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Employing for the case (11,8,3)/(1,0,0)

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Abstract

In this article we employ the mapping cone (mapp.co.) to obtain the exactness of the sequence (seq.) of the Lascoux (L.) elements of the Weyl module (W.m.) resolution (res.) by in usage the diagrams (diag.) of mapp.co. and employing the Capelli identities for the case (11,8,3)/(1,0,0).

Subject Classification (2020): *13D07.*

Keywords: *Mapping cone, Capelli identities, Lascoux elements.*

1. Introduction

This research employ the same idea in [1,2] for (11,8,3)/(1,0,0), where the W.m. is $\mathcal{K}_{\lambda/\mu}\mathcal{F} = \mathrm{Im}(\mathcal{DF} \to \Lambda\mathcal{F})$ for the partition λ/μ, we depended this work for Capelli identities, [3, 4].

We can applied that for $\mathcal{SL}(2,\mathfrak{U})$, $\mathfrak{U}$ =81, 729, 125, 3125, 121, 196, PG(3,8), [5-6], some finite groups and fractional class [7-8], Also for the studies by authors in [9-12].

2. The Result with discussion

For (11,8,3)/(1,0,0) the seq. of res. L. elements is

$$\mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}\mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$$

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$\mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \ \to \oplus \to \oplus$
$\mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \ \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$
$\to \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$

Define the following maps from Figure 1.

$h_1(s) = \partial_{21}(s)$; $s \in \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F}$,
$f_1(s) = \partial_{32}(s)$; $s \in \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F}$,
$h_2(s) = \partial_{21}^{(2)}(s)$; $s \in \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$, and
$g_2(s) = \partial_{32}(s)$; $s \in \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$

$$\begin{tikzcd}
0 \arrow[r] & \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \arrow[r, "h_1"] \arrow[d, "f_1"'] \arrow[dr, phantom, "\mathrm{A}"] & \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \arrow[d, "g_1"] \\
0 \arrow[r] & \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \arrow[r, "h_2"] \arrow[d, "f_2"'] \arrow[dr, phantom, "\mathrm{B}"] & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \arrow[d, "g_2"] \\
0 \arrow[r] & \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \arrow[r, "h_3"] & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \arrow[d, "d'"] \\
 & & \mathcal{K}_{(10,8,3)}(\mathcal{F}) \arrow[d] \\
 & & 0
\end{tikzcd}$$

Figure 1
The diagram for (11,8,3)/(1,0,0) the seq. of res. L. elements

We need define g_1 which make sub diag. A in Figure 1 is a commute. We define it as $g_1(s) = \left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(s)$

Proposition 2.1: The sub diag. A in Figure 1 is a commute.

Proof: $(h_2 \circ f_1)(s) = \left(\partial_{21}^{(2)}\,\partial_{32}\right)(s)$

From Capelli identities $\partial_{21}^{(2)}\,\partial_{32} = \partial_{32}\,\partial_{21}^{(2)} - \partial_{21}\partial_{31}$. Thus

$(h_2 \circ f_1)(s) = \partial_{32}\,\partial_{21}^{(2)} - \partial_{21}\partial_{31} = \left(\frac{1}{2}\partial_{32}\partial_{21}\partial_{21} - \partial_{31}\,\partial_{21}\right)(s)$
$= \left(\frac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)\partial_{21}(s) = (g_1 \circ h_1)(s)$

Now from the sub diag. A, we have the subsequence

$$0 \to \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \xrightarrow{\varphi_3} \begin{matrix} \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \\ \oplus \\ \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \end{matrix}$$

$$\xrightarrow{\delta_1} \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$$

Where $\varphi_3(s) = \big(-\partial_{21}(s), \partial_{32}(s)\big)$ and

$$\delta_1(s_1, s_2) = \partial_{21}^{(2)}(s_2) + \left(\tfrac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(s_1)$$

Proposition 2.2: The following subsequence is a subcomplex

$$0 \to \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \xrightarrow{\varphi_3} \begin{array}{c} \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \\ \oplus \\ \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \end{array} \xrightarrow{\delta_1} \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$$

Proof: $(\delta_1 \circ \varphi_3)(s) = \lambda_1\big(-\partial_{21}(s), \partial_{32}(s)\big)$

$= \big(\partial_{21}^{(2)}\partial_{32}\big)(s) - \big(\partial_{32}\,\partial_{21}^{(2)}\big)(s) + (\partial_{31}\partial_{21})(s)$

By using Capelli identities we get

$(\delta_1 \circ \varphi_3)(s)=\big(\partial_{32}\,\partial_{21}^{(2)}\big)(s) - (\partial_{21}\partial_{31})(s) - \big(\partial_{32}\,\partial_{21}^{(2)}\big)(s) + \big(\partial_{31}^{(2)}\big)(s) = 0$

Now consider the seq.

