

Unveiling strong domination number on some graphs

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Abstract

Let $G = (V, E)$ be the given graph, a subset $D \subseteq V$ is defined as the dominating set if every vertices in the set $(V-D)$ is adjacent to at least one vertex in the set D . Then the set D is known as the Dominating set. Let $xy \in E$ be an edge in Graph G then x strongly dominates y if $\deg x \geq \deg y$. Set D is known as a strong dominating set of G if every vertex of $(V-D)$ is strongly dominated by some element in set D . Strong domination number is the minimum cardinality of strong dominating set it is represented by $\gamma_s(G)$. In this paper we investigated the strong domination number of crown graph, sunflower graph, vertex switching of gear graph, vertex switching of wheel graph. Applications of strong domination numbers extend to security frameworks and alert-based systems, emphasizing their importance in net- work reliability and control.

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1. Introduction

Let G be a simple and connected graph, where $V(G)$ and $E(G)$ represent the vertex set and edge set of G , respectively. A dominating set in a graph G is a subset $A \subseteq V(G)$ such that every vertex in $V(G) \setminus A$ is adjacent to at least one vertex in A . The minimum cardinality of a dominating set is called the domination number of G , denoted by $\gamma(G)$. A subset $U \subseteq V(G)$ is called a strong dominating set if every vertex $v \in V(G) \setminus U$ is strongly dominated by at

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least one vertex $u \in U$, where u is strongly adjacent to v , that is, $\deg(u) \geq \deg(v)$. The minimum cardinality of a strong dominating set is called the strong domination number of G , denoted by $\gamma_s(G)$.

Domination theory is an important area of graph theory that has attracted considerable attention due to its theoretical significance and applications in communication networks, social network analysis, linear algebra, and related fields. The study of domination parameters provides valuable insights into the structural properties of graphs and the identification of efficient subsets of vertices for monitoring, control, and coverage. Several studies have investigated different domination parameters and their relationships in various classes of graphs.

The relationship between weak and strong domination numbers was investigated by Boutrig and Chellali [5], providing important insights into the mathematical properties of these domination parameters. Durgun and Lokcu [6] studied the strong domination number of some graphs, including paths and cycles. Further contributions by Karkar and Vaidya [11] focused on the strong domination number of path-related graphs and the strong domination number of graphs in general [12], while Pratap [13] examined the strong domination number of jump graphs.

In addition to strong domination, several researchers have explored other variants of domination. Rahbani, Abdollahzadeh, and Sadeghi [1] studied locating Roman domination edge-critical graphs, highlighting the role of edge-critical conditions in Roman domination. Bouchou and Omar [4] investigated double Roman domination in graphs, introducing additional structural considerations in domination theory. Alishahi and Mojdeh [3] studied global restrained Roman domination in graphs, incorporating global constraints into domination conditions. Alikhani, Ghanbari, and Zaherifar [2] investigated strong dominating sets of graphs, further extending research in this area.

The study of domination and related graph parameters has also been extended to vertex switching, bondage numbers, and other graph structures. Ghodasara and Rokad [7] examined cordial labeling in the context of vertex switching of special graphs. Hemalatha and Jeyanthi [8] investigated the domination number and bondage number for some families of graphs. Jayachandran and Sugumaran [9] contributed to the study of domination numbers of some graphs. These investigations demonstrate the diversity of domination-related problems and their importance in understanding graph structures.

Edge-related domination parameters have also received attention in the literature. Jony and Parthipan [10] studied the isolated edge detour domination number of some standard graphs, thereby contributing to the development of edge-based domination concepts. Sweetly and Joseph [15] investigated the radio number of W_n^2 graphs, while Sweetly and Joseph [16] studied the radio number of graphs of diameter three. Although radio domination parameters differ from

classical and strong domination, these studies illustrate the broader development of graph parameters associated with distance, coverage, and structural constraints.

Recent research has continued to explore domination parameters and their applications. Leel et al. [17] studied the domination number in the context of some new graphs. Gupta, Gupta, and Srivastav [18] investigated the domination number for some STAR-related graphs. Sigarreta [14] examined total domination on some graph operators, further enriching the study of domination in graph theory.

