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# Independent domination stability in graphs

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## Abstract

A non-empty subset $S \subseteq V$ of the vertex set of a simple graph $G = (V, E)$ is called an independent dominating set if every vertex not in $S$ is adjacent to at least one vertex in $S$, and no two vertices in S are adjacent to each other. The independent domination number of $G$, denoted by $\gamma_i(G)$, is the smallest possible size of such a set. The independent domination stability (or simply id-stability) of $G$ is defined as the minimum number of vertices that must be removed from the graph to alter its independent domination number. In this paper, we explore various properties of independent domination stability in graphs. Specifically,

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we establish several bounds and determine the id-stability for certain graph operations involving two graphs.

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***Keywords:*** *Dominating set, Independent domination number, Stability, Operation.*

## 1. Introduction

The various different domination concepts are well-studied now, however new concepts are introduced frequently and the interest is growing rapidly. We recommend two fundamental books [1, 2]. Let $G = (V(G), E(G))$ be a simple of finite orders graph, i.e., graphs are undirected with no loops or parallel edges and with finite number of vertices. For any vertex $v \in V(G)$, the open neighborhood of $v$ is the set $N(v) = \{u \in V(G) | uv \in E(G)\}$ and the closed neighborhood of $v$ is the set $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V(G)$, the open neighborhood of $S$ is $N(S) = \bigcup_{v \in S} N(v)$ and the closed neighborhood of $S$ is $N[S] = N(S) \cup S$. The private neighborhood $pn(v, S)$ of $v \in S$ is defined by $pn(v, S) = N(v) - N(S - \{v\})$. Equivalently, $pn(v, S) = \{u \in V | N(u) \cap S = \{v\}\}$. Each vertex in $pn(v, S)$ is called a private neighbor of $v$. The degree of a vertex $v$ is denoted by $\deg(v)$ and is equal to $|N(v)|$. A leaf of a tree is a vertex of degree 1, while a support vertex is a vertex adjacent to a leaf. A double star is a tree that contains exactly two vertices that are not end-vertices; necessarily, these two vertices are adjacent. A non-empty set $S \subseteq V(G)$ is a dominating set if $N[S] = V$, or equivalently, every vertex in $V(G) \backslash S$ is adjacent to at least one vertex in $S$ and the minimum cardinality of all dominating sets of $G$ is called the domination number of $G$ and is denoted by $\gamma(G)$. For further information on this subject and another kind of domination numbers, readers are referred to [1, 2, 3, 4, 5].

A domination-critical vertex in a graph $G$ is a vertex whose removal decreases the domination number. It is easy to observe that for any graph $G$ we have $\gamma(G) - 1 \leq \gamma(G + e) \leq \gamma(G)$ for every edge $e \not\in E(G)$. Sumner and Blitch in [6] have defined domination critical graphs. A graph $G$ is said to be domination critical, or $\gamma$-critical, if $\gamma(G + e) = \gamma(G) - 1$ for every edge $e$ in the complement $G^c$ of $G$. A graph is said to be domination stable, or $\gamma$-stable, if $\gamma(G) = \gamma(G + e)$ for every edge $e$ in the complement $G^c$ of $G$. For detailed information and results regarding the concept of domination critical graphs, we refer interested readers to the papers [7, 6, 8]. Bauer et al introduced the concept of domination stability in graphs in 1983 [9]. After then, was studied by Rad, Sharifi and Krzywkowski in [10]. Stability for different types of domination parameters has been investigated in the literature, for example, in [11, 12, 3, 13, 14]. This subject has considered and studied for another graph parameters. For example see [15, 16].

Recall that independent set in a graph $G$ is a set of pairwise non-adjacent vertices. A maximum independent set in $G$ is a largest independent set and its size is called independence number of $G$ and is denoted by $\alpha(G)$. An independent dominating set of $G$ is a set that is both dominating and independent in $G$. Equivalently, an independent dominating set is a maximal independent set. The independent domination number of $G$, denoted by $\gamma_i(G)$, is the minimum size of all independent dominating sets of $G$. An independent dominating set of cardinality $\gamma_i(G)$ is called a $\gamma_i$-set. A graph $G$ is independent domination critical, or $i$-critical if $\gamma_i(G-v) \neq \gamma_i(G)$ for every $v \in V(G)$. The independent domination and independent domination critical graphs have been reported in [17, 18, 19, 20]. This paper is organized as follows.

