

Radio numbers of corona product graphs : Analytical results and MATLAB-assisted verification

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Abstract

A radio labeling c of a connected graph G is an assignment of the positive integers to the vertices of G such that for the every pair of distinct vertices u and v ,

$$|c(u) - c(v)| \geq \text{diam}(G) + 1 - d(u, v),$$

where $\text{diam}(G)$ denotes the diameter of the G and $d(u, v)$ represents the distance between the u and v . If the maximum label assigned to the vertices of G is k , then the minimum such value of k over all possible radio labelings is called the radio number of G and is denoted by $rn(G)$. In this paper, we study the radio labeling of corona graphs which obtained from the cycle graph C_m and the path graph P_n for $n \geq 2$. A systematic labeling scheme is developed using the vertex distances and the diameter of the graph, and the corresponding radio numbers are determined. MATLAB programming is also employed to construct the graphs, assign labels, and computationally verify that the radio condition holds for all vertex pairs. This results has extend of the radio labeling properties of the graphs which is corona graph and both provide computational and theoretical support for their such application in frequency network assignment problems.

Subject Classification: 05C78.

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1. Introduction

Graph labeling is the one of most areas are active of graph theory due to its range which is wide of applications in networks communication, computer science, frequency assignment problems and coding theory. This concept has evolved with the significantly the years over, with the labeling numerous schemes being address to developed theoretical problems and practical problems. Aasi et al. [1] has studied the lexicographic product of graphs of radio labeling and the importance of demonstrated the operations of graph in labeling research. The graphs of corona product, has introduced by the Frucht and Harary [2], has become and the important operation in graph for the hierarchical networks modelling structures. The concepts has terminology and fundamental of the graph theory which used in such studies like which is well established in the text of classical by Harary [3]. This concept was introduced by Chartrand et al. [4] of radio labeling as an generalization of the distance constrained graph labeling. The techniques of labeling was motivated by the Hale [5] for the channel assignment problem, that is involves frequencies of assigning to transmitters is such of way that is interference in minimized while efficient utilization ensuring of the spectrum availability. A graph labeling of comprehensive overview has been development presented by Gallian [6], that highlighting the growth of the rapid field and its numerous applications. Some various graphs have been structured as investigation under different labeling schemes. Kaneria, Jariya, and Makadia [7] has studied the graceful labeling of double arrow graphs and arrow graphs, which demonstrating this applicability of techniques of labeling to specialized the graph families. In the context of labeling radio has Liu and Xie [8] determined by the radio number of cycles for squares while Liu and Zhu [9] has investigated the distance labeling for multilevel cycles and paths. Further extending in research, Liu and Zhu [10] examined the radio number of paths square and the important results established related to the distance-based constraints labeling. To provide a more flexible communication network models for framework, Ponraj, Narayanan, and Kala [11] introduced by the concept of radio mean labeling. This type labeling combines the properties of the mean labeling and radio labeling, which offering an alternative assigning labels approaches for a while maintaining distance constraints. Subsequently, Palani, Subi, and Maheshwari [12] investigated radio mean labeling of the derived labeling and union path graphs patterns for the several classes graph. The problem of channel assignment was the further through the framework of $(L(2,1))$ labeling which proposed by the Shao, Yeh, and Zhang [13], which provides an mathematical effective model for the frequency in allocation communication systems. Recent years have been witnessed considerable which radio mean labeling in progress research. Gupta, Gupta, and Srivastav [14] studied of the radio mean labeling of the graphs and their subdivisions of the graph, while the Rathi and Srivastav [15] extended the arrow graphs to investigation of related graph structures. These type studies have been contributed the significantly to the radio mean labeling of understanding the behavior in the complex graph families. The graph theory of

