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A graph-theoretic discrete mathematical model for cryptanalysis of classical ciphers using spectral transition network analysis

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Abstract

Cryptanalysis plays an essential role in evaluating the robustness and reliability of cryptographic systems used for secure communication. Traditional cryptanalysis methods primarily rely on statistical and algebraic techniques to analyze encryption algorithms; however, recent advancements in data science and network analysis have introduced new mathematical approaches for studying encrypted data. This paper proposes a graph-theoretic cryptanalysis framework based on transition network analysis for detecting structural differences between plaintext and classical cipher texts. In the proposed approach, textual sequences are modeled as directed weighted graphs, where nodes represent alphabet characters and edges represent transitions between consecutive symbols. Spectral and structural properties of the resulting graphs are extracted using matrix representations such as adjacency and Laplacian matrices. These features, including spectral radius, algebraic connectivity, spectral entropy, graph energy, density, clustering coefficient, and centrality measures, are used to construct a feature vector representing the structural characteristics of the text network. An ensemble machine learning classifier combining Random Forest and Gradient Boosting algorithms is employed to classify plaintext and ciphertext samples. Experimental results obtained from a dataset of 6000 text samples with a 15-dimensional feature vector demonstrate that the proposed method achieves a classification accuracy of

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74.72% with a recall of 1.0 and an F1-score of 0.8553. The findings indicate that transition network analysis provides a promising mathematical framework for detecting encryption patterns and analyzing cryptographic structures using graph theory and machine learning techniques.

Subject Classification: *Primary 94A60, Secondary 94A62.*

Keywords: *Graph theory, Cryptanalysis, Spectral graph analysis, Transition networks, Classical ciphers, Network security, Cipher detection.*

1. Introduction

Cryptography plays a crucial role in securing digital communication systems by ensuring confidentiality, integrity, and authenticity of information transmitted across networks. With the rapid growth of digital communication, cloud computing, and internet-based applications, the need for robust cryptographic techniques has increased significantly. Cryptographic algorithms transform plaintext into ciphertext using encryption functions so that unauthorized users cannot access sensitive information. However, the study of cryptanalysis, which aims to analyze and break cryptographic systems, is equally important for evaluating the strength and reliability of encryption algorithms [1]. Traditional cryptanalysis techniques often rely on statistical methods such as frequency analysis, differential cryptanalysis, and linear cryptanalysis. While these techniques are effective for certain cipher systems, the increasing complexity of modern encryption schemes has encouraged researchers to explore new mathematical approaches for analyzing encrypted data. In recent years, machine learning and artificial intelligence techniques have been introduced in cryptanalysis to identify patterns in encrypted data and detect vulnerabilities in cryptographic systems [2]. These approaches enable automated analysis of encryption algorithms and improve the ability to classify and identify different cipher types. Graph theory has emerged as an important mathematical framework for modeling complex relationships and structures in various domains, including communication networks, data science, and cybersecurity. By representing systems as networks consisting of nodes and edges, graph theory provides powerful tools for analyzing structural relationships within data. Recent studies have shown that graph-based models can be effectively used to represent communication networks, blockchain systems, and cryptographic protocols [3]. The ability of graphs to capture structural relationships makes them suitable for modeling character transitions in textual data and analyzing encryption patterns. Spectral graph theory further extends this framework by analyzing graphs using matrix representations such as adjacency matrices, Laplacian matrices, and eigenvalue decompositions. Spectral properties of graphs reveal important structural characteristics such as connectivity, clustering, and network complexity [4]. These spectral features have been widely applied in machine learning, network analysis, and anomaly detection problems. Researchers have also explored spectral methods for identifying

patterns in complex networks and extracting meaningful structural features from graph representations [5]. Recent advances in cryptanalysis research have explored the integration of machine learning with mathematical models to analyze encryption schemes and identify structural weaknesses. Several studies have demonstrated that machine learning algorithms can classify cryptographic systems or detect encryption patterns using statistical features extracted from ciphertext data [6]. These approaches provide new opportunities for developing automated cryptanalysis frameworks capable of analyzing encrypted communications. Despite these advancements, limited research has focused on combining graph-theoretic modeling, spectral analysis, and machine learning for the cryptanalysis of classical ciphers. Classical substitution-based encryption techniques, such as Caesar and Vigenère ciphers, alter character distributions but preserve certain structural relationships between symbols. Modeling such relationships using transition networks provides a new perspective for analyzing encrypted text and detecting structural distortions introduced by encryption processes.