The strong domination number has practical significance in several fields, particularly communication networks, social network analysis, and wireless sensor networks. In communication networks, strong domination parameters can be used to model and analyse the efficiency and reliability of network structures, supporting the identification of influential or monitoring nodes. In social network analysis, domination concepts can be applied to identify a set of individuals capable of influencing or controlling other individuals in a network. Similarly, in wireless sensor networks, strong domination can provide a mathematical framework for studying sensor placement, monitoring, and network coverage while considering the relative importance or connectivity of nodes.

Motivated by the above studies, the present work focuses on the investigation of strong domination numbers for selected classes of graphs. The objective is to derive structural results and mathematical expressions for the strong domination number of the graphs under consideration, thereby contributing to the existing literature on domination theory.

2. Definitions

Definition 1: In graph G , switching a vertex v means first eliminating all edges connected to v , and then linking v to all vertices that were not connected to it before. The graph formed after this transformation is written as G .

Definition 2: In a graph G with an edge uv in the edge set E , vertex u is considered to strongly dominate vertex v if the degree of u (denoted as $\deg(u)$) is greater than or equal to the degree of v (denoted as $\deg(v)$) i.e. $\deg(u) \geq \deg(v)$. Each vertex in V naturally dominate itself. A set S of $V(G)$ is called a strong dominating set if every vertex which is not present in S is strongly dominated by some vertex which is present in S . The smallest number of vertices forming such a set is the strong domination number, denoted $\gamma_{sd}(G)$.

Definition 3: A crown graph consists of $2n$ vertices and $n(n-1)$ edges with two sets of vertices $x_1, x_2, \dots, x_n$ and $y_1, y_2, \dots, y_n$ and with an edge from x_i to y_j whenever $i \neq j$. It is denoted by S^n .

Definition 4: A sunflower graph consists of odd number of vertices $2n+1 \forall n \geq 3$, say $a_0, a_1, a_2, \dots, a_n; b_1, b_2, \dots, b_n$ and $4n$ edges. Degree of the central vertex is n i.e.

$\deg(a_0) = n$, the vertices $a_1, a_2, \dots, a_n$ have degree 5 and the vertices $b_1, b_2, \dots, b_n$ have degree 2. It is denoted by Sf_n .

Definition 5: A gear graph can be described as a modified wheel graph where an additional vertex is inserted between every pair of neighboring graph vertices along the outer cycle. The resulting gear graph G_n has $2n+1$ vertices and $3n$ edges.

Definition 6: A wheel graph is created by linking a sole universal vertex to each vertex in a cycle. It is denoted by W_n with $n+1 \forall n \geq 3$ vertices and $2(n-1)$ edges.

3. Main Results

Corollary 1: For the crown graph S_n^o , $\gamma_{st}(S_n^o)$ is 2 $\forall n \geq 3$

Proof: Crown graph is denoted by S^o with $2n$ vertices and $n(n-1)$ edges. Through the construction of S_n^o , there are two vertices with the highest degree that collectively cover all other vertices in S_n^o , there by having the highest degree in S_n^o . So $\gamma_{st}(S_n^o)$ is 2 $\forall n \geq 3$.

Example:

- 1) SDN of Crown graph S^o is 2 as illustrated below in Figure 1.

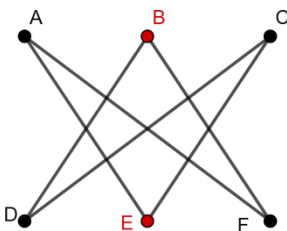


Figure 1
Crown graph S^o

The red-colored vertices in Figure 1 denote the required set.

- 2) SDN of Crown graph S^o is 2 as shown in the Figure 2.

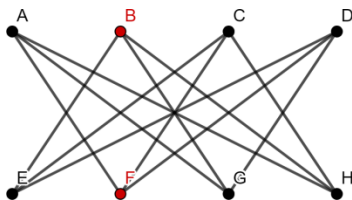


Figure 2
Crown graph S^o

The red-colored vertices in Figure 2 denote the required set.

Theorem 1: Strong Domination number of Sunflower graph Sf_n exists $\forall n \geq 3$

Proof: Sunflower graph consists of an odd number of vertices i.e. $2n+1$ vertices and $4n$ edges. Let a_o be the apex vertex of Sunflower graph $Sf_n \forall n \geq 3$. Let $a_1, a_2, a_3, \dots, a_n$ be rim vertices of the wheel W_n in Sf_n . Let $b_1, b_2, b_3, \dots, b_n$ be the outer vertices of Sf_n with degree 2 each. The rim vertices of the wheel W_n in Sf_n contribute to the strong dominating set with the apex vertex.