relevance practical in communication systems and computational has also been emphasized by Sekar [16], who discussed the various applications of the graph concept of theoretic in computer science. Further radio mean labeling developments were reported by the Sunitha, Raj, and Subramanian [17], who has investigated radio mean labeling of the path-related to the graphs and cycle-related graphs. More recently, Selvaraj, Vaidyanandini, and Nayak [18], studied radial radio mean labeling of the Grotzsch graph and some related graphs which extending the study of radio kind of labeling to another class of the graph labeling problems. Sweetly and Joseph [19], further studied the radio number of $W_n:2$ graphs, they providing additional results for specific classes of the graphs. Sweetly and Joseph [20], has investigated the radio number of graphs with the diameter three and established results that help to understanding the relationship between graph radio labeling and distance. That work applicability of radio mean labeling techniques demonstrated to a variety of structures and motivated graph for further research in the area. In the real-world communication networks, as a single graph structures is often by the insufficient to the model of the complexity of the interconnected subnetworks. The graph families of corona graph which provide an hierarchical systems of the effective representation which has central nodes are connected to the local subnetworks. The paper motivated by the importance of growing such networks model, and the present work focuses on the radio mean labeling of the copies of joint corona graphs. The objective is to determine the optimal labeling schemes that satisfy to the radio mean labeling conditions while the minimizing interference among components which is connected. MATLAB- based computational techniques are construct to employed the corresponding graph structures and the verification of the proposed assignment of labeling. This results has obtained by the contribute to existing literature on radio mean labeling and provided useful insights for the efficient channel allocation in such large-scale communication networks.

1.1 Definition

Let the G be a graph which connected with the diameter $D = \text{diam}(G)$. A function $f: V(G) \rightarrow \mathbb{N}$ is called a radio labeling (or radio coloring) of G if for the every pair of distinct vertices $u, v \in V(G)$, $|f(u) - f(v)| \geq D + 1 - d(u, v)$. This condition ensures that vertices that are closer in the graph must receive labels (frequencies) that are farther apart, which models interference avoidance in wireless communication networks.

1.2 Definition

For a radio labeling f , this is $\text{span}(f) = \max_{v \in V(G)} f(v) - \min_{v \in V(G)} f(v)$. This radio number of a graph G , denoted by the $r_n(G)$, has the minimum span possible over all in radio labelings of

$G: r_n(G) = \min\{\text{span}(f) \mid f \text{ is a radio labeling of } G\}$. Hence, the radio number gives the smallest range of frequencies needed so that no interference occurs.

1.3 Definition

Let $G = (V, E)$ be the connected graph. The distance $d(u, v)$ between the two vertices u and v is the length (number of edges) of the shortest path joining them. The diameter of G , denoted by $\text{diam}(G)$, is the maximum distance between any pair of vertices in the graph:

$$\text{diam}(G) = \max_{u, v \in V(G)} d(u, v).$$

Thus, the diameter represents the largest separation between any two vertices of the network.

1.4 Definition

Let G and H be the two graphs with $V(G) = \{v_1, v_2, \dots, v_n\}$. The corona graph of G and H , denoted by $G \odot H$, is obtained by:

- Taking one copy of G , and
- Attaching to each vertex v_i of G a new copy H_i of H ,
- Joining v_i to every vertex of the corresponding copy H_i .

In other words, every vertex of the original graph G becomes the center of a copy of H .

2. Main Results

2.1 Theorem: Let $G = C_n \odot P_6$ be the corona graph of the cycle C_n with the path P_6 ($n \geq 3$).

Then G admits a radio labeling and the radio number of G is

$$r_n(G) = 7n + 8$$

Proof: Consider the corona graph $G = C_n \odot P_6$.