In the next section, we introduce the independent domination stability ($id$-stability) and present some of its properties. In Section 3, we obtain some bounds for the independent domination stability of graphs. In Section 4, we study the independent domination stability of some operations of two graphs.

## 2. Introduction to $id$-stability

We now define an independent analogue of domination stability. The independent domination stability or $id$-stability of a graph $G$, denoted $st_{id}(G)$, is the minimum number of vertices that must be removed from $G$ to change its independent domination number. To establish a standard notation, we denote the independent domination stability by $st_{id}(G)$. The $i^-$-stability of $G$, denoted by $st_{id}^-(G)$, is the minimum number of vertices whose removal decreases the independent domination number. Likewise, the $i^+$-stability of $G$, denoted by $st_{id}^+(G)$, is the minimum number of vertices whose removal increases the independent domination number. Thus the independent domination stability of a graph $G$ is $st_{id}(G) = \min\{st_{id}^-(G), st_{id}^+(G)\}$. Note that considering the null graph $K_0$, which has no vertices, as a graph, the independent domination stability of a non-trivial graph is always defined. For instance, $st_{id}(G) = |V(G)|$, occurs if and only if $G = K_n$.

Now we determine domination stability for some classes of graphs. To aid our discussion, we first state some straightforward observations as follows.

**Observation 2.1:**

- If $G$ is a star or a double star, then $st_{id}(G) = 1$.
- If $K_{m,n}$ is a bipartite complete graph where $m \geq n \geq 2$, then $st_{id}(K_{m,n}) = 1$.

The friendship (or Dutch-Windmill) graph $F_n$ is a graph that can be constructed by the coalescence of $n$ copies of the cycle graph $C_3$ of length 3 with a common vertex. The Friendship Theorem of Erdős, Rényi, and Sós [21], states that graphs with the property that every two vertices have exactly one neighbour in common are exactly the friendship graphs. Let $n$ and $q \geq 3$ be any positive integer and $F_{q,n}$ be the *generalized friendship graph* formed by a collection of $n$ cycles (all of order $q$), meeting at a common vertex. (see Figure 1).

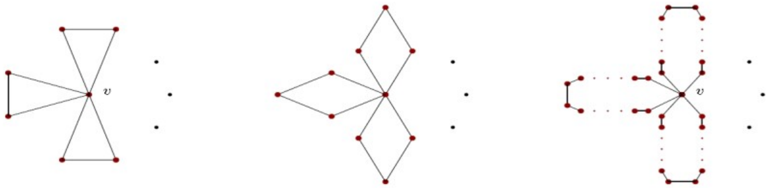

**Figure 1**
**The flowers $F_n, F_{4,n}$ and $F_{q,n}$, respectively.**

The $n$-book graph $(n \geq 2)$ (Figure 2) is defined as the Cartesian product $K_{1,n} \square P_2$. We call every $C_4$ in the book graph $B_n$, a page of $B_n$. All pages in $B_n$ have a common side $v_1 v_2$. We shall compute the distinguishing stability number of $B_n$. The following observation gives the independent domination number of the friendship graph and the book graph.

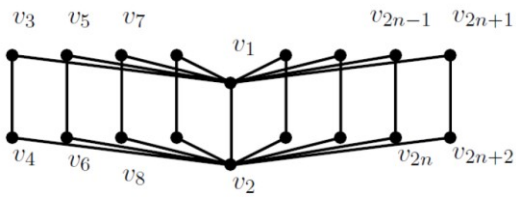


**Figure 2**
**Book graph $B_n$.**

**Observation 2.2:**

(i) If $F_n$ is a friendship graph, then $\gamma_i(F_n) = 1$.

(ii) $\gamma_i(F_{4,n}) = \gamma_i(F_{5,n}) = \gamma_i(F_{6,n}) = n + 1$

(iii) If $B_n$ is a book graph, then $\gamma_i(B_n) = n$.