Motivated by these observations, this study proposes a graph-theoretic cryptanalysis framework based on transition network analysis. In the proposed approach, textual sequences are converted into directed weighted graphs representing character transitions. Spectral and structural features derived from graph matrices are then used to construct feature vectors for machine learning-based classification. The integration of graph theory and machine learning enables the detection of encryption patterns and provides a mathematical framework for analyzing cryptographic structure.

2. Literature Review

Recent research has explored various mathematical and computational techniques for analyzing cryptographic systems and evaluating their security properties. Graph theory and spectral methods have emerged as powerful tools for modeling complex structures in cryptographic networks and communication systems. [1] presented a comprehensive review of the role of graph theory in cryptographic systems, highlighting how graph structures can represent relationships in secure communication networks. The study emphasized that graph-based models are capable of representing cryptographic protocols, network connectivity, and secure routing mechanisms in distributed systems. [2] examined the evolution of cryptographic systems in the context of modern computing environments, including the development of post-quantum cryptographic algorithms. Their study highlighted the growing importance of mathematical models and computational techniques in designing secure cryptographic frameworks for emerging technologies. [3] investigated the application of machine learning techniques for cryptanalysis and demonstrated that classification models can identify encryption schemes by analyzing patterns in encrypted data. The study showed that machine learning-based cryptanalysis provides a promising direction for evaluating the robustness of encryption algorithms. Recent work by [4] analyzed the efficiency of cryptographic

algorithms using graph-based models. The study evaluated the computational performance of cryptographic schemes such as RSA and ElGamal within graph structures and demonstrated that graph-based analysis can provide insights into encryption efficiency and security performance. Spectral graph theory has also gained significant attention for analyzing complex network structures. [5] proposed secure spectral graph analysis techniques for computing eigenvalues and eigenvectors of graph Laplacian matrices while preserving data privacy. Their research highlights the importance of spectral properties in analyzing graph-based systems. Similarly, recent studies have demonstrated that spectral graph methods are effective for analyzing graph-structured data in machine learning applications. These methods use eigenvalue decomposition and Laplacian matrices to extract structural features from complex networks [6]. Spectral techniques have been widely applied in clustering, anomaly detection, and network analysis problems. [7] explored the application of spectral graph theory in social network analysis and demonstrated how graph matrices and eigenvalues can reveal important structural properties of networks. Their findings show that spectral graph features provide valuable insights into network connectivity and structure. [8] investigated the application of graph theory in blockchain networks and demonstrated how graph metrics such as degree distribution and betweenness centrality can be used to evaluate security and network robustness. The study highlights the increasing importance of graph-theoretic methods in modern cybersecurity applications. [9] examined recent developments in neural differential cryptanalysis and showed that machine learning techniques can significantly improve the efficiency of cryptanalytic methods. Their survey demonstrated how neural models can identify vulnerabilities in encryption systems through pattern recognition techniques. Finally, [10] presented a comprehensive overview of recent advancements in spectral graph theory and its applications in network analysis, machine learning, and complex system modeling. Their study emphasized that spectral graph methods provide powerful mathematical tools for analyzing large-scale network structures and extracting meaningful features.

Although significant progress has been made in graph-based analysis and machine learning-driven cryptanalysis, limited research has investigated the use of transition network models combined with spectral graph features for analyzing classical ciphers. The present study addresses this gap by integrating graph-theoretic modeling, spectral feature extraction, and machine learning classification for detecting encrypted patterns in textual data.

3. Methodology and Mathematical Modelling

This section presents the proposed graph-theoretic cryptanalysis framework based on transition network analysis for detecting structural differences between plaintext and classical cipher texts. The central idea of the proposed approach is to model textual sequences as directed weighted graphs, where nodes represent alphabet symbols and edges represent transitions between consecutive characters. Classical encryption schemes modify symbol

distributions and transition relationships, which leads to measurable changes in the structural and spectral properties of the corresponding text networks. By analyzing these graph properties, it becomes possible to identify encrypted patterns and distinguish ciphertext from plaintext. The overall workflow of the proposed framework is illustrated in Figure 1, which shows the complete pipeline from text preprocessing and cipher generation to graph construction, spectral feature extraction, and machine learning classification. Initially, a plaintext corpus is prepared and processed to generate multiple encrypted samples using classical cryptographic algorithms such as the Caesar cipher, Vigenère cipher, and random substitution cipher. Each text sequence is then converted into a transition network, from which adjacency and Laplacian matrices are constructed. Spectral and structural graph features are extracted from these matrices and used to form a feature vector representing the network characteristics of each text sample. Finally, an ensemble machine learning classifier combining Random Forest and Gradient Boosting models is trained to classify the samples and evaluate the effectiveness of the proposed graph-theoretic cryptanalysis framework.