$$\begin{tikzcd}
0 \arrow[r] & \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} & \\
{} \arrow[r, "\varphi_3"] & \begin{array}{c} \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \\ \oplus \\ \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \end{array} \arrow[r, "\delta_1"] \arrow[d, "\delta_2"'] \arrow[dr, phantom, "C"] & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \arrow[d, "g_2"] \\
& \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \arrow[r, "h_3"] & \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \arrow[d, "d'"] \\
& & \mathcal{K}_{(10,8,3)}(\mathcal{F}) \arrow[d] \\
& & 0
\end{tikzcd}$$

Figure 2
The sub diagram for (11,8,3)/(1,0,0) the seq. of res. L. elements

Now define the maps by $h_3(s) = \partial_{21}(s)$; $s \in \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$, $\delta_2(s_1, s_2) = \partial_{32}^{(2)}(s_1) + \left(\tfrac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)(s_2)$;

$s_1 \in \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_1\mathcal{F}$, $s_2 \in \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$,

Proposition 2.3: The sub diag. C in Figure 2 is commute.

Proof: $(g_2 \circ \delta_1)(s_1, s_2) = g_2\left(\partial_{21}^{(2)}(s_2) + \left(\tfrac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(s_1)\right)$

$$= \left(\partial_{32}\,\partial_{21}^{(2)}\right)(\mathscr{s}_2) + \left(\,\partial_{32}^{(2)}\partial_{21} - \partial_{32}\partial_{31}\right)(\mathscr{s}_1)$$

From Capelli identities we have

$$(\mathscr{g}_2 \circ \delta_1)(\mathscr{s}_1, \mathscr{s}_2)$$

$$= \left(\partial_{21}^{(2)}\partial_{32}\right)(\mathscr{s}_2) + (\partial_{21}\partial_{31})(\mathscr{s}_2) + \left(\partial_{21}\,\partial_{32}^{(2)}\right)(\mathscr{s}_1) + (\partial_{32}\partial_{31})(\mathscr{s}_1)$$

$$= \left(\tfrac{1}{2}\partial_{21}\partial_{21}\partial_{32}\right)(\mathscr{s}_2) + (\partial_{21}\partial_{31})(\mathscr{s}_2) + \left(\partial_{21}\,\partial_{32}^{(2)}\right)(\mathscr{s}_1)$$

$$= \partial_{21}\left(\left(\tfrac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)(\mathscr{s}_2) + \partial_{32}^{(2)}(\mathscr{s}_1)\right) = (\hbar_3 \circ \delta_2)(\mathscr{s}_1, \mathscr{s}_2)$$

Theorem 2.6: *The following seq. is exact.* $\mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_1\mathcal{F}$

$0 \to \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \xrightarrow{\varphi_3} \oplus \xrightarrow{\varphi_2} \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$

$\mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_2\mathcal{F} \;\oplus\; \xrightarrow{\varphi_1} \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_3\mathcal{F} \xrightarrow{d'_{(10,8,3)(\mathcal{F})}}, \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_3\mathcal{F}$

$\mathcal{K}_{(10,8,3)}(\mathcal{F}) \to 0$, where, $\varphi_3(\mathscr{s}_1, \mathscr{s}_2) =$

$\left(-\,\partial_{21}^{(2)}(\mathscr{s}_2) - \left(\tfrac{1}{2}\partial_{32}\partial_{21} - \partial_{31}\right)(\mathscr{s}_1), \partial_{32}^{(2)}(\mathscr{s}_1) + \left(\tfrac{1}{2}\partial_{21}\partial_{32} + \partial_{31}\right)(\mathscr{s}_2)\right),$

$\varphi_1(\mathscr{s}_1, \mathscr{s}_2) = \partial_{32}(\mathscr{s}_1) + \partial_{21}(\mathscr{s}_2)$

Proof: Since the diag.s A and C in a diag. s (1) and (2) are commute and the maps of place polarization are injective [3] then the maps $\hbar_1$ and $\hbar_2$ are injective, [4]. But $\delta_1 \circ \varphi_3 = 0$ so the mapping cone satisfied then

$0 \;\to\; \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \xrightarrow{\varphi_3} \mathcal{D}_{11}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_1\mathcal{F} \oplus \xrightarrow{\delta_1} \mathcal{D}_{10}\mathcal{F} \otimes \mathcal{D}_9\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}, \mathcal{D}_{12}\mathcal{F} \otimes \mathcal{D}_7\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$ is exact.

