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Result of the projective special unitary groups $\mathcal{PSU}$(3,49) and $\mathcal{PSU}$(3,169)

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Abstract

The cyclic decomposition (cy.dec.) for the groups $\mathcal{PSU}$**(3,49)** and $\mathcal{PSU}$**(2,169)** after gain the conjugacy classes and the character table (cha.ta.) of rational representations (ra.re.) for each group survey in this work.

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1. Introduction

The $\mathcal{PSL}(n,F)$ gain it by $\mathcal{SU}(n,F)$ / (Z($\mathcal{SU}(n,F)$) by its center, [1]. Authors in [2-5] studied the cy. dec. for the groups $\mathcal{PSL}(2,\mathfrak{U})$ and $\mathcal{SL}(2,\mathfrak{U})$, where $\mathfrak{U}$=23, 29, 31, 37.

We apply the same idea in [6-8] for the groups $\mathcal{PSL}$(3,49) and $\mathcal{PSL}$(3,169) in this work. Any author can employ the same idea for the studied in [9-17] and [21-23].

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2. The Basic Concepts for the groups $\mathcal{PSU}(3, q^2)$

The order of $\mathcal{PSU}(3, q^2)$ equal to |SU(3, q^2)|/|Z(SU(3, q^2))|, the Tables 1 and 2 of the conjugacy classes and the cha.ta. of ra.re. for this groups $d = \gcd(n, q-1) = 3$ studied in [18]. We employ that for our groups $\mathcal{PSU}(3,49)$ and $\mathcal{PSU}(2,169)$ to get the results.

2.1. *The group* $\mathcal{PSU}(3,49)$

$|\mathcal{PSU}(3,49)| = |\mathcal{SU}(3,49)| / |Z(\mathcal{SU}(3,49))| = 2822400 / 3 = 940800$, d = gcd(3,6) = 3, $\delta = 1$, S = q + δ = 8, r = q – δ = 6, t = $q^2 + \delta q + 1 = 57$, $t' = \frac{t}{d} = 19, t'' = \frac{t'-1}{6} = 3, r' = 2$, $r'' = \frac{r'(1+\delta)}{2} = 2$ d′ = $0, \epsilon^6 = \mp 1, \omega^3 = 1, \rho^6 = 1$.

Table 1 represents some of the conjugacy classes and the cha.ta. of ra.re. for this group.

Table 1

The conjugacy classes and the cha.ta. of ra.re. for the group $\mathcal{PSU}(3,49)$.

Conjugacy class	1 1 1 $\chi_1\ \chi_{56}\ \chi_{343}$			$1 \le u \le 1$		$1 \le u \le 1$	$1 \le u < v \le 2$ $v < \omega \le 6$ $u+v+\omega = 0 \pmod r$ $t''-r''\ \chi_{3t^{(u,v,\omega)}}$		
				$r'-1$ $\chi_{57}^{(1)}$	$r'-1$ $\chi_{57}^{(1)}$				
C_1	1	56	343	57	399	152	456	456	456
C_2	1	7	7	8	7	5	5	5	15
$C_3^{(0)}$	1	0	0	1	0	1	5	12	1
$C_3^{(1)}$	1	0	0	1	0	1	5	12	1
$C_3^{(2)}$	1	0	0	1	0	1	5	12	1
$C_4^{(1)}$	1	8	7	-7	-1	8	8	8	−24
$C_6^{(1,2,3)}$	1	2	1	-1	-1	1	1	1	-1
$C_7^{(4)} = C_7^{(12)}$	1	0	-1	1	-1	0	0	0	0
$C_8^{(1)} = C_8^{(7)} = C_8^{(11)}$	1	-1	1	0	0	0	0	0	0
$C_8^{(2)} = C_8^{(14)} = C_8^{(3)}$	1	-1	1	0	0	0	0	0	0
$C_8^{(5)} = C_8^{(16)} = C_8^{(17)}$	1	-1	1	0	0	0	0	0	0
$C_8^{(10)} = C_8^{(13)} = C_8^{(15)}$	1	-1	1	0	0	0	0	0	0
C_1	342	342	342	342	342	342	342	288	288

