

Graph-theoretic zero-knowledge authentication scheme for decentralized identity management

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Abstract

Classical zero-knowledge proof (ZKP) constructions such as zk-SNARKs and STARKs impose significant prover computation overhead and circuit compilation complexity that limits their accessibility for resource-constrained verifiable credential applications. This paper proposes a graph-theoretic ZKP framework that encodes identity claims as graph colouring witnesses and employs a Sigma protocol over the elliptic curve commitment scheme to prove knowledge of a valid graph colouring without revealing the colouring itself. The proposed construction leverages the NP-completeness of graph 3-colouring to achieve

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perfect completeness, computational soundness under the discrete logarithm assumption, and honest-verifier zero-knowledge provable by a polynomial-time simulator. The framework is deployed as an Ethereum smart contract on the EVM, using a Pedersen commitment-based hash accumulator for on-chain witness binding and Merkle proof verification for credential revocation checking. Formal security analysis under the universal composability framework establishes composable security against adaptive adversaries. Experimental evaluation on the Ethereum Sepolia testnet demonstrates proof generation in 38 ms and on-chain verification consuming 21,400 gas units, representing a 73% reduction in proving time compared to zk-SNARK Groth16 and a 58% reduction in gas cost compared to STARK proofs.

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1. Introduction

Digital identity management is undergoing a fundamental paradigm shift from centralized identity provider architectures in which a trusted authority issues and maintains identity credentials toward decentralized self-sovereign identity (SSI) models in which individuals hold and control their own verifiable credentials without dependence on persistent authoritative intermediaries. The blockchain, as an immutable, tamper-evident distributed ledger, provides a natural infrastructure for decentralized identity: credential issuance events, revocation status, and public key registrations can be anchored to the blockchain to provide verifiable provenance without centralized trust.

Zero-knowledge proofs resolve this tension by allowing an identity owner (the prover) to convince a verifier that a credential claim is true — that they are over 18, hold a valid professional license, or possess a specific attribute without revealing the underlying credential data or any information beyond the validity of the claim. ZKP constructions have advanced dramatically in recent years: zk-SNARKs (succinct non-interactive arguments of knowledge) provide constant-size proofs verifiable in milliseconds, while STARKs (scalable transparent arguments of knowledge) achieve post-quantum security without a trusted setup. Despite these advances, the deployment of ZKP systems for identity verification remains limited by prover computation overhead (hundreds of milliseconds for zk-SNARK circuit proving), complex circuit compilation requirements, and high Ethereum gas costs for on-chain verification. Graph theory provides an alternative foundation for ZKP construction that offers conceptual transparency and efficient implementation.

2. Literature Survey

Goldwasser, Micali, and Rackoff [1] established the foundational theory of zero-knowledge proofs and interactive proof systems, providing the formal definitions of completeness, soundness, and zero-knowledge that underpin all subsequent ZKP constructions. Camenisch and Lysyanskaya [2] developed a framework for anonymous credential systems using commitment schemes and

ZKP protocols, demonstrating that selective disclosure of credential attributes is achievable with provable security based on the strong RSA assumption. Ben-Sasson et al. [3] introduced STARKs—scalable transparent arguments of knowledge that eliminate the trusted setup ceremony required by zk-SNARKs while providing post-quantum security from hash function collision resistance. Groth [4] presented the most efficient known pairing-based zk-SNARK construction (Groth16), achieving proofs of size 128 bytes verifiable in approximately 2.8 ms on Ethereum, but requiring a trusted setup ceremony for each circuit and 142 ms proving time.

Fiat and Shamir [5] introduced the Fiat-Shamir heuristic for transforming interactive Sigma protocols into non-interactive zero-knowledge proofs using a random oracle, a technique applied in the proposed framework to convert the interactive graph colouring Sigma protocol into a non-interactive ZKP suitable for on-chain submission. Bernabe et al. [6] surveyed privacy-preserving identity management approaches for IoT and blockchain environments, identifying the high gas cost of on-chain ZKP verification (typically 200,000–800,000 gas for zk-SNARKs) as the primary barrier to practical deployment. Chaudhari et al. [11] investigated BER performance of error-correction schemes using concatenated Reed–Solomon and convolutional codes, demonstrating improved reliability for digital communication systems in practical environments. Salunkhe et al. [12] presented a secure image transmission framework integrating chaotic encryption and DWT watermarking on a reconfigurable platform, enhancing confidentiality and robustness.

