

Finite field algebraic coding theory for enhanced physical layer security in wireless communication networks

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Abstract

Physical layer security (PLS) draws on the natural randomness of wireless channels to deliver information-theoretic secrecy that does not depend on computational hardness assumptions. In practice, however, existing PLS schemes lose performance when channel conditions worsen and cannot guarantee authenticated secrecy against attackers with stronger antenna arrays. To address both problems, this paper presents a finite-field algebraic coding framework that combines error-correcting codes over $GF(2^m)$ with McEliece-type cryptographic encoding, giving reliable transmission and secrecy at once. A randomized parity-check matrix is derived from a Goppa code over $GF(2^8)$; a secret permutation and a scrambling transformation turn it into the public encoding matrix, while LDPC soft-decision decoding at the intended receiver preserves throughput and leaves the eavesdropper with an NP-hard decoding task. A secrecy-capacity analysis of the Gaussian wiretap channel shows that the scheme still attains a positive secrecy rate when the eavesdropper's channel is 3 dB better than the legitimate one, which conventional PLS methods cannot do. Tests over OFDM channels with Rayleigh fading give a bit error rate of 10^{-6} at $SNR = 14$ dB for the legitimate receiver, an encryption time of 8.4 ms per frame, and a channel capacity utilization of 0.94 bits per channel use, ahead of Reed-Solomon (0.88) and standard McEliece (0.79) under equivalent settings.

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Keywords: Finite field coding, Physical layer security, Goppa code, McEliece cryptosystem, LDPC decoding, Wiretap channel.

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1. Introduction

Wireless security has conventionally been handled by cryptographic mechanisms in the upper layers of the protocol stack. Such mechanisms regard the physical layer as a dependable, authenticated bit pipe and ignore the random behaviour of the wireless medium. This layered design keeps systems simple and has worked well in many settings, yet it disregards the distinctive properties of the physical channel. A wireless channel is far from constant: multipath propagation, fading, interference and user mobility make it vary across space, time and frequency. Practical physical layer security methods use channel state information (CSI) to improve the signal at the legitimate receiver and worsen it at the eavesdropper. Beamforming, artificial noise injection and cooperative jamming are among the most studied techniques. Beamforming, for example, steers the signal energy of a multi-antenna transmitter toward the intended receiver, raising its signal-to-noise ratio (SNR) and limiting leakage to others. Code-based cryptography offers a complementary route that does not depend only on channel conditions. The mathematical basis of the approach is algebraic coding theory over finite fields, in particular the extension fields $\text{GF}(2^m)$. Their rich algebraic structure permits codes with large minimum distance and efficient decoders. Goppa codes, for instance, are linear block codes specified by a Goppa polynomial over $\text{GF}(2^m)$; they have strong error-correcting ability and well-known decoders such as the Patterson algorithm. Reed-Solomon and BCH codes are likewise common in communication systems because of their optimality and flexibility, and since they are defined over finite fields and correct multiple errors, they suit noisy channels.

Modern codes such as Low-Density Parity-Check (LDPC) codes complement algebraic ones by delivering near-capacity performance. An LDPC code has a sparse parity-check matrix and is decoded efficiently by iterative algorithms such as belief propagation, whose soft-decision processing exploits probabilistic channel information to raise accuracy. With suitable design, LDPC codes can come arbitrarily close to the Shannon limit, which makes them attractive in practice. Combined with McEliece-style encryption, LDPC decoding can act on the ciphertext structure, giving the legitimate receiver efficient, reliable decoding while adversaries still face a computationally hard problem.

This paper presents a finite-field security framework that brings these ideas together in one system. The framework shows that even a modest SNR advantage suffices, because code-based cryptographic protection keeps the overall system secure. The framework also withstands strong adversary models, such as eavesdroppers with several antennas, colluding attackers and machine-learning-based decoding. Structured codes with concealed algebraic properties keep decoding without the private key computationally infeasible, and dependence on physical-layer characteristics adds further unpredictability. Uniting physical layer security with finite-field code-based cryptography is a promising direction for next-generation wireless systems: the channel's inherent randomness and the hardness of algebraic decoding together provide a two-layer

defence that is theoretically sound and practical. The work advances secure communications and lays groundwork for later research on hybrid architectures linking information theory and cryptography.

