Journal of Discrete Mathematical Sciences & Cryptography
ISSN 0972-0529 (Print), ISSN 2169-0065 (Online)
Vol. 29 (2026), No. 7, pp. 2699–2709
DOI : 10.47974/JDMSC-2388

CTATD Number for power graph of various Ladder Graphs

G. Mahadevan *
K. Priya †
Department of Mathematics
The Gandhigram Rural Institute (Deemed to be University)
Gandhigram 624302
Tamil Nadu
India

C. Sivagnanam §
Department of Mathematics
University of Technology and Applied Sciences
Sur
Sultanate of Oman

Abstract

The concept of the CTATD number has been recently further developed, expanding on the original work by Mahadevan, Priya, and Sivagnanam [4]. A subset $S \subseteq V$ is considered a CTATD set for a graph G if for each vertex $v \in V - S$, the intersection of v's neighborhood with S contains between one and two vertices, and for any set of three vertices in $<V - S>$, they must be connected along a path. The smallest size of such a CTATD set is referred to as CTATD(G). This paper specifically focuses on analyzing the CTATD number for various types of graphs, including DL_l, $O(DL_l)$, TL_l, $O(TL_l)$, SL_l, CL_l, ML_l, providing explicit values for their CTATD numbers instead of just their bounds.

Subject Classification: *05C69.*

Keywords: *Ladder graphs, Triple connected, Power graphs, [1, 2] dominating set.*

* *E-mail:* **drgmaha2014@gmail.com (Corresponding Author)**
† *E-mail:* **priyak250796@gmail.com**
§ *E-mail:* **choshi71@gmail.com**

Introduction

A simple, connected, undirected graph is denoted as $G = (Y(G), Z(G))$, where the order and size are represented by l and t respectively, as defined in [1], [2], [10], [14], [16], [17], [18], [19]. The concept of a [1, 2] set was initially explored by Chellali et al. [3], and later expanded upon by Yang and Wu [6]. The development of the $\gamma_t(G)$ parameter was contributed by Joseph et al. [5], while Priya et al. [7] were the first to introduce the CTATD-number for graphs.

The diagonal ladder graph DL_l [8] is generated by the strong product of two paths: P_2 (which has two vertices) and P_l (which has l vertices). An open diagonal ladder graph, referred to as $O(DL_l)$, is obtained by removing the edges $p_i d_i$ for each i, along with the edge $p_l d_l$. The triangular ladder graph TL_l [9] is created by modifying a ladder graph by adding the edges $d_i p_{i-1} for 2 \leq i \leq l$. An open triangular ladder graph is formed by removing the edges $d_1 p_1$ and $d_l p_l$ from the triangular ladder graph. This modified graph is denoted as $O(TL_l)$. A slanting ladder graph SL_l [11] is obtained by taking a triangular ladder graph and removing the edges $d_i p_i; 1 \leq i \leq l$. The circular ladder graph CL_l [12] is derived from a ladder graph by adding the edges $d_1 d_l$ and $p_1 p_l$, effectively connecting the first and last vertices of the top and bottom paths. The Möbius ladder graph ML_l [13] is formed by modifying a ladder graph with the addition of the edges $v_i u_i; 1 \leq i \leq l$, creating a twisted connection between the first and last vertices of the top and bottom paths. The author computed the CTATD(G)(complementary triple connected at most twin domination number) for the power graph of certain graphs in [4], [15]. This article includes Section 2, which discusses certain ladder graphs in standard form, and Section 3, which covers the power graphs of various ladder graphs. The article concludes with a summary.

2. CTATD- Number for variety of L_l

Theorem 1: *If* $l \geq 3$, *then* $CTATD(DL_l) = \left\lceil \frac{l}{3} \right\rceil$.

Proof: Let $Y(DL_l) = \{p_i, d_i : 1 \leq i \leq l\}$.