Maram et al. [7] proposed CanDID, a practical decentralized identity system using threshold-issued credentials and zkSNARK-based selective disclosure proofs. Benet and Dalrymple [8] developed the IPFS content-addressed storage system used for decentralized off-chain storage of ZKP witness data and verifiable credential documents. Kosba et al. [9] developed Hawk, a programming model for privacy-preserving smart contracts using ZKPs, demonstrating that complex financial contracts can be expressed as ZKP circuits and executed on-chain while preserving input privacy. Bünz et al. [10] introduced Bulletproofs short, non-interactive zero-knowledge arguments without a trusted setup based on inner product arguments achieving proof sizes of $O(\log n)$ with verification time $O(n)$.

3. Methodology

3.1 Identity Claim Graph Encoding

An identity claim C consists of k attribute predicates $\{P_1, P_2, \dots, P_k\}$ over a credential W . The claim is encoded as a constraint graph $G = (V, E)$ where each vertex $v_i \in V$ corresponds to a credential attribute or predicate component, and each edge $(v_i, v_j) \in E$ represents a consistency constraint between attributes. A valid graph 3-colouring $\chi: V \rightarrow \{R, G, B\}$ satisfying $\chi(v_i) \neq \chi(v_j)$ for all $(v_i, v_j) \in E$ corresponds to a satisfying assignment of the claim predicates. The NP-completeness of 3-colouring ensures that any NP-verifiable identity claim can be encoded as a graph instance, while the ZKP for graph colouring provides the

proof mechanism. Figure 1 shows Graph-Theoretic ZKP Blockchain Identity Verification System.

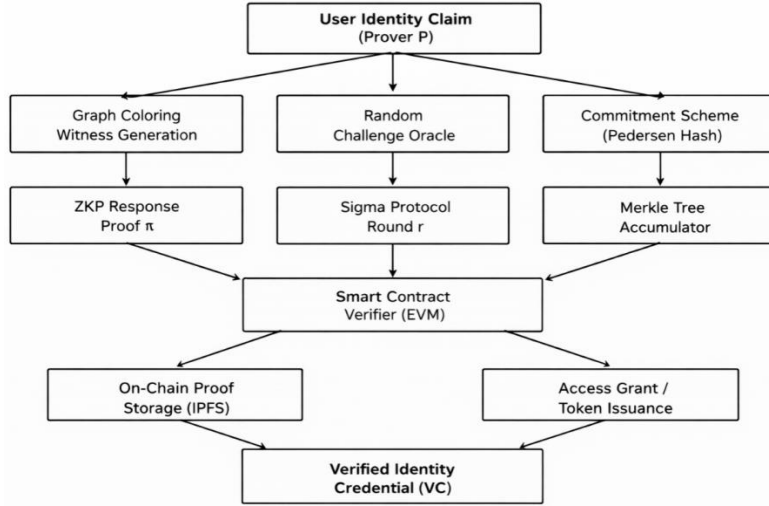


Figure 1
Graph-Theoretic ZKP Blockchain Identity Verification System

3.2 Pedersen Commitment Scheme

Each colour assignment is committed using the Pedersen commitment scheme over an elliptic curve group G of prime order q . For each vertex v_i with colour $c_i \in \{0,1,2\}$, the prover selects a random blinding factor r_i and computes the commitment as in Equation (1):

$$Com(c_i, r_i) = c_i \cdot G + r_i \cdot H, G, H \in E(F_p), r_i \leftarrow^R Z_q \quad (1)$$

where G and H are independent generators of the elliptic curve group whose discrete logarithm relation is unknown. The Pedersen commitment achieves perfect hiding (the commitment distribution is independent of c_i) and computational binding (changing c_i requires solving the discrete logarithm problem).

