(30) \quad T\overline{T}^T = \sum_{k=1,2}^n K_k(\mu) \tan^2 \gamma_k t$$

Here $\overline{T}^T, d\overline{T}^T$ are conjugate transposes of T, dT and (28) reduces to the flat measure carried by a torus i.e.

$$(31) \quad ds^2 = \sum_{k=1,2}^n dz_k d\bar{z}_k, \quad z_k = i\gamma_k t$$

which is the Kaehler condition that the metric approximate the Euclidean metric at each point of the torus ([16], Ch. 0.7). However (25) implies a translation to a normalized canonical form

$$(0, 1, \gamma_2, \dots, \gamma_n), \quad n \leq p$$

where $(\gamma_2, \dots, \gamma_n)$ are all positive. If this condition is not met we must add an angular momentum γ_0 equal to the greatest negative eigenvalue and then divide by $\gamma = (\gamma_j + \gamma_0)$, which makes the j -th translated eigenvalue unity and may be absorbed in t with only a virtual change to the wave function $e^{\mu t}$. However $e^{\mu t}$ must be corrected by a 'twist' $e^{i\gamma_0 t}$ that multiplies (30) by $\tan^2 \gamma_0 t$ and introduces the distorted metric

$$(32) \quad ds^2 = \sum_k \frac{\tan^2 \gamma_0 t \sec^4 \gamma_k t}{(1 + \tan^2 \gamma_0 t \tan^2 \gamma_k t)^2} dz_k d\bar{z}_k = \sum_k g_{k\bar{k}} dz_k d\bar{z}_k$$

which is dependent only on the normalized eigenvalues of μ and reduces to (31) when $\tan \gamma_0 t = 1$. Here $k = \gamma_k t, \bar{k} = -\gamma_k t$ are respectively coordinates of $X, -X^T$ in (23) while $i\gamma_k t$ are the coordinates of μ because of the one-to-one correspondence between γ_k and the state $[\lambda]_k$ provided by (5). According to (32) each section k has its own metric and in the case of ${}^9Li, {}^11C$ and their mirror pairs it is possible to find a preferred section with the eigenvalue $\gamma_k = \frac{1}{2}\gamma_0$ which yields Fig. 5 where the wave function $\sin t \sin \frac{1}{2}t$ indicates five singularities at the origin.

These may be reduced to four that Griffiths ([10], Table 1) demonstrates can generate at least a quintic algebraic variety V in the complex projective space P^4 . A section of this quintic is the quartic of (3) in P^3 . Then Fig. 1 is a cross-section of this quartic found by setting $t = 1$. There are 4 singularities at the origin and 4 more

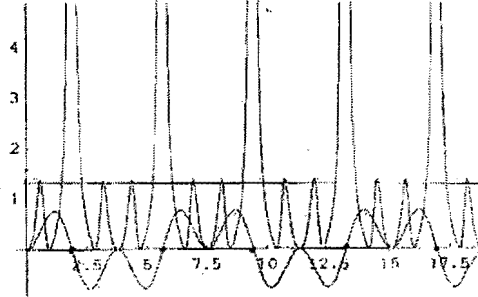


FIGURE 5. Metric of ${}^9\text{Li}$, ${}^{11}\text{C}$. $\text{Out}[1] = \frac{\sec(5t/6)^4 \tan(5t/3)^2}{(1 + \tan(5t/6)^2 \tan(5t/3)^2)^2}$, and $\text{Out}[2] = \sin(5t/6) \sin(5t/3)$.

at the apices of the 3-branes but $t = 1$ only picks up the point $(0,0,0,1)$ believed to be site of a neutron. Clearly there is some missing string information, but the four 3-branes correspond to $(x,y,z) = (-,+,+)$; $(+,-,+)$; $(+,+,-)$; $(-,-,-)$. Quadrupoles have been measured by M1, E2 transitions associated with the ground states of the nuclei ${}^9\text{Li}$, ${}^{11}\text{C}$ and their mirror partners by Ajzenberg-Selove([1] (1975), (1979)) . The nucleus ${}^{15}\text{N}$ exhibits ground state E3 transitions associated with an octupole and Fig. 4 shows the eight singularities at the origin which are clearly two sets of four generated by the section $\gamma_0 = \frac{16}{9}\gamma_0$. Experimental evidence appears in [1],(1976). In the case of ${}^9\text{Be}$ there is no preferred section because $\gamma_0 = 0$ but exploiting (24) and selecting μ_{34}, μ_{44} one finds the complementary wave functions

$$(33) \quad \Psi_3 = \frac{1}{16} \left(-\sin \frac{t}{2} + \sin t + 5 \sin \frac{5t}{2} + 5 \sin 5t \right)$$

$$(34) \quad \Psi_4 = \frac{1}{16} \left(-\sin \frac{t}{2} - \sin t + 5 \sin \frac{5t}{2} + 5 \sin 5t \right) = \Psi_3(2\pi - t).$$

Then according to Kobayashi and Nomizu([15], Ch.9) a parametric plot will give us geodesics in 3-space. These may be considered to be lines of electric flux or strings generated by the spinning protons and enclosing the rotating and spinning nucleons as depicted in Fig. 2 [12]. However because the nucleus as a whole is also pulsating and rotating, nucleons sketched in the figure will move with the geodesic so only average positions are drawn in a dipole.

4. CONCLUSION

In the last two sections we have endeavoured to relate the quadrupole, determined logically by the fundamental particle representation (1), to actual odd-A nuclei that are found experimentally to have this shape. Moreover they are in a string space and we have been able to draw the strings and 3-branes generated by the spinning protons which bind the nucleus thus dispensing with any appeal to the 'strong' force. Physically the binding agent appears as mesons that are the quanta of the string field.

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