$Z(DL_l) = \{p_i p_{i+1}, d_i d_{i+1}, p_i d_{i+1}, v d_i p_{i+1} 1 \leq i \leq l-1\} \cup \{p_i d_i : 1 \leq i \leq l\}$.

Let $A_1 = \{d_i : i \equiv 2 \ mod \ 3\}$.

Assume $S = \begin{cases} A_1 & if \ l = 3e \ or \ 3e+2 \\ A_1 \cup \{d_l\} & if \ l = 3e+1. \end{cases}$

If S is a CTATD-set of (DL_l) then $CTATD(DL_l) \leq |S| = \left\lceil \frac{l}{3} \right\rceil$. For any set D with a size of at most $f = \left\lceil \frac{l}{3} \right\rceil - 1$, the graph $< V - D >$ fails to be triple connected. Consequently, $|D| \geq f + 1 = \left\lceil \frac{l}{3} \right\rceil$. Completing the proof.

Example:

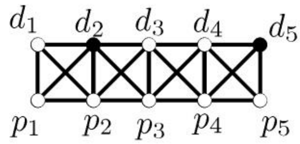

Figure 1
($D(L_5)$)- Diagonal ladder graph

Figure 1, l=5, then $\left\lceil\frac{l}{3}\right\rceil = \left\lceil\frac{5}{3}\right\rceil = 2$.

Theorem 2: *If* $l \geq 3$*, then* $CTATD(O(DL_l)) = \left\lceil\frac{l}{2}\right\rceil + 2$.

Proof: Let $Y(O(DL_l)) = \{p_i, d_i: 1 \leq i \leq l\}$ and $Z(O(DL_l)) = \{p_i p_{i+1}, d_i d_{i+1}, p_i d_{i+1}, d_i p_{i+1}: 1 \leq i \leq l-1\} \cup \{p_i d_i; 2 \leq i \leq l-1\}$. Let $B_1 = \{d_i: i = 3e + 2\} \cup d_1$.

$$\text{Assume } S = \begin{cases} B_1 \cup \{d_l\} & if\ l = 3e \\ B_1 \cup \{d_{l-1}, d_l\} & if\ l = 3e+1 \\ B_1 \cup \{d_{l-1}\} & if\ l = 3e+2. \end{cases}$$

If S is a CTATD-set of $(O(DL_q))$ then $CTATD(O(DL_q)) \leq |S| = \left\lceil\frac{l}{2}\right\rceil + 2$. For any set D with a size of at most $f = \left\lceil\frac{q}{2}\right\rceil + 1$, the graph $< V - D >$ fails to be triple connected.Consequently, $|D| \geq f + 1 = \left\lceil\frac{l}{2}\right\rceil + 2$. Completing the proof.

Theorem 3: *If* $l \geq 3$*, then*

$$CTATD(T(L_l)) = \begin{cases} 2\left\lceil\frac{l}{5}\right\rceil & ifl = 5e\ or\ 5e+3\ or\ 5e+4 \\ 2\left\lceil\frac{l}{5}\right\rceil - 1 & otherwise. \end{cases}$$

Proof: Let $Y(T(L_l)) = \{p_i, d_i: 1 \leq i \leq l\}$ and $Z(T(L_l)) = \{p_i p_{i+1}, d_i d_{i+1}, d_i p_{i+1}: i \in [1, l-1]\} \cup \{p_i d_i: i \in [1, l]\}$.

Let $B_1 = \{p_i: i = 5e + 4, d_i: i = 5e + 2\}$.

$$\text{Assume } S = \begin{cases} B_1 & if\ l = 5e\ or\ 5e+2\ or\ 5e+4 \\ B_1 \cup \{d_l\} & if\ l = 5e+1 \\ B_1 \cup \{p_l\} & if\ l = 5e+3. \end{cases}$$

