

Sum 3-equitable labeling of fan graphs : Theoretical results and algorithmic approach

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Abstract

The study explores the use of a MATLAB program to prove the theorem that fan graphs admit sum 3-equitable labeling. The labeling scheme is defined as a bijective transformation, which assigns integer values to the vertices of the graph. These values are then used to determine the edge values, which are computed on the sum of the values of the vertices the edge joins modulo three. This classification primarily requires the preservation of equitability, which means the edges labeled via the transformation must almost evenly lie in the three classes, such that the absolute value of edges in any two classes differs by no more than one.

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I. Introduction

Graph labeling is an important area of graph theory that deals with the assignment of labels or weights to the vertices, edges, or both vertices and edges of a graph. Since its development, graph labeling has found applications in several areas of science and technology, including crystallography, circuit design, communication networks, cryptography, and coding theory [1]. The subject has attracted considerable attention because of its wide range of theoretical and practical applications. A comprehensive account of various graph-labeling techniques and their developments can be found in the dynamic survey presented by Gallian [12].

Among the different types of graph labeling, cordial labeling was introduced as a weaker version of graceful and harmonious labeling by Cahit [6]. Subsequently, Cahit [2] introduced the concept of 3-equitable labeling, which provided a new direction for studying graph-labeling problems based on the number 3. Since then, several extensions and variants of 3-equitable labeling have been investigated. One such extension is 3-equitable total labeling, in which labels are assigned to both the vertices and edges of a graph subject to specified balancing conditions [3]. These developments have significantly expanded the scope of equitable labeling and have motivated researchers to investigate its applicability to different classes of graphs.

Several modifications of equitable labeling have also been proposed by incorporating different mathematical operations and structural properties. In particular, square-difference 3-equitable labeling has been studied for certain classes of graphs [5]. Similarly, sum 3-equitable labeling has been investigated for various graphs, providing further extensions of the conventional 3-equitable labeling concept [4]. The incorporation of prime-related conditions has led to another interesting variant known as 3-equitable prime cordial labeling, which has been studied for fundamental graph structures such as paths, cycles, and stars [7]. More recently, research on graph labeling has continued to explore specialized labeling classes, including super graceful labeling of co-maximal subgroup graphs of finite cyclic groups [13]. Such studies demonstrate the continuing expansion of graph labeling toward specialized graph structures and algebraic constructions.

Graph labeling has also been studied from a computational perspective. Computational approaches are particularly useful for verifying labeling conditions, identifying patterns, and reducing the possibility of human error in complex graph-theoretic investigations. MATLAB-based approaches have been successfully employed to investigate graph invariants and labeling-related problems. For example, MATLAB has been used to study the energy of simple graphs [8] and to develop algorithms for obtaining new results concerning graph energy [9]. Furthermore, MATLAB coding has been applied to the middle graph of cycles and related graphs in the context of sum-divisor cordial labeling [10]. These studies demonstrate the usefulness of computational techniques as an effective complement to theoretical graph-labeling investigations.

Recent developments in graph theory and combinatorics have further emphasized the study of structural properties of graphs and their associated labeling schemes. In particular, investigations involving line graph constructions and combinatorial structures for network optimization illustrate the broader applicability of graph-theoretic methods to computational and optimization problems [14]. Such developments provide additional motivation for employing algorithmic techniques in the study of graph-labeling problems.

The present study focuses specifically on **sum 3-equitable labeling of fan graphs**. Fan graphs constitute an important class of graphs obtained from a path by joining an additional vertex to every vertex of the path. The study of sum 3-equitable labeling for different graph structures provides motivation for examining this labeling concept in the case of fan graphs [4]. The objective of the present work is to computationally verify the theorem concerning the sum 3-equitable labeling of fan graphs by developing and implementing a MATLAB algorithm. The computational approach enables the labeling conditions to be systematically examined for different orders of fan graphs, thereby reducing manual effort and improving the accuracy and efficiency of the verification process.

The mathematical terminology and fundamental graph-theoretic concepts used in this work are based on the standard framework of graph theory presented by Harary [11]. Throughout this paper, all graphs under consideration are assumed to be finite, simple, connected, and undirected unless stated otherwise. The proposed MATLAB-based computational procedure is intended to provide an efficient mechanism for verifying the required labeling conditions and to establish the validity of the corresponding theorem for the considered class of fan graphs.

II. Terminology and Notations

A. Definition: Ternary Vertex Labeling

In ternary vertex labeling, every vertex is allocated labels from one of the three distinctive integers. These labels can be considered as the remainders obtained while dividing by 3. This is, in fact, using modular arithmetic. Usually, such vertex labeling provides the foundation for the calculation of edge labels, which is carried out by adding the two adjacent vertex labels in the ternary system.

B. Definition: 3-Equitable Labeling

A graph is said to exhibit 3-equitable labeling scheme when the assigned vertex labels meet the conditions:

- a. $|v_{\alpha}(n) - v_{\alpha}(l)| \leq 1$, and
- b. $|e_{\alpha}(n) - e_{\alpha}(l)| \leq 1$ for all $0 \leq n, l \leq 2$

C. Definition: Sum 3-Equitable Labeling

Edge labeling in the said pattern is given by:

$$\alpha^*(qr) = \begin{cases} 0 & \text{if } \alpha(q) + \alpha(r) \equiv 0 \pmod{3} \\ 1 & \text{if } \alpha(q) + \alpha(r) \equiv 1 \pmod{3} \\ 2 & \text{if } \alpha(q) + \alpha(r) \equiv 2 \pmod{3} \end{cases}$$

and $|e_{\alpha^*}(n) - e_{\alpha^*}(l)| \leq 1$ for all $0 \leq n, l \leq 2$. Sum 3-equitable graphs are those that preserve the sum 3-equitable labeling pattern.