**Proposition 2.3:**

(i) If $F_n$ is a friendship graph, then $st_{id}(F_n) = 1$.

(ii) For every $k \geq 3$, $st_{id}(F_{k,n}) = 1$.

(iii) $st_{id}(B_n) = 2$.

***Proof:***

(i) By removing the center vertex of $F_n$, we have $nK_2$ and obviously $\gamma_i(F_n) \neq \gamma_i(nK_2) = n$. So we have the result.

(ii) If $v$ is the center vertex of $F_{k,n}$, then $\gamma_i(F_{k,n} - v) = \gamma_i(P_{k-1}) = \left\lceil \frac{k+1}{3} \right\rceil \neq n + 1$, so we have the result.

(iii) By removing two non-central vertices $u, v$ in the page of $B_n$ we have $\gamma_i(B_n - \{u, v\}) = n - 1$.

**Theorem 2.4:** *There exist graphs $G$ and $H$ with the same independent domination number such that $|st_{id}(G) - st_{id}(H)|$ is very large.*

***Proof:*** Let $G = K_n$ with $\gamma_i(G) = 1$ and $st_{id}(G) = n$. Let $H$ denote the graph $K_{1,n-1}$ with $\gamma_i(H) = 1$ and $st_{id}(H) = 1$. We have $|st_{id}(G) - st_{id}(H)| = n - 1$ and so the result follows.

Before investigating the independent domination stability of paths ans cycles, we state the following observation.

**Observation 2.5:** For the path and cycle, $\gamma_i(P_n) = \gamma_i(C_n) = \left\lfloor \frac{n+2}{3} \right\rfloor$.

The following result establishes an upper bound on the independent domination stability in terms of $\delta(G)$.

**Observation 2.6:** For any graph $G$, $st_{id}(G) \leq \delta(G) + 1$.

We first determine the $i$-stability of paths.

**Proposition 2.7:** If $P_n$ is a path of order $n$, then $st_{id}(P_n) = \begin{cases} 2 & n \equiv 2(mod\ 3) \\ 1 & \text{otherwise} \end{cases}$.

***Proof:*** We consider the following cases:

**Case 1:** Suppose first that $n \equiv 0(mod\ 3)$. We observe that $\gamma_i(P_n - v) = \gamma_i(P_n) + 1$, where $v$ is a support vertex. So, $st_{id}(P_n) = 1$.

**Case 2:** Assume that $n \equiv 1(mod 3)$. If $v$ is a leaf, then $\gamma_i(P_n - v) = \gamma_i(P_n) - 1$, and it follows that $st_{id}(P_n) = 1$.

**Case 3:** Suppose that $n = 3k + 2$ for some integer $k$. By Observation 2.5, we have $\gamma_(P_n) = k + 1$. We show that the removal of $v$ does not change the independent domination number. For this purpose, we consider the following subcases.

**Subcase 3.1:** If $v$ is a leaf, then, $\gamma_i(P_n - v) = \gamma_i(P_{n-1}) = k + 1 = \gamma_i(P_n)$.

**Subcase 3.2:** Let $deg(v) = 2$ and $P_n - v$ consists of two connected components $P_{n_1}$ and $P_{n_2}$ such that $n_1 + n_2 = n - 1$. Without loss of generality, we first assume that $n_1 \equiv 0(mod 3)$ and $n_2 \equiv 1(mod 3)$, and so

$$\gamma_i(P_n - v) = \gamma_i(P_{n_1}) + \gamma_i(P_{n_2}) = \left\lfloor \frac{n_1+2}{3} \right\rfloor + \left\lfloor \frac{n_2+2}{3} \right\rfloor = \frac{n_1}{3} + \frac{n_2+2}{3} = k + 1 = \gamma_i(P_n).$$

Analogously, we can obtain that $\gamma_i(P_n - v) = \gamma_i(P_n)$ when $n_1 \equiv 2(mod 3)$ and $n_2 \equiv 2(mod\ 3)$.