If S is a CTATD-set of $(T(L_l))$ then

$$CTATD(T(L_l)) \leq |S| = \begin{cases} 2\left\lceil\frac{l}{5}\right\rceil & ifl = 5e\ or\ 5e+3\ or\ 5e+4 \\ 2\left\lceil\frac{l}{5}\right\rceil - 1 & otherwise. \end{cases}$$

$$f = \begin{cases} 2\left\lceil \frac{l}{5} \right\rceil - 1 & if l = 5e \ or \ e5 + 3 \ or \ 5e + 4 \\ 2\left\lceil \frac{l}{5} \right\rceil & otherwise, \end{cases}$$

the graph $< V - D >$ fails to be triple connected. Consequently,

$$|D| \geq f + 1 = \begin{cases} 2\left\lceil \frac{l}{5} \right\rceil & if l = 5e \ or \ e5 + 3 \ or \ 5e + 4 \\ 2\left\lceil \frac{l}{5} \right\rceil - 1 & otherwise. \end{cases}$$

Completing the proof.

Theorem 4: *If* $l \geq 3$, *then*

$$CTATD(O(TL_l)) = \begin{cases} 2\left\lceil \frac{l}{5} \right\rceil + 1 & if l = 5e + 1 \ or \ 5e + 2 \ or \ 5e + 2 \ or \ 5e + 4 \\ 2\left\lceil \frac{l}{5} \right\rceil + 2 & otherwise. \end{cases}$$

Proof: Let $Y(O(TL_l)) = \{p_i, d_i : 1 \leq i \leq l\}$ and $Z(O(DL_l)) = \{p_i p_{i+1}, d_i d_{i+1}, d_i p_{i+1} : 1 \leq i \leq l - 1\} \cup \{p_i d_i ; 2 \leq i \leq l - 1\}$. Let $B_1 = \{p_i : i = 5e + 2(mod\ 5); p_i : i = 5e + 4\}$.

Assume $S = \begin{cases} B_1 \cup \{d_l\} & if\ l = 5e \ or \ e5 + 2 \ or \ 5e + 3 \\ B_1 \cup \{d_{l-1}, d_l\} & if\ l = 5e + 1 \\ B_1 \cup \{p_l\} & if\ l = 5e + 4. \end{cases}$

If S is a CTATD-set of $(O(DL_l))$ then

$$CTATD(O(DL_l)) \leq |S|$$

$$= \begin{cases} 2\left\lceil \frac{l}{5} \right\rceil + 1 & if l = 5e + 1 \ or \ 5e + 2 \ or \ 5e + 3 \ or \ 5e + 4 \\ 2\left\lceil \frac{l}{5} \right\rceil + 2 & otherwise. \end{cases}$$

For any set D with a size of at most

$$f = \begin{cases} 2\left\lceil \frac{l}{5} \right\rceil & if l = 5e + 1 \ or \ 5e + 2 \ or \ 5e + 3 \ or \ 5e + 4 \\ 2\left\lceil \frac{l}{5} \right\rceil + 1 & otherwise, \end{cases}$$

the graph $< V - D >$ fails to be triple connected. Consequently,

$$|D| \geq f + 1 = \begin{cases} 2\left\lceil \frac{l}{5} \right\rceil + 1 & if l = 5e + 1 \ or \ 5e + 2 \ or \ 5e + 3 \ or \ 5e + 4 \\ 2\left\lceil \frac{l}{5} \right\rceil + 2 & otherwise. \end{cases}$$

Completing the proof.

Theorem 5: *If* $l \geq 3$*, then*

$$CTATD(S(L_l)) = \begin{cases} \left\lceil \frac{l}{2} \right\rceil + 2 & if\ l = 4e \\ \left\lceil \frac{l}{2} \right\rceil & if\ l = 4e+1\ or\ 4e+2 \\ \left\lceil \frac{l}{2} \right\rceil + 1 & otherwise. \end{cases}$$

Proof: Let $Y(S(L_l)) = \{p_i, d_i: 1 \leq i \leq l\}$ and $Z(S(L_l)) = \{p_i p_{i+1}, d_i d_{i+1}, d_i p_{i+1}: 1 \leq i \leq l-1\}$. Let $B_1 = \{d_i: i = 4e+2, p_i: i = 4e+1\}$.

