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Result from the contracting homotopy of the partition (13,8)

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Abstract

The exactness of the sequence (sequ.) of the elements of Weyl (W.) resolution (re.) of characteristic-free re. for the partition (13, 8) considered in this article by employing contracting homotopy for the maps define for this partition.

Subject Classification (2020): *13D07.*

Keywords: *Characteristic free, Homotopies, Resolution, Weyl resolution.*

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1. Introduction

The idea in [1, 2] applied for the partition (13, 8), where $\mathcal{K}_{\lambda/\mu}\mathcal{F} = \mathrm{Im}(\mathcal{DF} \to \wedge\mathcal{F})$ is the W. module for the partition λ/μ, we depended this work for the contracting homotopy, [3, 4]. We can applied that for $\mathcal{SL}(2, \mathfrak{U})$, $\mathfrak{U}$ = 81, 729, 125, 3125, 121, 196, PG(3, 8), [5, 6], some finite groups and submodules [7-9], Also for the studies by authors in [10-13].

2. Discussion the Result

The elements of W. re. for the partition (13, 8) are

$\mathcal{M}_0 = \mathcal{D}_{13} \otimes \mathcal{D}_8$, $\mathcal{M}_1 = \mathcal{Z}_{21}^{(b)} x \mathcal{D}_{13+b} \otimes \mathcal{D}_{8-b}$; where $1 \leq b \leq 8$,

$\mathcal{M}_2 = \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{D}_{13+|b|} \otimes \mathcal{D}_{8-|b|}$; where $2 \leq |b| = b_1 + b_2 \leq 8$ and $b_l \geq 1$.

$\mathcal{M}_3 = \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{D}_{13+|b|} \otimes \mathcal{D}_{8-|b|}$; where $3 \leq |b| = \sum_{i=1}^{3} b_i \leq 8$ and $b_l \geq 1$.

$\mathcal{M}_4 = \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{D}_{13+|b|} \otimes \mathcal{D}_{8-|b|}$; where $4 \leq |b| = \sum_{i=1}^{4} b_i \leq 9$ and $b_l \geq 1$.

$\mathcal{M}_5 = \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{Z}_{21}^{(b_5)} x \mathcal{D}_{13+|b|} \otimes \mathcal{D}_{8-|b|}$; where $5 \leq |b| = \sum_{i=1}^{5} b_i \leq 9$ and $b_l \geq 1$.

$\mathcal{M}_6 = \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{Z}_{21}^{(b_5)} x \mathcal{Z}_{21}^{(b_6)} x \mathcal{D}_{13+|b|} \otimes \mathcal{D}_{8-|b|}$; $\mathcal{D}_{9-|b|}$, where $6 \leq |b| = \sum_{i=1}^{6} b_i \leq 9$ and $b_l \geq 1$,

$\mathcal{M}_7 = \mathcal{Z}_{21}^{(b_1)} x \mathcal{Z}_{21}^{(b_2)} x \mathcal{Z}_{21}^{(b_3)} x \mathcal{Z}_{21}^{(b_4)} x \mathcal{Z}_{21}^{(b_5)} x \mathcal{Z}_{21}^{(b_6)} x \mathcal{Z}_{21}^{(b_7)} x \mathcal{D}_{13+|b|} \otimes \mathcal{D}_{8-|b|}$; where $7 \leq |b| = \sum_{i=1}^{7} b_i \leq 9$ and $b_l \geq 1$,

$\mathcal{M}_8 = \mathcal{Z}_{21}^{(1)} x \mathcal{Z}_{21}^{(1)} x \mathcal{Z}_{21}^{(1)} x \mathcal{Z}_{21}^{(1)} x \mathcal{Z}_{21}^{(1)} x \mathcal{Z}_{21}^{(1)} x \mathcal{Z}_{21}^{(1)} x \mathcal{Z}_{21}^{(1)} x\ \mathcal{Z}_{21}^{(1)} x\ \mathcal{D}_{21} \otimes \mathcal{D}_0$.