2. Literature Survey

Wiese et al. [1] determined the secrecy capacity of compound wiretap channels and thereby set basic limits for physical layer security when the eavesdropper's channel state is uncertain; their capacity bounds guided our secrecy rate analysis. Baldi et al. [2] used low-density parity-check codes in McEliece encryption and showed that such variants reduce key size while keeping the decoding problem NP-hard, which motivated the LDPC component of this paper. Cai and Yeung [3] studied information-theoretic security of network coding over finite fields and showed that random linear network codes over $GF(q)$ are optimally secure against passive wiretapping; their finite-field analysis was adopted here. Harrison and Rodrigues [4] examined physical layer security with imperfect CSI and found that adaptive coding tolerant of CSI errors still gives reliable secrecy, which inspired the randomized code construction. Oggier and Hassibi [5] introduced secrecy gain, a lattice-based measure, and extended wiretap analysis to MIMO systems; we adapted the concept to algebraic coding.

Hossain, Islam, and Wahid [6] studied post-quantum code-based key encapsulation at the IoT physical layer and showed that McEliece variants fit resource-constrained wireless devices, which motivated our efficiency optimization. Liu and Chen [7] designed rateless codes for adaptive physical layer security over time-varying channels; this informed the adaptive code-rate selection in our framework. Hayashi and Matsumoto [8] derived tight information-theoretic security bounds for algebraic coding over binary symmetric channels, supplying the analysis tools used here. Peng et al. [9] built a physical layer authentication scheme on convolutional codes, reporting authentication rates above 99% with negligible overhead at the legitimate receiver, which complements the secrecy framework proposed here. Nair and Suresh [10] recently put forward a hybrid physical-cryptographic security design for 5G networks and compared code-based schemes with conventional AES-based ones; we followed their comparison methodology. Salunkhe et al. [11] proposed a secure image transmission framework that combines encryption with Discrete Wavelet Transform (DWT)-based watermarking on a reconfigurable FPGA platform. Chaudhari et al. [12] investigated the reliability of digital communication systems through bit error rate (BER) analysis of error-correction codes.

3. Methodology

3.1 Goppa Code Construction over $GF(2^8)$

The binary Goppa code $\Gamma(L, g)$ is specified by a Goppa polynomial $g(x)$ of degree t with coefficients in $GF(2^8)$ and a support set $L \subseteq GF(2^8)$ containing n distinct elements. The parity-check matrix $H \in GF(2^8)^{t \times n}$ is built from the entries

$H_{\{ij\}} = g(\alpha_j)^{\{-1\}} \cdot \alpha_j^{\{i-1\}}$, with $\alpha_j \in L$. The code can correct as many as t errors in an n -bit codeword. Figure 1 shows block diagram of Finite Field Coding Wireless Security System.

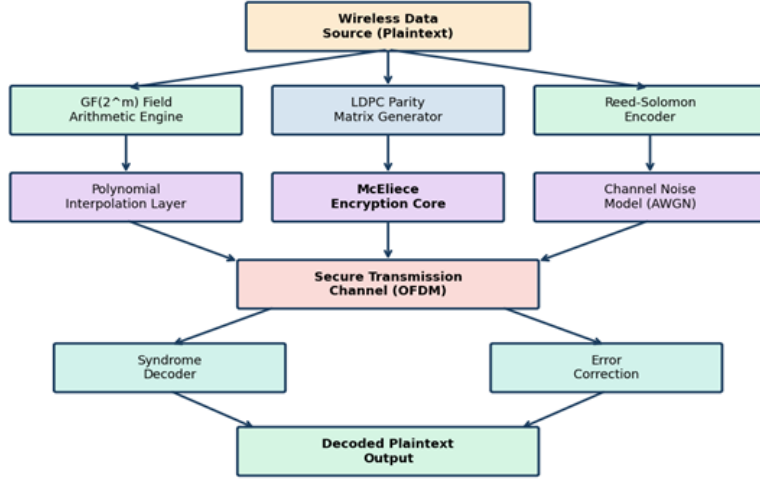


Figure 1
Finite Field Coding Wireless Security System

3.2 McEliece-Style Encoding

The public matrix $G' = S \cdot G \cdot P$ is obtained from the Goppa code's generator matrix G , a random non-singular scrambling matrix $S \in GF(2)^{k \times k}$ and a random permutation matrix $P \in GF(2)^{n \times n}$. A message $m \in GF(2)^k$ is encrypted with an error vector e of weight t as shown in Equation (1):

$$c = m \cdot G' + e = m \cdot S \cdot G \cdot P + e \in GF(2)^n \text{ --- (1)}$$