Now, by Subcases 3.1 and 3.2, we conclude that $st_i(P_n) \geq 2$. Moreover, recall that by Proposition 2.6, we have $st_{id}(P_n) \leq 2$. Consequently, $st_{id}(P_n) = 2$ and the result follows.

Next, we determine the $i$-stability of cycles.

**Proposition 2.8:** If $C_n$ is a cycle of order $n$, then $st_{id}(C_n) = \begin{cases} 3 & n \equiv 0(mod\ 3) \\ 2 & n \equiv 2(mod\ 3). \\ 1 & n \equiv 1(mod\ 3) \end{cases}$

***Proof:*** We consider the following cases:

**Case 1:** We assume that $n = 3k + 1$ for some integer $k$. Then for any vertex $v$ we have $\gamma_i(C_n - v) = \gamma_i(P_{n-1}) = k = \gamma_i(C_n) - 1$, and so $st_{id}(C_n) = 1$.

**Case 2:** Suppose that $n = 3k + 2$ for some integer $k$. For any vertex $v$ we have $\gamma_i(C_n - v) = \gamma_i(P_{n-1}) = k + 1 = \gamma_i(C_n)$, and it holds that $st_i(C_n) \geq 2$. Now we consider that $u$ and $v$ are two adjacent vertices. Then, $\gamma_i(C_n - u - v) = \gamma_i(P_{n-2}) = k = \gamma_i(C_n) - 1$, which implies $st_{id}(C_n) = 2$.

**Case 3:** Assume that $n = 3k$ for some integer $k$. It is easy to see that the removal of any vertex does not change the independent domination number. Since $C_n - v = P_{n-1}$, by Proposition 2.5 we have $st_i(P_{n-1}) = 2$, and so $st_i(C_n) = 3$.
W

**3. Bounds on the independent domination stability**

In this section, we establish some bounds for the independent domination stability of a graph.

Here, we study the relationship between the $id$-stabilities of graphs $G$ and $G - v$, where $v \in V(G)$. Also we obtain upper bounds for $st_{id}(G)$.

**Proposition 3.1:** Let $G$ be a graph and $v$ be a vertex of $G$, then

$$st_{id}(G) \leq st_{id}(G - v) + 1.$$

***Proof:*** If $\gamma_i(G) = \gamma_i(G - v)$, then $st_{id}(G) \leq st_{id}(G - v) + 1$. If $\gamma_i(G) \neq \gamma_i(G - v)$, then $st_{id}(G) = 1$, and so we have the result.

Using Proposition 3.1 and mathematical induction, we have

$$st_{id}(G) \leq st_{id}(G - v_1 - \cdots - v_s) + s,$$

where $1 \leq s \leq n - 2$ and $n = |V(G)|$. Only using this formula we can get different upper bounds for $st_{id}(G)$ of graph $G$. We state some these upper bounds in the following theorem which for the proof of each case, it is sufficient to remove the vertices of $G$ until the induced subgraph which stated in the hypothesis, appears. Next using $st_{id}(G) \leq st_{id}(G - v_1 - \cdots - v_s) + s$ and the value of $st_{id}(G - v_1 - \cdots - v_s)$, we can have the result.

**Theorem 3.2:** *Let $G \neq K_n$ be a simple graph of order $n \geq 2$.*

(i) $st_{id}(G) \leq n - 1$.

(ii) *If graph $G$ has the star graph $K_{1,t}$ as the induced subgraph with $t \geq 3$, then $st_{id}(G) \leq n - t$.*

(iii) *For any graph $G$, $st_{id}(G) \leq n - \Delta(G)$.*

**Theorem 3.3:** *If $G$ is a graph of order $n$, then $st_{id}(G) \leq n + 1 - 2\gamma_i(G)$.*

***Proof:*** Set $st_{id}(G) = k$. Thus for every $i$ vertices of $G$ $(1 \leq i \leq k-1)$, say $v_1, \dots, v_i$, we have $\gamma_{id}(G) = \gamma_{id}(G - v_1) = \cdots = \gamma_{id}(G - v_1 - \cdots - v_i)$. It is known that the independent domination number of a graph is at most equal to half of its order, so $\gamma_i(G) = \gamma_i(G - v_1 - \cdots - v_{k-1}) \leq \frac{n-k+1}{2}$.