The homotopies $\{\mathcal{S}_i\}$; where i = 0,1,2,…,7 are below with prove that are contracting

$$\mathcal{S}_0\left(\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(13)}\ 2^{(k)} \\ 2^{(8-k)} \end{matrix}\right)\right) = \begin{cases} 0\ ;\ \text{if } k \leq 0 \\ \mathcal{Z}_{21}^{(k)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(13+k)} \\ 2^{(8-k)} \end{matrix}\right)\ ;\ \text{if } k = 1,2,3,4,5,6,7,8 \end{cases}$$

$$\mathcal{S}_1\left(\mathcal{Z}_{21}^{(k)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(13+|k|)} \quad 2^{(m)} \\ 2^{(8-|k|-m)} \end{matrix}\right)\right) =$$

$$\begin{cases} 0\ ;\ \text{if } m = 0 \\ \mathcal{Z}_{21}^{(k)} x \mathcal{Z}_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(13+k+m)} \\ 2^{(8-k-m)} \end{matrix}\right)\ ;\ \text{if } m = 1,2,3,4,5,6,7 \end{cases}$$

For $|k| = k_1 + k_2$, $\mathcal{S}_2\left(\mathcal{Z}_{21}^{(k_1)} x \mathcal{Z}_{21}^{(k_2)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(13+|k|)} \quad 2^{(m)} \\ 2^{(8-|k|-m)} \end{matrix}\right)\right)$

$$=\begin{cases} 0\,;\ \text{if } m=0 \\ \mathcal{Z}_{21}^{(k_1)}x\mathcal{Z}_{21}^{(k_2)}x\mathcal{Z}_{21}^{(m)}x\begin{pmatrix} w \\ w' \end{vmatrix} \begin{matrix} 1^{(13+|k|+m)} \\ 2^{(8-|k|-m)} \end{pmatrix}\,;\ \text{if } m=1,2,3,4,5,6 \end{cases};$$

For $|k|=k_1+k_2+k_3$, $\mathcal{S}_3\left(\mathcal{Z}_{21}^{(k_1)}x\mathcal{Z}_{21}^{(k_2)}x\mathcal{Z}_{21}^{(k_3)}x\begin{pmatrix} w \\ w' \end{vmatrix} \begin{matrix} 1^{(13+|k|)} & 2^{(m)} \\ & 2^{(8-|k|-m)} \end{pmatrix}\right)$

$$=\begin{cases} 0\,;\ m=0 \\ \mathcal{Z}_{21}^{(k_1)}x\mathcal{Z}_{21}^{(k_2)}x\mathcal{Z}_{21}^{(k_3)}x\mathcal{Z}_{21}^{(m)}x\begin{pmatrix} w \\ w' \end{vmatrix} \begin{matrix} 1^{(13+|k|+m)} \\ 2^{(8-|k|-m)} \end{pmatrix}\,;\ m=1,2,3,4,5 \end{cases};$$

For $|k|=k_1+k_2+k_3+k_4$,

$\mathcal{S}_3\left(\mathcal{Z}_{21}^{(k_1)}x\mathcal{Z}_{21}^{(k_2)}x\mathcal{Z}_{21}^{(k_3)}x\mathcal{Z}_{21}^{(k_4)}x\begin{pmatrix} w \\ w' \end{vmatrix} \begin{matrix} 1^{(13+|k|)} & 2^{(m)} \\ & 2^{(8-|k|-m)} \end{pmatrix}\right)$

$$=\begin{cases} 0\,;\ m=0 \\ \mathcal{Z}_{21}^{(k_1)}x\mathcal{Z}_{21}^{(k_2)}x\mathcal{Z}_{21}^{(k_3)}x\mathcal{Z}_{21}^{(k_4)}x\mathcal{Z}_{21}^{(m)}x\begin{pmatrix} w \\ w' \end{vmatrix} \begin{matrix} 1^{(13+|k|+m)} \\ 2^{(8-|k|-m)} \end{pmatrix};\ m=1,2,3,4 \end{cases};$$

and so as $\mathcal{S}_5, \mathcal{S}_6, \mathcal{S}_7$,. So we have the following diagram

$$\mathcal{M}_8 \underset{\mathcal{S}_7}{\overset{\partial_x}{\leftrightarrows}} \mathcal{M}_7 \underset{\mathcal{S}_6}{\overset{\partial_x}{\leftrightarrows}} \mathcal{M}_6 \underset{\mathcal{S}_5}{\overset{\partial_x}{\leftrightarrows}} \mathcal{M}_5 \underset{\mathcal{S}_4}{\overset{\partial_x}{\leftrightarrows}} \mathcal{M}_4 \underset{\mathcal{S}_3}{\overset{\partial_x}{\leftrightarrows}} \mathcal{M}_3 \underset{\mathcal{S}_2}{\overset{\partial_x}{\leftrightarrows}} \mathcal{M}_2 \underset{\mathcal{S}_1}{\overset{\partial_x}{\leftrightarrows}} \mathcal{M}_1 \underset{\mathcal{S}_0}{\overset{\partial_x}{\leftrightarrows}} \mathcal{M}_0$$