D. Definition: Fan Graph

A fan graph is a graph in which a single central vertex is connected to every vertex in a path graph.

III. Results & Algorithm

To construct a MATLAB program for the theorem “The fan graph preserves sum 3-equitable labeling.”

Open the MATLAB editor window and type the following command. Save the function file as *fangraph*. When the program runs, we get the fan graph and the result on whether the given fan graph is a sum 3-equitable graph or not.

```
function Result=fangraph(m)
z=0;
o=0;
t=0;
s=ones(1,m-1);
r=2:m;
G=graph(s,r);
i=2;
while i<m
G=addege(G,i,i+1);
i=i+1;
end
Y=plot(G);
title(["Fan Graph for m=", num2str(m)])
%Vertex labeling
a=[];
j=1;
while j<m+1
labelnode(Y,j,j-1);
a(j)=j-1;
j=j+1;
end
%Edge Labeling
for l=2:m
if rem(a(1)+a(l),3)==0
labeledge(Y,1,l,0);
z=z+1;
```

```

elseif rem(a(1)+a(l),3)==1
labeledge(Y,1,l,1)
o=o+1;
elseif rem(a(1)+a(l),3)==2
labeledge(Y,1,l,2)
t=t+1;
end
end
%Edge Labeling
for n=2:m-1
if rem(a(n)+a(n+1),3)==0
labeledge(Y,n,n+1,0);
z=z+1;
elseif rem(a(n)+a(n+1),3)==1
labeledge(Y,n,n+1,1)
o=o+1;
elseif rem(a(n)+a(n+1),3)==2
labeledge(Y,n,n+1,2)
t=t+1;
end
end
fprintf("The number of edges with label '0'=%i\n",z)
fprintf("The number of edges with label '1'=%i\n",o)
fprintf("The number of edges with label '2'=%i\n",t)
%Checking the conditions
if abs(z-o)<=1 & abs(o-t)<=1 & abs(t-z)<=1
Result='The given fan graph is a sum 3-equitable graph.';
else
Result='The given fan graph is not a sum 3-equitable graph.';
end
end

```

Example 1: For $m=15$

Taking $m=15$

In the command window, write *fangraph(15)* and press enter. The result will be displayed as shown in below Figure 1 f.

```

>> fangraph(15)
The number of edges with label '0'=9
The number of edges with label '1'=9
The number of edges with label '2'=9

ans =

```

'The given fan graph is a sum 3-equitable graph.'

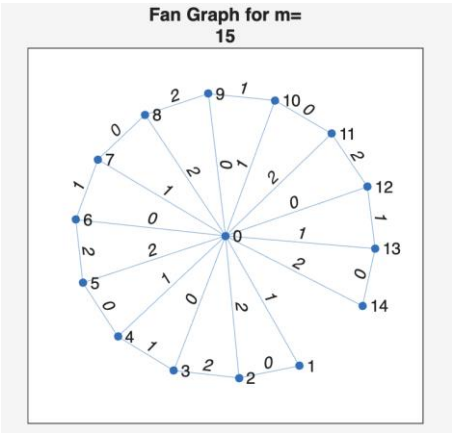


Figure 1
Fan Graph for $m = 15$

Example 2: For $m = 16$

Taking $m = 16$

In the command window, write `fan graph(16)` and press enter. The result will be displayed as shown in Figure 2.

```
>> fan graph(16)
```

The number of edges with label '0'=10

The number of edges with label '1'=9

The number of edges with label '2'=10

ans =

'The given fan graph is a sum 3-equitable graph.'

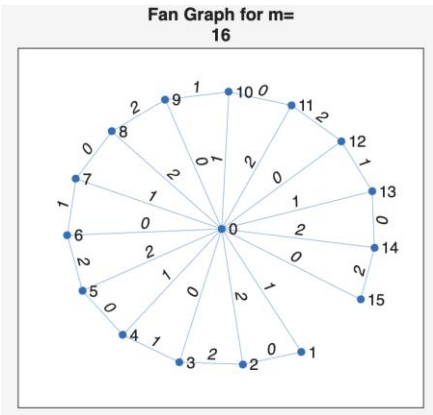


Figure 2
Fan Graph for $m=16$

IV. Conclusion

The study relied on elaborate MATLAB algorithms, which confirmed the theorem that fan graphs preserve the sum 3-equivalent labeling. The study assigned values to the vertices and edges within different fan graphs, which is proof that the different theoretical models are well incorporated. The results fit the predicted modulo three summations, which confirms that this particular labeling is correctly used in fan graphs with the mentioned modulo classes.

Additionally, the computer simulations allowed the graphs of the distributions of the edge labels to be obtained, and this helped to confirm the fairness and equality in the distribution of the labels. This clearly shows that the more vertices the graph has, the more equal the distribution of the edge labels is. Finally, the MATLAB program fully confirms the theoretical analysis and shows that sum 3-equitable labeling is valid computationally.

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