**Corollary 3.4:** *Let $G$ be a graph of order $n \geq 2$. If $st_{id}(G) = n - 1$, then $\gamma_i(G) = 1$.*

***Proof:*** It is a direct consequence of Theorem 3.3.

**Theorem 3.5:** *For any graph $G$ with $\gamma_i(G) \geq 2$ we have $st_{id}(G) \leq \frac{n}{\gamma_i(G)}$.*

***Proof:*** Let $I$ be an independent dominating set of $G$. We consider the following cases.

**Case 1:** We assume that $u$ is a vertex of $I$ which has no private neighbor in $V(G) - I$. Then the removal of $u$ in $V(G) - I$ decreases the independent domination number, which holds that $st_i(G) \leq \frac{n}{\gamma_i(G)}$.

**Case 2:** Suppose that $w$ is a vertex of $I$ with minimum number of private neighbors in $V(G) - I$. Then the removal of $w$ and its private neighbors in $V(G) - I$ decreases the independent domination number, which follows that $st_{id}(G) \leq \frac{n}{\gamma_i(G)}$.

**Proposition 3.6:** [18] If $G$ is an isolate-free graph of order $n$, then

$$\gamma_i(G) \leq n + 2 - \gamma(G) - \left\lceil \frac{n}{\gamma(G)} \right\rceil.$$

**Proposition 3.7:** [1] For every connected graph $G \neq K_1$ we have $\gamma(G) \leq \frac{n}{2}$.

**Proposition 3.8:** If $k \in \mathbb{N}$ and $\gamma_i(G) \geq k > 1$, then there is no isolate-free graph $G$ of order $n$ with $\gamma_i(G) \geq 2$ and $st_{id}(G) = n - k$.

***Proof:*** Assume that $G$ is an isolate-free graph with $\gamma_i(G) \geq 2$ and $st_{id}(G) = n - k$. By Proposition 3.6, we have $n > \frac{nk}{\gamma_i(G)} \geq \frac{k(\gamma_i(G) - 2 + \gamma(G) + \frac{n}{\gamma(G)})}{\gamma_i(G)}$. Using Proposition 3.7, it can be seen that $n - k = st_{id}(G) > \frac{k\frac{n}{2}}{\gamma_i(G)}$. Since $k \geq 2$, then $n - k = st_{id}(G) > \frac{n}{\gamma_i(G)}$, contradicting Theorem 3.5.

**Proposition 3.9:** For any graph $G$ with $\gamma_i(G) \geq 2$, we have

$$st_{id}(G) \leq \min\{\delta(G)+1, n-\delta(G)-1\}.$$

***Proof:*** Let $I$ be an independent dominating set. Suppose that there is a vertex $u \in I$ such that $epn(u,I) = \emptyset$, so $st_{id}(G) = 1$. Now, we assume that for each vertex $u \in I$ we have $epn(u,I) \neq \emptyset$. Suppose that $v \in I$, and let $X$ be the set of private neighbors of $v$ in $V(G) - I$. Therefore $\gamma_i([G(X \cup \{v\}]) = 1 < \gamma_i(G)$. By Observation 2.6, we have the result.

Here, we shall state Nordhaus–Gaddum type inequalities for the sum of the independent domination stability of a graph and its complement. Note that if $\gamma_i(G) = 1$, then $st_{id}(\overline{G}) = 1$, and thus we obtain the following result.

**Observation 3.10:** If $G$ is a graph of order $n$ with $\gamma_i(G) = 1$ or $\gamma_i(\overline{G}) = 1$, then $st_{id}(G) + st_i(\overline{G}) \leq n+1$, and this bound is sharp for the complete graphs.