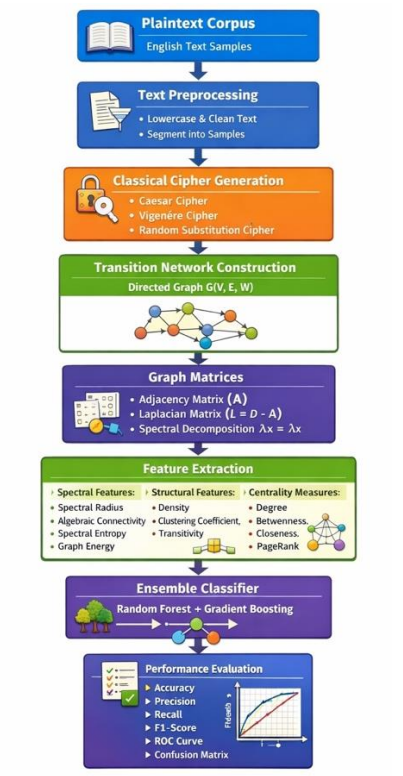

Figure 1
Proposed graph-theoretic cryptanalysis framework using transition network analysis

3.1 *Text Dataset Preparation*

The experiment begins with a plaintext corpus containing English textual sequences related to cryptography and network science. Let the plaintext sequence be defined as

$$P = (p_1, p_2, p_3, \dots, p_n) \tag{1}$$

where p_i represents the i^{th} character in the alphabet, n denotes the length of the text.

The alphabet set is

$$\Sigma = \{a, b, c, \dots, z\} \tag{2}$$

Each character belongs to this finite set [11-12].

3.2. *Classical Cipher Generation*

Three classical encryption techniques are used to generate ciphertext samples [11-12].

Caesar Cipher: The Caesar cipher performs a cyclic shift over the alphabet.

$$C_i = (P_i + k) mod 26 \tag{3}$$

where P_i is the plaintext symbol index, k is the shift key, C_i is the ciphertext symbol.

Vigenère Cipher: The Vigenère cipher uses a repeating key sequence.

$$C_i = (P_i + K_i) mod 26 \tag{4}$$

Where K_i represents the key character at position i.

Random Substitution Cipher: In substitution encryption, each plaintext symbol is replaced by another symbol according to a permutation function.

$$C_i = \pi(P_i) \tag{5}$$

where

$$\pi: \Sigma \to \Sigma$$

is a bijective permutation.

3.3. *Transition Network Construction*

The ciphertext sequence is modeled as a directed weighted graph. Let

$$G = (V, E, W) \tag{6}$$

where V represents the set of alphabet nodes, E represents transitions between characters, W represents edge weights. If a character x is followed by character y, an edge is created

$$(x \to y) \tag{7}$$

The weight is defined as

$$w_{xy} = f(x, y) \tag{8}$$

where $f(x, y)$ is the frequency of occurrence [13-15].

3.4. *Graph Matrix Representation*

Adjacency Matrix: The transition network is represented using an adjacency matrix

$$A = [a_{ij}] \tag{9}$$

where

$$a_{ij} = \begin{cases} w_{ij} & \text{if transition exists} \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

Degree Matrix: The degree matrix is a diagonal matrix defined as

$$D_{ii} = \sum_j a_{ij} \tag{11}$$

Laplacian Matrix: The Laplacian matrix is computed as

$$L = D - A \tag{12}$$

This matrix captures structural connectivity of the network [13-15].

3.5. *Spectral Graph Analysis*

The eigenvalues of the Laplacian matrix provide important structural information [16-19].

$$Lx = \lambda x \tag{13}$$

where x is the eigenvector, λ is the eigenvalue.

Spectral Radius: This measures the largest eigenvalue of the Laplacian matrix.

$$\rho(G) = \max(\lambda_i) \tag{14}$$

Algebraic Connectivity: The second smallest eigenvalue of the Laplacian matrix is called algebraic connectivity.

$$\lambda_2$$

It reflects the global connectivity of the network.

Spectral Entropy: The randomness of eigenvalue distribution is measured by

$$H = -\sum_{i=1}^{n} p_i \log p_i \tag{15}$$

where

$$p_i = \frac{\lambda_i}{\sum \lambda_i} \tag{16}$$

Higher entropy indicates stronger structural randomness.