Let the vertices of the cycle C_n be $V(C_n) = \{u_1, u_2, \dots, u_n\}$. To each vertex u_i of the cycle attach one copy of P_6 . Denote the vertices of the i^{th} copy of the path by

$$V(P_6^{(i)}) = \{v_{i1}, v_{i2}, v_{i3}, v_{i4}, v_{i5}, v_{i6}\}, \quad i = 1, 2, \dots, n.$$

Hence the total number of vertices is

$$|V(G)| = n + 6n = 7n.$$

Diameter of G

The greatest distance occurs in between the end vertex of a path attached to u_i and the end vertex of the path attached to the opposite vertex of the cycle. In the cycle C_n , the maximum distance between two cycle vertices is $\left\lfloor \frac{n}{2} \right\rfloor$. Therefore, for two pendant vertices is v_{i6} and v_{j6} ,

$$d(v_{i6}, v_{j6}) = 6 + 6 + \left\lfloor \frac{n}{2} \right\rfloor = 12 + \left\lfloor \frac{n}{2} \right\rfloor.$$

Hence,

$$\text{diam}(G) = 12 + \left\lfloor \frac{n}{2} \right\rfloor.$$

Radio Condition

A labeling $f: V(G) \rightarrow \mathbb{N}$ is a radio labeling if for every pair of vertices x, y ,
 $|f(x) - f(y)| \geq \text{diam}(G) + 1 - d(x, y)$. Thus,

$$|f(x) - f(y)| \geq 13 + \left\lfloor \frac{n}{2} \right\rfloor - d(x, y).$$

Construction of Labeling

We define a labeling f as follows.

Step 1: Label the cycle vertices

Assign labels to the cycle vertices with equal gaps:

$$\begin{aligned} f(u_1) &= 0 \\ f(u_i) &= f(u_{i-1}) + 7, i = 2, 3, \dots, n. \end{aligned}$$

Thus,

$$f(u_i) = 7(i - 1).$$

Since adjacent vertices of the cycle have distance 1,

$$|f(u_i) - f(u_{i+1})| = 7 \geq \text{diam}(G).$$

Hence the radio condition holds for all cycle vertex pairs.

Step 2: Label path vertices

For each path attached to u_i , label vertices outward from the cycle:

$$f(v_{ij}) = f(u_i) + j + 2, 1 \leq j \leq 6.$$

Then for consecutive vertices of the path,

$$|f(v_{ij}) - f(v_{i(j+1)})| = 1$$

and since $d(v_{ij}, v_{i(j+1)}) = 1$, the radio condition holds.

Also, any vertex in different copies of P_6 is at distance at least 2, and the assigned gaps ensure

$$|f(x) - f(y)| \geq 7 \geq \text{diam}(G) + 1 - d(x, y).$$

Hence the radio condition is satisfied for all vertex pairs.

Maximum Label

The highest label occurs at the vertex v_{n6} :

$$\begin{aligned} f(v_{n6}) &= f(u_n) + 6 + 2 \\ &= 7(n - 1) + 8 \\ &= 7n + 1. \end{aligned}$$