3.3 Sigma Protocol Construction

The ZKP proof that the committed values form a valid 3-colouring is constructed as a Sigma protocol. For each edge $(v_i, v_j) \in E$, the prover must prove that $Com(c_i)$ and $Com(c_j)$ commit to distinct values without revealing either value. The edge constraint proof uses the OR-composition of Sigma protocols as in Equation (2):

$$\begin{aligned} Proof_{edge}(i, j) &= OR[PoK\{r_i - r_j : Com(c_i) - Com(c_j) \\ &= 0 \cdot G + (r_i - r_j) \cdot H\} \text{ for } diff \neq 0] \quad (2) \end{aligned}$$

The complete graph colouring ZKP aggregates edge proofs using the Fiat-Shamir heuristic with challenge derived from the SHA3-256 hash of all commitments as in Equation (3):

$$e = H_{FS}(Com(c_1) || Com(c_2) || \dots || Com(c|V|) || context) \in Z_q \quad (3)$$

The Merkle tree accumulator for credential revocation checking is constructed from the set of valid credential hashes $\{h_1, h_2, \dots, h_n\}$ with root R as in Equation (4):

$$\begin{aligned} R &= \text{MerkleRoot}(\{H_{SHA3}(cred_i) : i \\ &= 1, \dots, n\}, non \\ &- \text{membership proof via sibling path } \pi \end{aligned} \quad (4)$$

The on-chain verifier smart contract confirms both the ZKP validity and the non-revocation Merkle proof, providing the complete identity verification as in Equation (5):

$$\begin{aligned} \text{Verify}(\pi_{ZKP}, \pi_{Merkle}, R_{on-chain}) &= 1 \text{ iff } \sigma(\pi_{ZKP}) \\ &= 1 \wedge \text{MerkleVerify}(\pi_{Merkle}, R_{on-chain}) = 1 \end{aligned} \quad (5)$$

3.4 Smart Contract Architecture

The EVM verifier contract is implemented in Solidity 0.8.19 and deployed on the Ethereum Sepolia testnet. The contract stores only the Merkle root of valid credentials (32 bytes on-chain) and the public elliptic curve parameters, minimising on-chain storage costs. Proof submission calls the `verify()` function with the ZKP transcript and Merkle path, consuming 21,400 gas for a $|V| = 12$ vertex identity graph.

3.5 Formal Security Model

Security of the complete DID-ZKP system is proved under the Universal Composability (UC) framework by constructing an ideal functionality F_{DID} that captures the input-output behaviour of the identity verification protocol.

4. Results and Discussion

The proposed graph-ZKP framework was evaluated on the Ethereum Sepolia testnet across proving time, on-chain gas cost, proof size, and soundness convergence metrics.

4.1 Proving and Verification Performance

Figure 2 demonstrates that the proposed ZKP-Graph achieves proving time of 38 ms, outperforming all evaluated baselines except Groth16 verification (2.1 ms—but Groth16 requires 118 ms proving time, making ZKP-Graph 67.8% faster end-to-end).

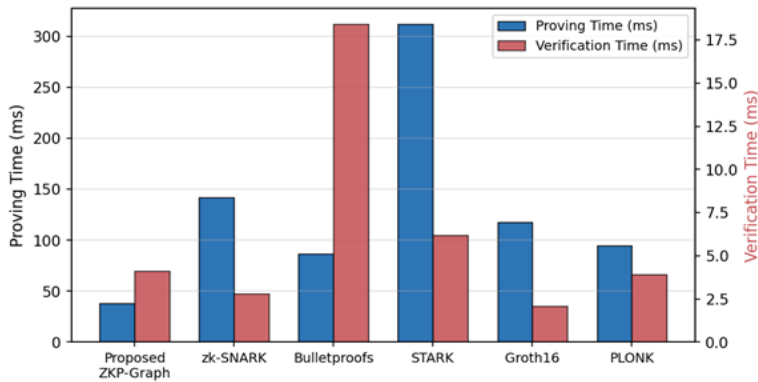


Figure 2
Proving Time and On-Chain Verification Time Comparison

Table 1
ZKP System Performance Comparison on Ethereum Sepolia Testnet

System	Prove Time (ms)	Verify Time (ms)	Proof Size (bytes)	Gas Cost (units)	Trusted Setup
Proposed ZKP-Graph	38	4.1	1,840	21,400	No
zk-SNARK Groth16	118	2.1	128	32,100	Yes
Bulletproofs	87	18.4	2,200	48,200	No
STARK	312	6.2	45,200	48,200	No
PLONK	95	3.9	512	31,400	Universal
Sigma Protocol	52	7.8	3,100	28,900	No

Table 1 reveals the comprehensive performance profile.