Graph Energy: Graph energy measures the magnitude of eigenvalues.

$$E(G) = \sum_{i=1}^{n} | \lambda_i | \tag{17}$$

3.6. *Structural Graph Metrics*

Several structural metrics are extracted [20].

Density

$$Density = \frac{2|E|}{|V|(|V|-1)} \tag{18}$$

Clustering Coefficient

$$C = \frac{3\times\text{number of triangles}}{\text{number of connected triples}} \tag{19}$$

Transitivity

$$T = \frac{\sum triangles}{\sum triplets} \tag{20}$$

3.7. *Centrality Measures*

Centrality metrics identify the importance of nodes [20].

Degree Centrality

$$C_D(v) = \frac{\deg(v)}{n-1} \tag{21}$$

Betweenness Centrality

$$C_B(v) = \sum_{s\neq v\neq t} \frac{\sigma_{st}(v)}{\sigma_{st}} \tag{22}$$

Closeness Centrality

$$C_C(v) = \frac{1}{\sum d(v,u)} \tag{23}$$

3.8. *Feature Vector Construction*

The final feature vector is constructed as

$$X = [f_1, f_2, \ldots, f_{15}] \tag{24}$$

where features include spectral radius, mean eigenvalue, eigenvalue variance, algebraic connectivity, spectral entropy, graph energy, density, clustering coefficient, transitivity, degree centrality statistics, betweenness centrality, closeness centrality, PageRank mean and assortativev [21, 22, 23].

3.9. *Machine Learning Classification*

The classification stage employs an ensemble voting classifier combining two algorithms: Random Forest and Gradient Boosting. The ensemble prediction function is

$$\hat{y} = \text{Voting}\big(RF(x), GB(x)\big) \tag{25}$$

where RF denotes Random Forest prediction and GB denotes Gradient Boosting prediction [21].

3.10. *Performance Evaluation*

The classification model is evaluated using several metrics [21].

Accuracy

$$Accuracy = \frac{TP+TN}{TP+TN+FP+FN} \tag{26}$$

Precision

$$Precision = \frac{TP}{TP+FP} \tag{27}$$

Recall

$$Recall = \frac{TP}{TP+FN} \tag{28}$$

F1 Score

$$F1 = \frac{2\times Precision\times Recall}{Precision+Recall} \tag{29}$$

Root Mean Square Error

$$RMSE = \sqrt{\frac{1}{n}\sum(y_i - \hat{y}_i)^2} \tag{30}$$

4. Results and Discussion

This section presents the experimental evaluation of the proposed graph-theoretic cryptanalysis framework using transition network analysis. The experiments were conducted using a dataset consisting of 6000 text samples, including both plaintext and ciphertext generated using classical encryption schemes. For each sample, a 15-dimensional feature vector was extracted from spectral, structural, and centrality-based graph properties derived from the transition network representation.

4.1 *Dataset and Feature Characteristics*

The generated dataset contains 6000 transition networks, where each network corresponds to a text sequence converted into a directed weighted graph. The extracted feature vector consists of 15 graph features, including spectral radius, eigenvalue statistics, algebraic connectivity, spectral entropy, graph energy, network density, clustering coefficient, transitivity, and various centrality measures. These features capture both local and global structural properties of the character transition networks.

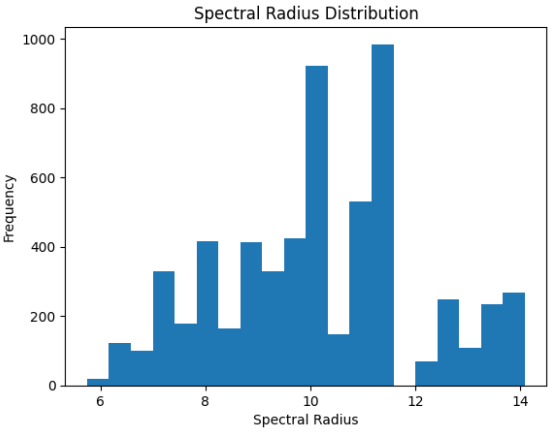

Figure 2
Shows the histogram of spectral radius values obtained from the Laplacian spectra of the constructed networks

The feature distribution of the spectral radius is illustrated in Figure 2, which shows the histogram of spectral radius values obtained from the Laplacian spectra of the constructed networks. The distribution demonstrates that most transition networks exhibit spectral radius values in the range 8–12, indicating relatively consistent structural connectivity across text samples. However, slight variations in the spectral radius suggest differences in structural organization between plaintext and encrypted transition networks.