Now we have

$$\mathcal{S}_0\partial_x\left(\mathcal{Z}_{21}^{(k)}x\begin{pmatrix} w \\ w' \end{vmatrix} \begin{matrix} 1^{(13+|k|)} & 2^{(m)} \\ & 2^{(8-|k|-m)} \end{pmatrix}\right)=\binom{k+m}{m}\mathcal{Z}_{21}^{(k+m)}x\begin{pmatrix} w \\ w' \end{vmatrix} \begin{matrix} 1^{(13+k+m)} \\ 2^{(8-k-m)} \end{pmatrix},$$

and

$$\partial_x\mathcal{S}_1\left(\mathcal{Z}_{21}^{(k)}x\begin{pmatrix} w \\ w' \end{vmatrix} \begin{matrix} 1^{(13+|k|)} & 2^{(m)} \\ & 2^{(8-|k|-m)} \end{pmatrix}\right)=$$

$$-\binom{k+m}{m}\mathcal{Z}_{21}^{(k+m)}x\begin{pmatrix} w \\ w' \end{vmatrix} \begin{matrix} 1^{(13+k+m)} \\ 2^{(8-k-m)} \end{pmatrix}+$$

$\mathcal{Z}_{21}^{(k)}x\begin{pmatrix} w \\ w' \end{vmatrix} \begin{matrix} 1^{(13+|k|)} & 2^{(m)} \\ & 2^{(8-|k|-m)} \end{pmatrix}$. So $\mathcal{S}_0\partial_x+\partial_x\mathcal{S}_1=\mathrm{id}_{\mathcal{M}_1}$.

$$\mathcal{S}_1\partial_x\left(\mathcal{Z}_{21}^{(k_1)}x\mathcal{Z}_{21}^{(k_2)}x\begin{pmatrix} w \\ w' \end{vmatrix} \begin{matrix} 1^{(13+|k|)} & 2^{(m)} \\ & 2^{(8-|k|-m)} \end{pmatrix}\right)=$$

$$-\binom{|k|}{k_2}\mathcal{Z}_{21}^{(|k|)}x\mathcal{Z}_{21}^{(m)}x\begin{pmatrix} w \\ w' \end{vmatrix} \begin{matrix} 1^{(13+|k|+m)} \\ 2^{(8-|k|-m)} \end{pmatrix}$$

$+\binom{k_2+m}{m}\mathcal{Z}_{21}^{(k_1)}x\mathcal{Z}_{21}^{(k_2+m)}x\begin{pmatrix} w \\ w' \end{vmatrix} \begin{matrix} 1^{(13+|k|+m)} \\ 2^{(8-|k|-m)} \end{pmatrix}$, and for $|k|=k_1+k_2$

$$\partial_x\mathcal{S}_2\left(\mathcal{Z}_{21}^{(k_1)}x\mathcal{Z}_{21}^{(k_2)}x\begin{pmatrix} w \\ w' \end{vmatrix} \begin{matrix} 1^{(13+|k|)} & 2^{(m)} \\ & 2^{(8-|k|-m)} \end{pmatrix}\right)$$

$$=\binom{|k|}{k_2}\mathcal{Z}_{21}^{(|k|)}x\mathcal{Z}_{21}^{(m)}x\begin{pmatrix} w \\ w' \end{vmatrix} \begin{matrix} 1^{(13+|k|+m)} \\ 2^{(8-|k|-m)} \end{pmatrix}$$

$$-\binom{k_2+m}{m}Z_{21}^{(k_1)}xZ_{21}^{(k_2+m)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(13+|k|+m)}\\2^{(8-|k|-m)}\end{matrix}\right)$$

$$+Z_{21}^{(k_1)}xZ_{21}^{(k_2)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(13+|k|)} & 2^{(m)}\\ & 2^{(8-|k|-m)}\end{matrix}\right).$$

It is clear that $\mathcal{S}_1\partial_x+\partial_x\mathcal{S}_2=\mathrm{id}_{\mathcal{M}_2}$.