Assume $S = \begin{cases} B_1 \cup \{d_{l-1}, d_l\} & if\ l = 4e \\ B_1 \cup \{d_l\} & if\ l = 4e+1\ or\ 4e+3 \\ B_1 & if\ l = 4e+2. \end{cases}$

If S is a CTATD-set of $(S(L_l))$ then

$$CTATD(S(L_l)) \leq |S| = \begin{cases} \left\lceil \frac{l}{2} \right\rceil + 2 & if\ l = 4e \\ \left\lceil \frac{l}{2} \right\rceil & if\ l = 4e+1\ or\ 4e+2 \\ \left\lceil \frac{l}{2} \right\rceil + 1 & otherwise. \end{cases}$$

For any set D with a size of at most

$$f = \begin{cases} \left\lceil \frac{l}{2} \right\rceil + 1 & if\ l = 4e \\ \left\lceil \frac{l}{2} \right\rceil - 1 & if\ l = 4e+1\ or\ 4e+2 \\ \left\lceil \frac{l}{2} \right\rceil & otherwise, \end{cases}$$

the graph $< V - D >$ fails to be triple connected. Consequently,

$$|D| \geq f + 1 = \begin{cases} \left\lceil \frac{l}{2} \right\rceil + 2 & if\ l = 4e \\ \left\lceil \frac{l}{2} \right\rceil & if\ l = 4e+1\ or\ 4e+2 \\ \left\lceil \frac{l}{2} \right\rceil + 1 & otherwise. \end{cases}$$

Completing the proof.

Theorem 6: *If* $l \geq 3$*, then*

$$CTATD(C(L_l)) = \begin{cases} \left\lceil \frac{l}{2} \right\rceil & if\ l = 4e\ or\ 4e+2\ or\ 4e+3 \\ \left\lceil \frac{l}{2} \right\rceil - 1 & otherwise. \end{cases}$$

Proof: Let $Y(C(L_l)) = \{p_i, d_i: 1 \leq i \leq l\}$ and $Z(C(L_l)) = \{p_i p_{i+1}, d_i d_{i+1}: 1 \leq i \leq l-1\} \cup \{p_i d_i; 1 \leq i \leq l\} \cup \{d_l d_1, p_l p_1\}$. Let $B_1 = \{d_i: i = 4e+1, p_i: i = 4e+3\}$.

Assume $S = \begin{cases} B_1 & if\ l = 4e\ or\ 4e+1\ or\ 4e+3 \\ B_1 \cup \{d_l\} & if\ l = 4e+2. \end{cases}$

If S is a CTATD-set of $(C(L_l))$ then

$$CTATD(C(L_l)) \leq |S| = \begin{cases} \left\lceil \frac{l}{2} \right\rceil & if\ l = 4e\ or\ 4e+2\ or\ 4e+3 \\ \left\lceil \frac{l}{2} \right\rceil - 1 & otherwise. \end{cases}$$

For any set D with a size of at most

$$f = \begin{cases} \left\lceil \frac{l}{2} \right\rceil - 1 & if\ l = 4e\ or\ 4e+2\ or\ 4e+3 \\ \left\lceil \frac{l}{2} \right\rceil - 2 & otherwise, \end{cases}$$

the graph $< V - D >$ fails to be triple connected. Consequently,

$$|D| \geq f + 1 = \begin{cases} \left\lceil \frac{l}{2} \right\rceil & ifl = 4e\ or\ 4e+2\ or\ 4e+3 \\ \left\lceil \frac{l}{2} \right\rceil - 1 & otherwise. \end{cases}$$

Completing the proof.