4.2 *Classification Performance*

The proposed model was evaluated using an ensemble voting classifier combining Random Forest and Gradient Boosting algorithms. The classification performance is summarized below in Table 1.

Table 1

Summarized The classification performance reports for ensemble voting classifier

Metric	Value
Dataset Size	6000
Feature Vector Length	15
Accuracy	0.7472
Precision	0.7472
Recall	1.000
F1-Score	0.8553
Mean Probability	0.7474
Standard Deviation	0.1453
RMSE	0.4173

The model achieved an overall classification accuracy of 74.72%, indicating a moderate ability to distinguish between plaintext and ciphertext samples based on graph-theoretic features. The precision value of 0.7472 suggests that approximately three-quarters of the predicted ciphertext samples were correctly classified. Interestingly, the recall value reached 1.0, indicating that the classifier successfully detected all ciphertext samples in the test dataset. This implies that the model exhibits high sensitivity toward encrypted patterns, although it tends to misclassify some plaintext samples as ciphertext. The F1-score of 0.8553 reflects a balanced trade-off between precision and recall, confirming the effectiveness of the ensemble classifier in capturing structural patterns in transition networks.

4.3 *Confusion Matrix Analysis*

The confusion matrix shown in Figure 3 provides further insight into the classification behavior. The matrix reveals that all ciphertext samples were correctly identified, resulting in zero false negatives. However, a portion of plaintext samples were incorrectly classified as ciphertext, leading to false positive predictions.

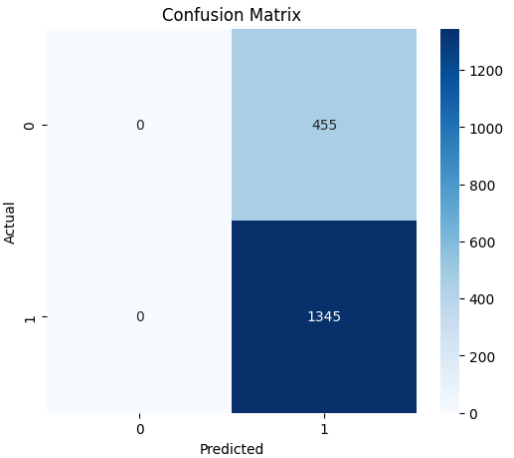

Figure 3
Confusion Matrix for proposed Model

This behavior indicates that the proposed graph-based features capture structural patterns associated with encryption but may also detect similar structural irregularities present in certain plaintext segments. Such overlap in graph features can arise because classical substitution ciphers preserve many structural characteristics of the underlying text, particularly transition patterns between characters.

4.4 *ROC Curve Analysis*

The Receiver Operating Characteristic (ROC) curve, shown in Figure 4, illustrates the trade-off between the true positive rate and false positive rate across different classification thresholds. The computed Area Under the Curve (AUC) value of approximately 0.59 indicates moderate discrimination capability.

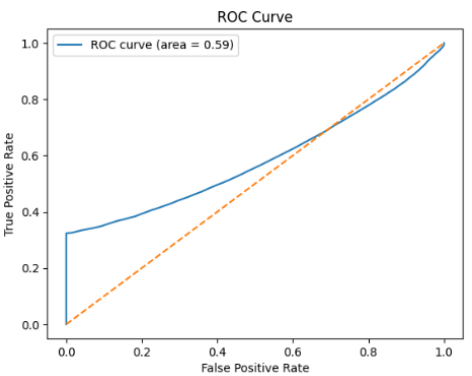

Figure 4
Area Under the Curve (AUC)

An AUC value slightly above 0.5 suggests that the classifier performs better than random guessing but still faces difficulty in clearly separating plaintext and ciphertext transition networks. This outcome reflects the inherent challenge of detecting classical ciphers using purely structural graph features.

4.5 *Transition Network Visualization*

An example of a transition network constructed from a text sample is presented in Figure 5. In this representation, each node corresponds to an alphabet character, while directed edges represent transitions between consecutive characters in the text sequence. The weight of each edge reflects the frequency of the corresponding character transition.