From Proposition (2.3) the diag. Q is commute and the map $\hbar_3$ is injective [4], then we have the diag. (2) commute with exact rows. But

$(\varphi_2 \circ \varphi_3)(\mathscr{s}_1) = \varphi_2(-\partial_{21}(\mathscr{s}), \partial_{32}(\mathscr{s}))$, $\mathscr{s} \in \mathcal{D}_8\mathcal{F} \otimes \mathcal{D}_6\mathcal{F} \otimes \mathcal{D}_2\mathcal{F}$

$=\Big(\left(-\,\partial_{21}^{(2)}\partial_{32}\right)(\mathscr{s}_1) + \left(\partial_{32}\,\partial_{21}^{(2)} - \partial_{31}\partial_{21}\right)(\mathscr{s}_1), -\left(\partial_{32}^{(2)}\partial_{21}(\mathscr{s}_1) +\right.$

$\left.\left(\partial_{21}\,\partial_{32}^{(2)} + \partial_{31}\partial_{32}\right)(\mathscr{s}_1)\right) =$

$\Big(\left(-\,\partial_{21}^{(2)}\partial_{32}\right)(\mathscr{s}_1) + \left(\partial_{21}^{(2)}\partial_{32}\right)(\mathscr{s}_1) + (\partial_{21}\partial_{31})(\mathscr{s}_1) - (\partial_{21}\partial_{31})(\mathscr{s}_1),$

$-\left(\partial_{32}^{(2)}\partial_{21}(\mathscr{s}_1) + \left(\partial_{32}^{(2)}\partial_{21}\right)(\mathscr{s}_1) - (\partial_{32}\partial_{31})(\mathscr{s}_1) + (\partial_{32}\partial_{31})(\mathscr{s}_1)\right) = (0,0)$

So, $\varphi_2 \circ \varphi_3 = 0$ and as the same way $\varphi_1 \circ \varphi_2 = 0$. Thus the exactness satisfy.

Conclusion

From the above work by proving the contracting homotopy for the maps define for the case (11,8,3)/(1,0,0), we gain that the seq. of res. L. elements is exact.

References

[1] N. Y. Khudair and H. R. Hassan, "Application of the Two Rowed Weyl Module in the Case of Partitions (7,7) and (7,7)/(1,0)," *Iraqi Journal of Science,* vol. 61, no. 5, pp. 1123–1135 (May 2020), doi: 10.24996/ijs.2020.61.5.21.

[2] A. A. Abbas and H. R. Hassan, "Resolution for the Two-Rowed Weyl Module in the Cases of (6,5)/(1,0) and (6,5)/(2,0)," *Iraqi Journal of Science,* vol. 61, no. 2, pp. 416–421 (Feb. 2020), doi: 10.24996/ijs.2020.61.2.20.

[3] W. K. Jaber and N. S. Jasim, "Outcome for the Case (11,10; 1)," *Baghdad Science Journal,* vol. 22, no. 2, pp. 664–676 (2025), doi: 10.21123/bsj.2024.9905.

[4] H. R. Hassan and N. S. Jasim, "Weyl Module Resolution Res (6,6,4;0,0) in the Case of Characteristic Zero," *Iraqi Journal of Science,* vol. 62, no. 4, pp. 1344–1348 (Apr. 2021), doi: 10.24996/ijs.2021.62.4.30.

[5] S. J. Kadhum, A. I. Abdul-Nabi, and N. S. Jasim, "Compute some concepts in PG(3,8)," *J. Discrete Math. Sci. Cryptogr.,* vol. 25, no. 2, pp. 599–604 (2022), doi: 10.1080/09720529.2021.1982489.

[6] S. J. Kadhum, N. S. Jasim, and A. I. Abdul-Nabi, "Results for the (b,t)-Blocking Sets in PG(2,8)," *AIP Conference Proceedings,* vol. 2845, no. 1, pp. 50044–50050 (2023), doi: 10.1063/5.0170047.

[7] H. B. Shelash and A. R. Ashrafi, "The Number of Subgroups of a Given Type in Certain Finite Groups," *Iran. J. Math. Sci. Inform.,* vol. 16, no. 2, pp. 73–87 (2021), doi: 10.52547/ijmsi.16.2.73.

[8] Z. E. Abdulnaby and A. E. Abdul-Nabi, "Integral transforms defined by a new fractional class of analytic function in a complex Banach space," *J. Phys. Conf. Ser.,* vol. 1294, no. 3, Art. no. 032008 (2019), doi: 10.1088/1742-6596/1294/3/032008.

[9] A. A. Aswhad, "Approximate Solution of Emden-Fowler Equation Using the Galerkin Method," *Journal of University of Anbar for Pure Science,* vol. 17, no. 2, pp. 200–203 (2023), doi: 10.37652/juaps.2023.178798.

[10] O. M. Salah, G. J. M. Mahdi, and I. A. A. Al-Latif, "A modified ARIMA model for forecasting chemical sales in the USA," *J. Phys. Conf. Ser.,* vol. 1879, no. 3, Art. no. 032008 (2021), doi: 10.1088/1742-6596/1879/3/032008.

[11] R. G. Abtan and R. S. Haraj, "Hyperfactored of the arrangements A(G24) and A(G27)," *J. Phys. Conf. Ser.,* vol. 1818, no. 1, Art. no. 012144 (2021).

[12] A. A. Abdulah and R. G. A. Al-Aleyawee, "Some new result on Shi arrangement," *AIP Conference Proceedings,* vol. 2834, no. 1, Art. no. 080133 (2023).

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