Theorem 7: *If* $l \geq 3$, *then*

$$CTATD(M(L_l)) = \begin{cases} \left\lceil \frac{l}{2} \right\rceil - 1 & l = 4e\ or\ 4e+2\ or\ 4e+3 \\ \left\lceil \frac{l}{2} \right\rceil + 1 & otherwise. \end{cases}$$

Proof: Let $Y(M(L_l)) = \{p_i, d_i : 1 \leq i \leq l\}$ and $Z(M(L_l)) = \{p_i p_{i+1}, d_i d_{i+1} : 1 \leq i \leq l-1\} \cup \{p_i d_i ; 1 \leq i \leq l\} \cup \{d_l p_1, p_l d_1\}$. Let $B_1 = \{d_i : i = 4e+1, p_i : i = 4e+3\}$.

Assume $S = \begin{cases} B_1 \cup \{p_l\} & if\ l = 4e \\ B_1 & if\ l = 4e+1\ or\ 4e+2\ or\ 4e+3. \end{cases}$

If S is a CTATD-set of $(M(L_l))$ then

$$CTATD(M(L_l)) \leq |S| = \begin{cases} \left\lceil \frac{l}{2} \right\rceil - 1 & ifl = 4e+1\ or\ 4e+2\ or\ 4e+3 \\ \left\lceil \frac{l}{2} \right\rceil + 1 & otherwise. \end{cases}$$

For any set D with a size of at most

$$f = \begin{cases} \left\lceil \frac{l}{2} \right\rceil - 2 & if\ l = 4e+1\ or\ 4e+2\ or\ 4e+3 \\ \left\lceil \frac{l}{2} \right\rceil & otherwise. \end{cases}$$

the graph $< V - D >$ fails to be triple connected.Consequently,

$$|D| \geq f + 1 = \begin{cases} \left\lceil \frac{l}{2} \right\rceil - 1 & ifl = 4e+1\ or\ 4e+2\ or\ 4e+3 \\ \left\lceil \frac{l}{2} \right\rceil + 1 & otherwise. \end{cases}$$

Completing the proof.

3. G^k of CTATD- Number for variety of L_l

Theorem 8: *If* $l \geq 3$, *then* $CTATD(DL_l)^k) = \left\lceil \frac{l}{2k+1} \right\rceil$.

Proof: Let $Y(DL_l)^k = \{p_i, b_i: 1 \leq i \leq l\}$. $Z(DL_l)^k = \{p_i p_{i+1}, b_i b_{i+1}, p_i b_{i+1}, b_i p_{i+1} 1 \leq i \leq q-1\} \cup \{p_i b_i: 1 \leq i \leq l\} \cup \{p_i p_j, b_i b_j, p_i b_j, b_i p_j: d(p_i p_j) \leq k, d(b_i b_j) \leq k, d(p_i b_j) \leq k, d(b_i p_j) \leq k\}$. Let $B_1 = \{b_i: i \equiv k+1 (mod\ 2k+1)\}$.

Assume $S = \begin{cases} B_1 & if\ 0, k+1 \leq l \leq 2k \\ B_1 \cup \{b_l\} & if \equiv 1,2,3 \ldots, k (mod\ 2k+1). \end{cases}$

If S is a CTATD-set of $((DL_l)^k)$ then $CTATD((DL_l)^k) \leq |S| = \left\lceil \frac{l}{2k+1} \right\rceil$. For any set D with a size of at most $f = \left\lceil \frac{l}{2k+1} \right\rceil - 1$, the graph $< V - D >$ fails to be triple connected.Consequently, $|D| \geq f + 1 = \left\lceil \frac{l}{2k+1} \right\rceil$. Completing the proof.

Theorem 9: *If* $l \geq 3$, *then* $CTATD(TL_l)^k) = \left\lceil \frac{l}{2k} \right\rceil$.

Proof: Let $Y(T(L_l)^k) = \{p_i, b_i: 1 \leq i \leq l\}$ and $Z(T(L_l)^k) = \{p_i p_{i+1}, b_i b_{i+1}, b_i p_{i+1}: i \in [1, l-1]\} \cup \{p_i b_i: i \in [1, l]\} \cup \{p_i p_j, b_i b_j, p_i b_j, b_i p_j: d(p_i p_j) \leq k, d(b_i b_j) \leq k, d(p_i b_j) \leq k, d(b_i p_j) \leq k\}$.