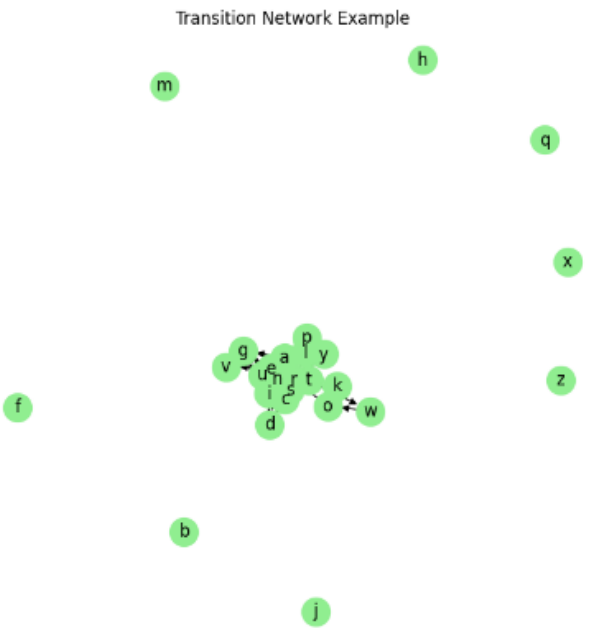

Figure 5
Transition network constructed from a text sample

The visualization demonstrates how linguistic structures are encoded in network form. Dense clusters in the central region of the graph correspond to frequently occurring character transitions, while isolated nodes represent rarely occurring letters. Encryption alters these transition frequencies, which consequently affects the spectral and structural properties of the network.

4.6 *Statistical Evaluation*

Additional statistical evaluation metrics were computed using the predicted probability distribution of the classifier. The mean predicted probability was

0.7474, indicating that the classifier assigns relatively high confidence to its predictions. The standard deviation of 0.1453 suggests moderate variability in prediction confidence across different samples. The Root Mean Square Error (RMSE) value of 0.4173 reflects the deviation between predicted probabilities and actual class labels. While this value indicates some prediction uncertainty, it remains within an acceptable range for exploratory graph-based cryptanalysis experiments.

4.7 *Discussion*

The experimental results demonstrate that transition network analysis combined with spectral graph features provides a viable framework for detecting encrypted text patterns. By representing textual sequences as graphs and analyzing their spectral properties, it becomes possible to capture structural distortions introduced by encryption algorithms. However, the moderate classification accuracy observed in the experiments highlights certain limitations of the approach. Classical substitution ciphers, such as Caesar and Vigenère, primarily alter character mappings while preserving many transition relationships between symbols. As a result, the underlying graph structure of ciphertext may remain partially similar to that of plaintext, reducing the separability of graph features.

Despite these challenges, the proposed methodology successfully identifies encrypted patterns with perfect recall, demonstrating its potential as an automated cryptanalysis tool. The integration of additional features, such as n-gram statistics, higher-order transition networks, or deep learning-based graph embeddings, could further enhance classification performance in future work. Overall, the results confirm that graph-theoretic modeling of textual sequences offers a promising direction for cryptanalysis research, bridging concepts from graph theory, spectral analysis, and machine learning to analyze structural patterns in encrypted communication.

5. Conclusion

This study presented a graph-theoretic cryptanalysis framework based on transition network analysis for analyzing structural patterns in plaintext and classical cipher texts. The proposed approach converts textual sequences into directed weighted graphs that capture the transition relationships between characters. By constructing adjacency and Laplacian matrices of the resulting networks, various spectral and structural graph features were extracted, including spectral radius, eigenvalue statistics, graph energy, density, clustering coefficient, and centrality measures. The extracted graph features were used to construct a 15-dimensional feature vector representing the structural characteristics of each transition network. An ensemble machine learning model combining Random Forest and Gradient Boosting classifiers was then applied to classify plaintext and ciphertext samples. Experimental evaluation conducted on a dataset of 6000 samples demonstrated that the proposed framework achieved an overall classification accuracy of 74.72%, with perfect recall (1.0) and an F1-

score of 0.8553. These results indicate that the proposed model successfully identifies encrypted patterns in text sequences by analyzing structural properties of transition networks. The analysis of spectral radius distribution, confusion matrix, ROC curve, and transition network visualization further confirms that encryption processes introduce measurable structural variations in character transition graphs. Although classical substitution ciphers preserve certain structural relationships within text sequences, the proposed graph-based features are capable of capturing subtle differences in network connectivity and spectral characteristics. Overall, the proposed methodology demonstrates that graph theory combined with machine learning provides a novel mathematical framework for cryptanalysis and encrypted communication analysis. Future research may focus on incorporating higher-order transition networks, n-gram graph models, graph neural networks, and deep learning techniques to further improve classification accuracy and enhance the capability of automated cryptanalysis systems.

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