**Theorem 3.11:** *If $G$ is a graph with $\gamma_i(G) \geq 2$ and $\gamma_i(\overline{G}) \geq 2$, then*

$$st_{id}(G) + st_{id}(\overline{G}) \leq \begin{cases} n & n = 2k \\ n-1 & n = 2k+1 \end{cases}$$

***Proof:*** If $n = 2k+1$ for some integer $k$, then by Theorem 3.5 we have $st_{id}(G) + st_{id}(\overline{G}) \leq \frac{n-1}{2} + \frac{n-1}{2} = n-1$. Now, we suppose that $n = 2k$. Using Proposition 3.9, we observe that

$$\begin{aligned} st_{id}(G) + st_{id}(\overline{G}) \; &\leq \; min\{\delta(G)+1, n-\delta(G)-1\} + min\{\delta(\overline{G}) \\ &\qquad +1, n-\delta(\overline{G})-1\} \\ &\leq \frac{n}{2} + \frac{n}{2} = n. \end{aligned}$$

## 4. $id$-stability of some operations of two graphs

In this section, we study the independent domination stability of some operations of two graphs. First we consider the join of two graphs. The join $G_1 + G_2$ of two graphs $G_1$ and $G_2$ with disjoint vertex sets $V_1$ and $V_2$ and edge sets $E_1$ and $E_2$ is the graph union $G_1 \cup G_2$ together with all the edges joining $V_1$ and $V_2$.

**Observation 4.1:** If $G_1$ and $G_2$ are nonempty graphs, then

$$\gamma_i(G_1 + G_2) = \min\{\gamma_i(G_1), \gamma_i(G_2)\}.$$

***Proof:*** By the definition, every $\gamma_i$-set $D_1$ of $G_1$ (or $\gamma_i$-set $D_2$ of $G_2$), is a $\gamma_i$-set of $G_1 + G_2$. So we have result.

By Observation 4.1, we have the following result.

**Theorem 4.2:** *If $G_1$ and $G_2$ are nonempty graphs, then*

$$st_{id}(G_1 + G_2) = \min\{st_{id}(G_1), st_{id}(G_2)\}.$$

Here, we recall the definition of lexicographic product of two graphs. For two graphs $G$ and $H$, let $G[H]$ be the graph with vertex set $V(G) \times V(H)$ and such that vertex $(a, x)$ is adjacent to vertex $(b, y)$ if and only if $a$ is adjacent to $b$ (in $G$) or $a = b$ and $x$ is adjacent to $y$ (in $H$). The graph $G[H]$ is the lexicographic product (or composition) of $G$ and $H$, and can be thought of as the graph arising from $G$ and $H$ by substituting a copy of $H$ for every vertex of $G$ ([22]).

The following theorem gives the independent domination number of $G[H]$.

**Theorem 4.3:** *If $G$ and $H$ are two graphs, then*

$$\gamma_i(G[H]) = \gamma_i(G)\gamma_i(H).$$

***Proof:*** An independent dominating set in $G[H]$ of minimum cardinality, arises by choosing an independent dominating set in $G$ of cardinality $\gamma_i(G)$, and then, within each copy of $H$ in $G[H]$, choosing an independent dominating set in $H$ with cardinality $\gamma_i(H)$. Thus, we have the result.

By Theorem 4.3, we have the following result.

**Corollary 4.4:** *If $G$ and $H$ are two graphs, then*

$$st_{id}(G[H]) = min\{st_{id}(G), st_{id}(H)\}.$$

Now, we obtain the $id$-stability of corona of two graphs. We first state and prove the following theorem.

**Theorem 4.5:** *If $G$ and $H$ are two graphs, then $\gamma_i(G \circ H) = |V(G)|\gamma_i(H)$.*

***Proof:*** An independent dominating set in $G \circ H$ of minimum cardinality, arises by choosing an independent dominating set with minimum cardinality in each copy of $H$ in $G \circ H$. So we have the result.

By Theorem 4.5, we have the following result.

**Corollary 4.6:** *If $G$ and $H$ are two graphs, then $st_{id}(G \circ H) = 1$.*

***Proof:*** By Theorem 4.5, $\gamma_i(G \circ H) = |V(G)|\gamma_i(H)$, so by removing one vertex from $G$, the order of $G$ and so $\gamma_i(G \circ H)$ will be changed. Therefore, we have the result.

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