When n is odd in $Sf_n, \forall n \geq 3$ then strong dominating set consists of $a_0, a_1, a_3, a_5, \dots, a_{n-2}, a_{n-1}$ vertices.

When n is even in $Sf_n, \forall n \geq 3$ then strong dominating set consists of $a_0, a_1, a_3, a_5, \dots, a_{n-1}$ vertices. Then SDN is given as:

Condition I: $\gamma_{st}(Sf_n) = 2$ for $n=3,4$ **Condition II:** $\gamma_{st}(Sf_n) = 3$ for $n=5$ **Condition III:** $\gamma_{st}(Sf_n) = 4$ for $n=6$

Condition IV: When n is odd, $\gamma_{st}(Sf_n)$ is $\frac{n+3}{2}, \forall n \geq 7$

Condition V: When n is even, $\gamma_{st}(Sf_n)$ is $\frac{n}{2} + 1, \forall n \geq 8$

Example

1) SDN of Sunflower graph Sf_5 is 3 as illustrated below in Figure 3.

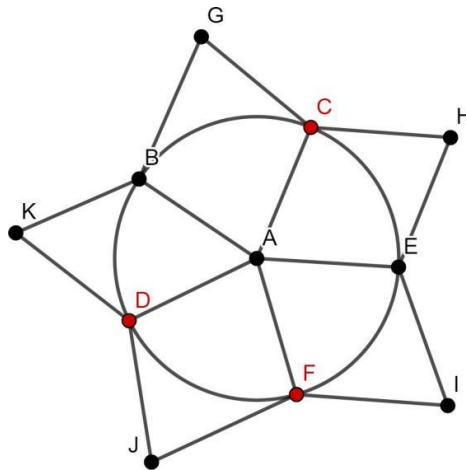


Figure 3
Sunflower graph Sf_5

The red-colored vertices in Figure 3 denote the required set.

2) SDN of Sunflower graph Sf_8 is 5 as shown in the Figure 4.

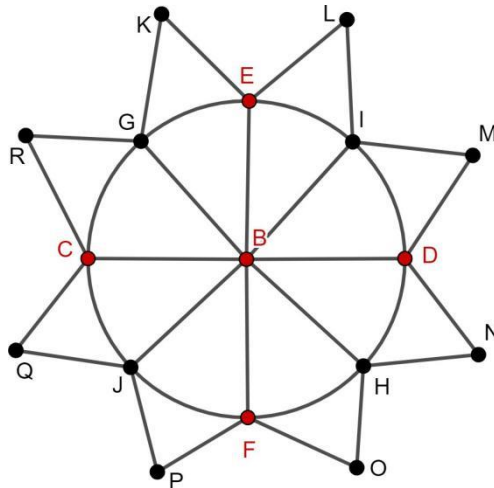


Figure 4
Sunflower graph Sf_8

The red-colored vertices in Figure 4 denote the required set.

Theorem 2: Strong domination number exists for the graph obtained by vertex switching of any rim vertex of degree 3 and degree 2 of gear graph.

Proof: Gear graph is denoted by G_n with $2n+1$ vertices $\forall n \geq 3$ and $3n$ edges. Let a_0 be the apex vertex and $\sum_{i=1}^n a_i$ be other vertices of the gear graph. Then there arises two cases

- 1) When i is odd then $\deg(a_i)$ is 3
- 2) When i is even then $\deg(a_i)$ is 2

The graph resulting from the vertex switching of any rim vertex of degree 3 is isomorphic to each other. Similarly the graph obtained by vertex switching of any rim vertex of degree 2 is isomorphic to each other. Let $(G_n)_{a_i}$ represent the vertex switching of G_n concerning the vertex a_i , where i takes values of 1 and 2.