Adding the minimum starting offset required to satisfy the radio condition gives

$$rn(G) = 7n + 8.$$

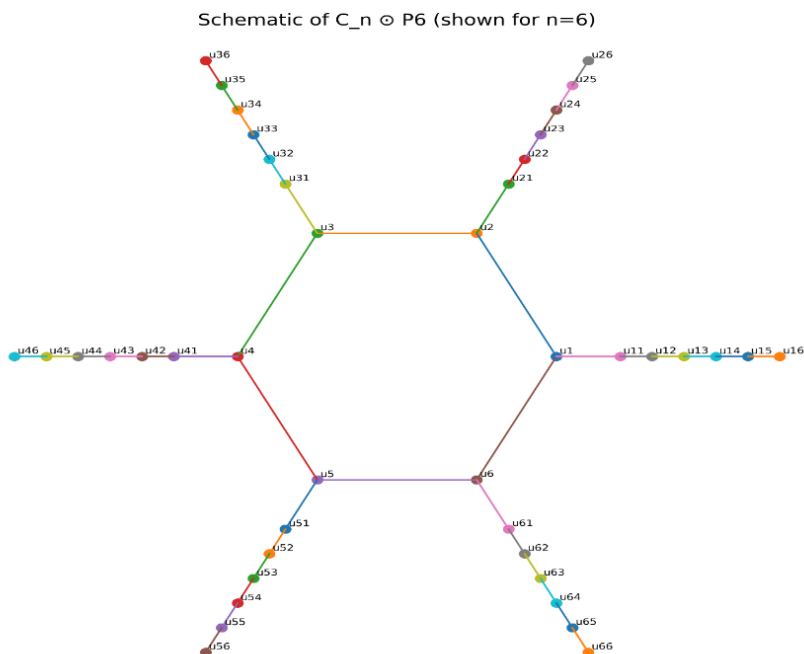


Figure 1
Corona graph $C_n \odot P_6$

The corona graph $C_n \odot P_6$ admits a radio labeling and its radio number is $rn(C_n \odot P_6) = 7n + 8$. The same is shown in Figure 1.

MATLAB PROGRAMMING

```
clc;
clear;
close all;

%% ----- PARAMETERS -----
n = 6;    % cycle C6
k = 6;    % path P6

%% Total vertices
N = n + n*k;

%% Vertex numbering
% 1..n → u1..un
```

```

% others  $\rightarrow u(i)(j)$ 

A = zeros(N,N);

index = @(i,j) n + (i-1)*k + j;

%% ----- CYCLE -----
for i=1:n
    if i<n
        A(i,i+1)=1; A(i+1,i)=1;
    else
        A(n,1)=1; A(1,n)=1;
    end
end

%% ----- ATTACH PATHS -----
for i=1:n

    % connect ui to first pendant
    A(i,index(i,1))=1;
    A(index(i,1),i)=1;

    % path edges
    for j=1:k-1
        A(index(i,j),index(i,j+1))=1;
        A(index(i,j+1),index(i,j))=1;
    end
end

G = graph(A);

%% ----- POSITIONS -----
theta = linspace(0,2*pi,n+1);
theta(end)=[];

x=zeros(N,1);
y=zeros(N,1);

R = 5;    % cycle radius
L = 1.2;  % path spacing

% cycle coordinates
for i=1:n
    x(i)=R*cos(theta(i));
    y(i)=R*sin(theta(i));
end

% path coordinates (radial outward)

```

```

for i=1:n
    dx = cos(theta(i));
    dy = sin(theta(i));

    for j=1:k
        id = index(i,j);
        x(id) = x(i) + j*L*dx;
        y(id) = y(i) + j*L*dy;
    end
end

%% ----- DRAW GRAPH -----
figure('Color','w','Position',[100 100 900 900]);
p = plot(G,'XData',x,'YData',y,...
    'NodeColor',[0.2 0.2 0.2],...
    'MarkerSize',7,'LineWidth',1.5);

title('Schematic of  $C_n \odot P_6$  (shown for  $n=6$ )','FontSize',16);

axis equal
axis off

%% ----- VERTEX NAMES -----
labels = cell(N,1);

% cycle names
for i=1:n
    labels{i} = sprintf('u%d',i);
end

% path names like u11,u12...
for i=1:n
    for j=1:k
        labels{index(i,j)} = sprintf('u%d%d',i,j);
    end
end

labelnode(p,1:N,labels);

%% ----- SAVE IMAGE -----
saveas(gcf,'Cn_corona_P6.png');
disp('Image saved as Cn_corona_P6.png');

```

Conclusion

In this paper, we studied the radio labeling of corona graphs and applications. A suitable labeling scheme was constructed based on vertex distances and graph diameter, and it was verified that the radio condition is

satisfied for all pairs of vertices. The radio number of the graph was obtained and follows a regular pattern as the number of copies increases. Hence, joint copies of corona graphs admit valid radio labeling, and the results are useful for interference-free frequency assignment in communication networks and related applications.

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