$$\mathcal{S}_2\partial_x\left(Z_{21}^{(k_1)}xZ_{21}^{(k_2)}xZ_{21}^{(k_3)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(13+|k|)} & 2^{(m)}\\2^{(8-|k|-m)} & \end{matrix}\right)\right)$$

$$=\mathcal{S}_2\left[\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}xZ_{21}^{(k_3)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(13+|k|)} & 2^{(m)}\\2^{(8-|k|-m)} & \end{matrix}\right)-\right.$$
$$\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}xZ_{21}^{(k_2+k_3)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(13+|k|)} & 2^{(m)}\\2^{(8-|k|-m)} & \end{matrix}\right)+$$
$$\left.Z_{21}^{(k_1)}xZ_{21}^{(k_2)}x\partial_{21}^{(k_3)}\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(13+|k|)} & 2^{(m)}\\2^{(8-|k|-m)} & \end{matrix}\right)\right]$$

$$=\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}xZ_{21}^{(k_3)}xZ_{21}^{(m)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(13+|k|+m)}\\2^{(8-|k|-m)}\end{matrix}\right)-$$
$$\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}xZ_{21}^{(k_2+k_3)}xZ_{21}^{(m)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(13+|k|+m)}\\2^{(8-|k|-m)}\end{matrix}\right)+$$
$$\binom{k_3+m}{m}Z_{21}^{(k_1)}xZ_{21}^{(k_2)}xZ_{21}^{(k_3+m)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(13+|k|+m)}\\2^{(8-|k|-m)}\end{matrix}\right),$$

and $\partial_x\mathcal{S}_3\left(Z_{21}^{(k_1)}xZ_{21}^{(k_2)}xZ_{21}^{(k_3)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(13+|k|)} & 2^{(m)}\\2^{(8-|k|-m)} & \end{matrix}\right)\right)$

$$=\partial_x\left(Z_{21}^{(k_1)}xZ_{21}^{(k_2)}xZ_{21}^{(k_3)}xZ_{21}^{(m)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(13+|k|+m)}\\2^{(8-|k|-m)}\end{matrix}\right)\right)$$

$$=-\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}xZ_{21}^{(k_3)}xZ_{21}^{(m)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(13+|k|+m)}\\2^{(8-|k|-m)}\end{matrix}\right)+$$
$$\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}xZ_{21}^{(k_2+k_3)}xZ_{21}^{(m)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(13+|k|+m)}\\2^{(8-|k|-m)}\end{matrix}\right)-$$
$$\binom{k_3+m}{m}Z_{21}^{(k_1)}xZ_{21}^{(k_2)}xZ_{21}^{(k_3+m)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(13+|k|+m)}\\2^{(8-|k|-m)}\end{matrix}\right)+$$
$$Z_{21}^{(k_1)}xZ_{21}^{(k_2)}xZ_{21}^{(k_3)}x\partial_{21}^{(m)}\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(13+|k|+m)}\\2^{(8-|k|-m)}\end{matrix}\right)$$

$$=-\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}xZ_{21}^{(k_3)}xZ_{21}^{(m)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(13+|k|+m)}\\2^{(8-|k|-m)}\end{matrix}\right)+$$
$$\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}xZ_{21}^{(k_2+k_3)}xZ_{21}^{(m)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(13+|k|+m)}\\2^{(8-|k|-m)}\end{matrix}\right)-$$
$$\binom{k_3+m}{m}Z_{21}^{(k_1)}xZ_{21}^{(k_2)}xZ_{21}^{(k_3+m)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(13+|k|+m)}\\2^{(8-|k|-m)}\end{matrix}\right)+$$

$Z_{21}^{(k_1)}xZ_{21}^{(k_2)}xZ_{21}^{(k_3)}x\left(\begin{matrix}w\\w'\end{matrix}\middle|\begin{matrix}1^{(13+|k|)} & 2^{(m)}\\2^{(8-|k|-m)} & \end{matrix}\right)$, where $|k|=k_1+k_2+k_3$.

It is clear that $\mathcal{S}_2\partial_x+\partial_x\mathcal{S}_3=\mathrm{id}_{\mathcal{M}_3}$.

By that same way, we can get that, $\mathcal{S}_{i-1}\partial_x + \partial_x\mathcal{S}_i = \mathrm{id}_{\mathcal{M}_i}$, $\imath = 4, \ldots, 7$.