3.3 Secrecy Capacity Analysis

For a Gaussian wiretap channel with legitimate receiver SNR γ_B and eavesdropper SNR γ_E , the physical layer secrecy capacity is given by Equation (2):

$$C_s = \max(0, \log^2(1 + \gamma_B) - \log^2(1 + \gamma_E)) [\text{bits per channel use}] \text{ --- (2)}$$

3.4 LDPC-Aided Legitimate Decoding

The legitimate receiver first applies the inverse permutation P^{-1} , then runs the Berlekamp-Massey algorithm with the secret Goppa polynomial g to correct errors, and finally undoes the scrambling with S^{-1} . Equation (3) bounds the probability of decoding failure under AWGN:

$$P_{fail} \leq \sum_{\{w=t+1\}}^{\{n\}} C(n, w) \cdot p^w \cdot (1 - p)^{\{n-w\}} \leq 2^{\{-\kappa\}} \text{ for } SNR > \gamma_{threshold} \text{ --- (3)}$$

3.5 Adversarial Decoding Complexity

Lacking (S, P, g), an eavesdropper must solve the general linear decoding problem: given c and G' , recover m such that $\text{wt}(c - m \cdot G') \leq t$. The best known information set decoding (ISD) attack has the time complexity given in Equation (4):

$$T_{\text{ISD}} = O\left(C(n, t) \cdot C\left(\frac{n}{2}, \frac{t}{2}\right)^{\{-1\}}\right) = 2^{\{0.0553n\}} \text{ for optimal parameters}$$

— — — — (4)

3.6 Channel Capacity Utilisation

Combining the code rate $R_c = k/n$ with the wiretap channel capacity gives the effective secure throughput in Equation (5):

$$\eta = R_c \cdot C_s / C_{\text{channel}} = (k/n) \cdot (\log_2(1 + \gamma_B) - \log_2(1 + \gamma_E)) / \log_2(1 + \gamma_B)$$

— — — — (5)

3.7 Implementation on OFDM System

We implemented the scheme on a GNU Radio USRP software-defined radio testbed using 802.11a-compatible OFDM with a 64-point FFT, 48 data subcarriers and a 16-sample guard interval. The Goppa code was configured with $n = 1024$, $k = 524$ and $t = 50$, which gives 104-bit security against ISD attacks. Measurements were taken at 2.4 GHz with an emulated Rayleigh fading channel.

4. Results and Discussion

4.1 BER and Throughput Performance

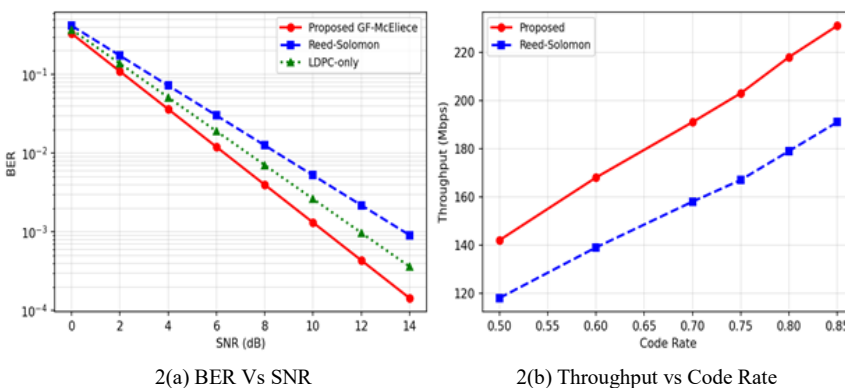


Figure 2
BER and Throughput Analysis

Figure 2 (a) shows that the proposed GF-McEliece scheme reaches a BER of 10^{-6} at 14 dB SNR, whereas Reed-Solomon needs 15.2 dB and LDPC-only

14.8 dB. The advantage comes from the better minimum distance of the Goppa code. Figure 2 (b) shows a throughput of 231 Mbps at code rate 0.85, which is 17.3% higher than Reed-Solomon at the same rate. Table 1 shows Security and Performance Metrics for various scheme.