Condition I: SDN for the graph resulting from the vertex switching of any rim vertex of degree 3 of the gear graph is given by:

- i) $\gamma_{st}((G_n)_{a_i}) = 3$ for $n = 3, 4, 5$ where $\deg(a_i)$ is 3
- ii) $\gamma_{st}((G_n)_{a_i}) = 4 \forall n \geq 6$ where $\deg(a_i)$ is 3

Condition II: SDN for the graph resulting from the vertex switching of any rim vertex of degree 2 of the gear graph is given by:

- i) $\gamma_{st}((G_n)_{a_i}) = 2 \quad \forall n \geq 3$ where $\deg(a_i)$ is 2

Example:

- 1) SDN of the graph obtained by vertex switching of Gear graph G_6 with respect to rim vertex B i.e. $\gamma_{st}(G_6)_B$ of degree 3 is 4 as shown in the Figure 5.

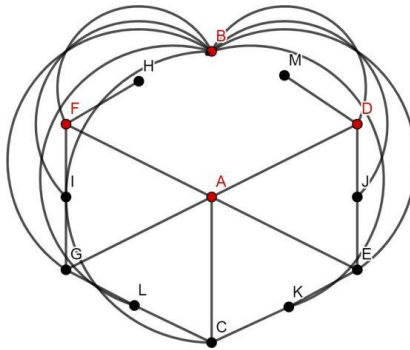


Figure 5
Vertex switching of Gear graph $(G_6)_B$

The red-colored vertices in Figure 5 denote the required set.

- 2) SDN of the graph obtained by vertex switching of Gear graph G_7 with respect to rim vertex I i.e. $\gamma_{st}(G_7)_I$ of degree 2 is 2 as shown in the Figure 6.

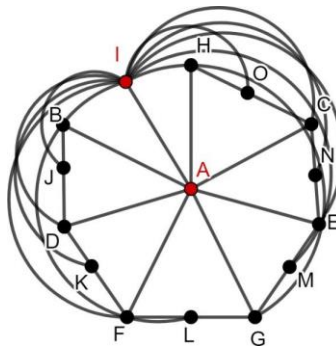


Figure 6
Vertex switching of Gear graph $(G_7)_I$

The red-colored vertices in Figure 6 denote the required set.

Theorem 3: *The strong domination number is defined for the graph obtained through vertex switching at any rim vertex of the wheel graph.*

Proof: Wheel graph is denoted by W_n with $n+1$ vertices $\forall n \geq 4$ and $2(n-1)$ edges. Let a_0 be the apex vertex with degree $n-1$ and $a_1, a_2, a_3, \dots, a_n$ be rim vertices of the wheel graph with degree

- Let $(W_n)_{ai}$ represent the vertex switching of W_n concerning the vertex a_i , where $i \in \mathbb{N}$. Therefore, the strong domination number for the graph generated through vertex switching of any rim vertex in the wheel graph can be represented as:

Condition I: $\gamma_{st}((W_n)_{ai}) = 1$ when $n = 5$

Condition II: $\gamma_{st}((W_n)_{ai}) = 2 \forall n \geq 6$

Example

- SDN of the graph obtained by vertex switching of Wheel graph W_7 with respect to rim vertex G i.e. $\gamma_{st}(W_7)_G$ is 2 as shown in the Figure 7.

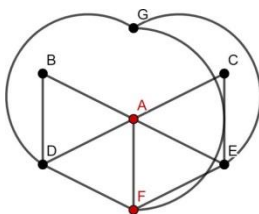


Figure 7
Vertex switching of Wheel graph $(W_7)_G$

The red-colored vertices in Figure 7 denote the required set.

- SDN of the graph obtained by vertex switching of Wheel graph W_{10} with respect to rim vertex B i.e. $\gamma_{st}(W_{10})_B$ is 2 as shown in the Figure 8.

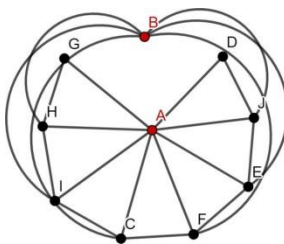


Figure 8
Vertex switching of Wheel graph $(W_{10})_B$

The red-colored vertices in Figure 8 denote the required set.

4. Concluding Remarks

Strong Domination in graphs establishes a connection between dominating sets and the vertex degrees. The literature contains strong domination numbers for certain standard graphs, and our research focuses on determining the strong domination number for graphs like crown graph, sun-flower graph, vertex switching of gear graph and vertex switching of wheel graph.

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