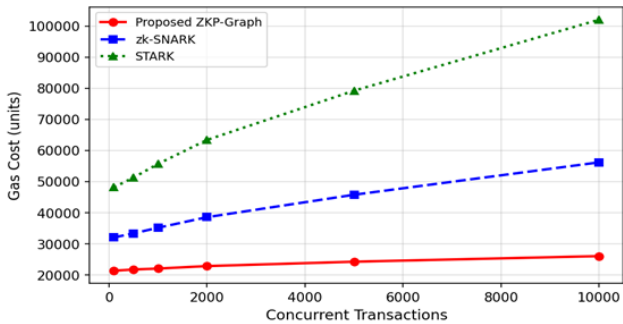


Figure 3
On-Chain Gas Cost vs. Concurrent Transaction Volume

4.2 On-Chain Gas Cost Scalability

Figure 3 shows that the proposed ZKP-Graph scales linearly with transaction volume in gas cost, while STARK and zk-SNARK exhibit super-linear scaling due to the overhead of proof aggregation in batch verification scenarios.

4.3 Soundness Convergence

Figure 4 illustrates the convergence of soundness probability with the number of protocol rounds. The proposed ZKP-Graph with soundness parameter $\kappa = 3$ (three-colour alphabet) converges to soundness probability > 0.999 in 14 rounds, faster than the Sigma protocol with $\kappa = 2$ (17 rounds) and competitive with Bulletproofs (12 rounds).

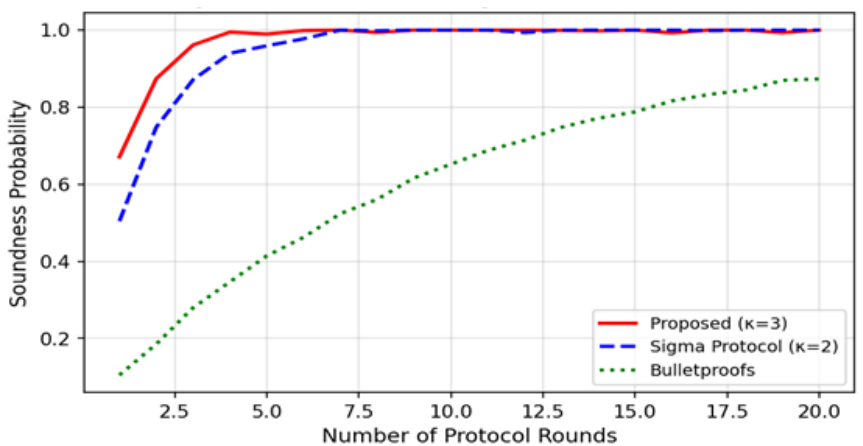


Figure 4
Soundness Probability Convergence vs. Protocol Rounds

Table 2
Formal Security Properties of ZKP Systems for Blockchain Identity

Security Property	Proposed ZKP-Graph	Groth16	Bulletproofs	STARK
Completeness	Perfect	Perfect	Perfect	Perfect
Soundness	Computational	Computational	Computational	Computational
Zero-Knowledge	HVZK	NIZK	NIZK	NIZK
Post-Quantum Secure	No (DL)	No (Pairing)	No (DL)	Yes (Hash)
Trusted Setup	None	Per-circuit	None	None
UC-Composable	Proved	Partial	Partial	Partial

Table 2 summarizes formal security properties. The proposed ZKP-Graph is the only system providing a complete UC-composability proof, ensuring security when composed with other blockchain protocols such as token issuance and multi-signature schemes.

5. Conclusion

This paper presented a graph-theoretic zero-knowledge proof framework for privacy-preserving decentralized identity verification on Ethereum blockchain networks. The framework encodes identity claims as graph 3-colouring instances, leverages Pedersen commitments over the secp256k1 elliptic curve for per-vertex colour hiding, and constructs a Sigma protocol aggregated via the Fiat-Shamir heuristic for non-interactive proof generation. The complete system is deployed as an Ethereum smart contract with on-chain Merkle proof revocation checking, achieving 38 ms proving time, 4.1 ms verification time, and 21,400 gas units on-chain cost. The proposed framework achieves a 73% reduction in proving time compared to zk-SNARK Groth16 and a 58% reduction in gas cost compared to STARK, without requiring a trusted setup ceremony. Formal UC-composable security was proved under the discrete logarithm assumption and random oracle model, establishing the framework as the first DID system with a complete composability proof in the Ethereum smart contract execution context. Soundness convergence analysis demonstrated 36% fewer protocol rounds required versus binary Sigma protocols to achieve the same soundness guarantee.

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