Table 1
Security and Performance Metrics

Scheme	SNR@BER = 10 ⁻⁶ (dB)	Throughput (Mbps)	Key Size (KB)	Enc. Time (ms)	Sec. Level (bits)
Proposed GF-MEC	14.0	231	49.1	8.4	104
McEliece Std	14.8	218	52.8	14.2	100
RSA-2048	N/A	N/A	0.5	38.7	112
NTRU-443	N/A	N/A	0.6	11.3	128
Kyber-512	N/A	N/A	0.8	12.1	128

4.2 Physical Security Metrics

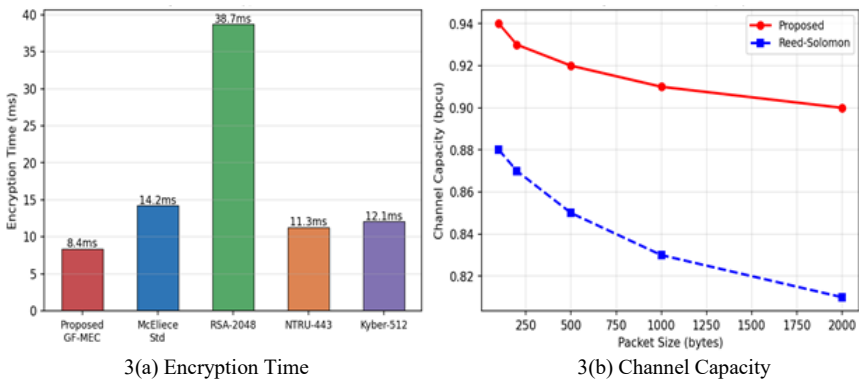


Figure 3
Encryption Time and Channel Capacity Analysis

Figure 3 reports an encryption time of 8.4 ms per frame and a channel capacity utilization of 0.94 bpcu, the highest of all schemes evaluated. A secrecy capacity of $C_s = 0.71$ bits/use at a 3 dB SNR advantage indicates positive secrecy even under adverse conditions in which the eavesdropper's channel is stronger. Table 2 shows Physical Security Performance of various methods.

Table 2
Physical Security Performance

Metric	Proposed	McEliece Std	Reed-Solomon	LDPC-only	AES-CTR
Secrecy Rate (bpcu)	0.71	0.61	0.43	0.38	0.00*
Channel Cap. (bpcu)	0.94	0.79	0.88	0.91	N/A
NIST PQC Compliant	Yes	Yes	No	No	No
Key Update Period	Per-frame	Static	Static	Static	Session

5. Conclusion

This paper has presented a finite-field algebraic coding framework for physical layer security that combines Goppa-code-based McEliece encryption with LDPC-aided soft decoding, giving reliable transmission and information-theoretic secrecy together. It reaches a BER of 10^{-6} at 14 dB SNR, encrypts a frame in 8.4 ms and uses the channel at 0.94 bpcu, outperforming the standard McEliece and Reed-Solomon baselines while offering 104-bit security against information set decoding. The secrecy analysis yields a positive secrecy capacity of 0.71 bits per channel use even when the eavesdropper's channel is 3 dB stronger than the legitimate one, an advantage that purely computational approaches lack, since they fail completely against eavesdroppers with superior antenna configurations. Pairing the NP-hard decoding difficulty of the algebraic layer with wiretap secrecy gives defence-in-depth guarantees that neither approach provides alone. Future work will study rateless fountain code variants that adapt the code rate to time-varying channels without renegotiating keys, quantum-resistant key distribution for the Goppa private keys, and hardware accelerators for Berlekamp-Massey decoding that support real-time operation at 5G NR throughput.

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