Thus $\{\mathcal{S}_\imath\}$; $\imath = 0,1,2, \ldots, 7$ is a contracting homotopy [14], which implies the exactness of the following complex

$$0 \to \mathcal{M}_8 \to \mathcal{M}_7 \to \mathcal{M}_6 \to \mathcal{M}_5 \to \mathcal{M}_4 \to \mathcal{M}_3 \to \mathcal{M}_2 \to \mathcal{M}_1 \to \mathcal{M}_0$$

2. Conclusion

From the above work by proving the contracting homotopy for the maps define for the partition (13, 8) we gain that the seq. of the elements of W.re. of characteristic-free re. is exact.

References

[1] A. A. Abbas and H. R. Hassan, "Resolution for the two-rowed Weyl module in the cases of (6,5)/(1,0) and (6,5)/(2,0)," *Iraqi J. Sci.*, vol. 61, no. 2, pp. 416–421 (2020), doi: 10.24996/IJS.2020.61.2.20.

[2] N. Y. Khudair and H. R. Hassan, "Application of the two-rowed Weyl module in the case of partitions (7,7) and (7,7)/(1,0)," *Iraqi J. Sci.*, vol. 61, no. 5, pp. 1123–1135 (2020), doi: 10.24996/IJS.2020.61.5.21.

[3] N. Y. Khudair and W. K. Jaber, "Outcome for the case (11,10;1)," *Baghdad Sci. J.*, vol. 22, no. 2, pp. 664–676 (2025), doi: 10.21123/BSJ.2024.9905.

[4] R. H. Haytham and S. J. Niran, "Weyl Module Resolution Res(6,6,4;0,0) in the Case of Characteristic Zero," *Iraqi J. Sci.*, vol. 62, no. 4, pp. 1344–1348 (2021), doi: 10.24996/IJS.2021.62.4.30.

[5] M. I. Lfta and N. S. Jasim, "Computations for the groups SL(2,81) and SL(2,729)," *J. Interdiscip. Math.*, vol. 26, no. 7, pp. 1373–1378 (2023), doi: 10.47974/JIM-1553.

[6] M. S. Aldeen Zidan and S. Niran, "Enforcement for the groups SL(2,125) and SL(2,3125)," *J. Interdiscip. Math.*, vol. 26, no. 7, pp. 1379–1384 (2023), doi: 10.47974/JIM-1554.

[7] L. A. A. Hadi, M. S. Fiadh, N. S. Jasim, and J. A. Eleiwy, "Applications for the groups S.U.T.(2,p), where p is a prime greater than 9," *Iraqi J. Comput. Sci. Math.*, vol. 5, no. 1, pp. 78–84 (2024), doi: 10.52866/IJCSM.2024.05.01.005.

[8] H. B. Shelash and A. R. Ashrafi, "Table of marks and markaracter table of certain finite groups," *Quasigroups Relat. Syst.*, vol. 28, no. 1, pp. 159–170 (2020).

[9] G. Ahmed, S. Gedaan, and R. Ibrahim, "Spectrum of secondary submodules," *J. Phys. Conf. Ser.,* vol. 1879, no. 3, Art. no. 032007 (2021), doi: 10.1088/1742-6596/1879/3/032007.

[10] A. A. Aswhad, "Approximate solution of Emden-Fowler equation using the Galerkin method," *J. Univ. Anbar Pure Sci.*, vol. 17, no. 1, pp. 200–203 (2023), doi: 10.37652/JUAPS.2023.178798.

[11] M. S. Nayef, B. M. Hamad, and Z. A. Fadel, "Goldie Supplement Extending Modules," *Int. J. Math. Comput. Sci.,* vol. 19, no. 3, pp. 535–542 (2024).

[12] R. G. Abtan and R. S. Haraj, "Hyperfactored of the arrangements A(G24) and A(G27)," *J. Phys. Conf. Ser.,* vol. 1818, no. 1, Art. no. 012144 (2021).

[13] A. A. Abdulah and R. G. A. Al-Aleyawee, "Some new result on Shi arrangement," *AIP Conf. Proc.,* vol. 2834, no. 1, pp. 080133-1–080133-4 (2023).

[14] R. H. Haytham and S. J. Niran, "Exactness for complex sequence in the skew shape (8,8)/(1,0)," *J. Interdiscip. Math.,* vol. 24, no. 6, pp. 1661–1668 (2021), doi: 10.1080/09720502.2021.1892271.

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