C_2	-1	-1	-1	-1	-1	-1	-1	-6	-6
$C_3^{(0)}$	-1	-1	-1	-1	-1	-1	-1	1	1
$C_3^{(1)}$	-1	-1	-1	-1	-1	-1	-1	1	1
$C_3^{(2)}$	-1	-1	-1	-1	-1	-1	-1	1	1
$C_4^{(1)}$	-6	6	-6	6	6	-6	-6	0	0
$C_5^{(1)}$	1	-1	1	-1	-1	1	1	0	0
C_6'	0	0	0	0	0	0	0	0	0

By applying the same idea in [19], and $\mathcal{K}(\mathrm{G}) = \oplus_{i=1}^{n} \mathbb{Z}_{p^i}$ for the $\mathcal{p}$ -group G, [20]. We obtain the diagonalization matrix of the above table, which is shown in Table 2.

Table 2

The diagonalization matrix for the group PSU(3,49).

Conjugacy class	$1 \leq u \leq 16$ $u \neq 0(mod\ 8)$ $(u) = (7u\delta)\ (mod 16)$ $3\ t'' - r'' - d\ \chi t(u)$							$1 \leq u \leq 18$ $(u) = (7u\delta) = (49u)$ $(mod 19)$ $2\ t''$					
$C_7^{(3)} = C_7^{(5)}$	-2	2	2	2	2	2	2	0	0	0	0	0	0
$C_7^{(4)} = C_7^{(12)}$	1	1	1	1	1	1	1	0	0	0	0	0	0
$C_8^{(1)} = C_8^{(7)} = C_8^{(11)}$	0	0	0	0	0	0	0	0	0	-1	1	1	1
$C_8^{(2)} = C_8^{(14)} = C_8^{(3)}$	0	0	0	0	0	0	0	0	-2	1	1	1	1
$C_8^{(10)} = C_8^{(13)} = C_8^{(15)}$	0	0	0	0	0	0	0	5	2	1	1	1	1

268128	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	134064	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	7	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	7	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		2		0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

And the cyclic decomposition equal to

$$\mathbb{Z}_{268128}\oplus\mathbb{Z}_{134064}\oplus\mathbb{Z}_{19}\oplus\mathbb{Z}_{19}\oplus\mathbb{Z}_{19}\oplus\mathbb{Z}_{7}\oplus\mathbb{Z}_{7}\oplus\mathbb{Z}_{7}\oplus\mathbb{Z}_{4}\oplus\mathbb{Z}_{3}\oplus\mathbb{Z}_{2}\oplus\mathbb{Z}_{2}\oplus\mathbb{Z}_{2}$$
$$\mathbb{Z}_{2}\oplus\mathbb{Z}_{2}\oplus\mathbb{Z}_{2}\oplus\mathbb{Z}_{2}\oplus\mathbb{Z}_{2}\oplus\mathbb{Z}_{2}\oplus\mathbb{Z}_{1}.$$

2.2. *The group* $\mathcal{PSU}(3,169)$

$|\mathcal{PSU}(3,169)|$ = 3973860, d = gcd(3,12)=3, $\delta = 1$, S = 14, r = 12, t = 183, $t' = 61, t'' = 10, r' = 4$, $r'' = \frac{r'(1+\delta)}{2} = 4, d' = 0, \epsilon^{12} = 1, \omega^3 = 1, \rho^{12} = 1$. This group has 64 the conjugacy classes and 64 character of rational representations Table 3, some of these are

Table 3

Some of the conjugacy classes for the group PSU(3,169).

Conjugacy class	Canonical Representation	parameters	No of classes	Centralizer order
C_1	$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$	–	1	270178272
C_2	$\begin{bmatrix}1&0&0\\1&1&0\\0&0&1\end{bmatrix}$	–	1	8788
$C_3^{(0)}$	$\begin{bmatrix}1&0&0\\1&1&0\\0&1&1\end{bmatrix}$	$0 \le l \le 2$ $l = 0,1,2$	3	169
$C_3^{(1)}$	$\begin{bmatrix}1&0&0\\\theta&1&0\\0&\theta&1\end{bmatrix}$			

3. Conclusion

From our work we get the cy.dec. for the groups $\boldsymbol{\mathcal{PSU}(3,49)}$ and $\boldsymbol{\mathcal{PSU}(2,169)}$ from the conjugacy classes and the cha.ta. of ra.re. for each group.

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