Let $B_1 = \{p_i: i \equiv k+1 (mod\ 2k)\}$.

Assume $S = \begin{cases} B_1 & if\ 0, k+1 \leq l \leq 2k-1 \\ B_1 \cup \{p_l\} & if\ l \equiv 1, 2, 3 \ldots, k\ (mod\ 2k). \end{cases}$

If S is a CTATD-set of $((TL_l)^k)$ then $CTATD((TL_l)^k) \leq |S| = \left\lceil \frac{l}{2k} \right\rceil$. For any set D with a size of at most $f = \left\lceil \frac{l}{2k} \right\rceil - 1$, the graph $< V - D >$ fails to be triple connected.Consequently, $|D| \geq f + 1 = \left\lceil \frac{l}{2k} \right\rceil$. Completing the proof.

Theorem 10: *If* $l \geq 3$, *then* $CTATD(SL_l)^k) = \left\lceil \frac{l}{2k-1} \right\rceil$.

Proof: Let $Y(S(L_l)^k) = \{p_i, b_i: 1 \leq i \leq l\}$ and $Z(S(L_l)^k) = \{p_i p_{i+1}, b_i b_{i+1}, b_i p_{i+1}: 1 \leq i \leq l-1\} \cup \{p_i p_j, b_i b_j, b_i p_j: d(p_i p_j) \leq k, d(b_i b_j) \leq k, d(b_i p_j) \leq k\}$. Let $B_1 = \{p_i: i \equiv k+1 (mod\ 2k-1)\}$. Assume

$$S = \begin{cases} B_1 & if\ l \equiv 0, k+i\ (i \equiv 1, 2, 3 \ldots, k-2) \\ B_1 \cup \{p_l\} & if\ l \equiv 1,2,3 \ldots, k\ (mod\ 2k-1). \end{cases}$$

If S is a CTATD-set of $((SL_l)^k)$ then $CTATD((SL_l)^k) \leq |S| = \left\lceil \frac{l}{2k-1} \right\rceil$. For any set D with a size of at most $f = \left\lceil \frac{l}{2k-1} \right\rceil - 1$, the graph $< V - D >$ fails to be triple connected. Consequently, $|D| \geq f + 1 = \left\lceil \frac{l}{2k-1} \right\rceil$. Completing the proof.

Theorem 11: *If* $l \geq 3$, *then* $CTATD(CL_l)^k)$

$$= \begin{cases} \left\lceil \frac{l}{2k} \right\rceil & if\ l \equiv 0, 2k, 2k+1, 2k+2, .., 3k+1\ (mod\ 4k) \\ \left\lceil \frac{l}{2k} \right\rceil - 1 & if\ l \equiv 1, 2, 3..., 2k-1\ (mod\ 4k). \end{cases}$$

Proof: Let $Y(C(L_l)^k) = \{p_i, b_i: 1 \leq i \leq l\}$ and $Z(C(L_l)^k) = \{p_i p_{i+1}, b_i b_{i+1}: 1 \leq i \leq l-1\} \cup \{p_i b_i; 1 \leq i \leq l\} \cup \{b_l b_1, p_l p_1\} \cup \{p_i p_j, b_i b_j, p_i b_j, : d(p_i p_j) \leq k, d(b_i b_j) \leq k, d(b_i p_j) \leq k\}$. Let $B_1 = \{p_i: i = 4k + k; b_i: i = 4k + 3k\}$.

Assume

$$S = \begin{cases} B_1 & if\ l \equiv 0, k, k+1, ..., 2k-1, 3k, 3k+1, .., 4k-1 (mod\ 4k) \\ B_1 \cup \{p_l\} & if\ l \equiv 1, ..., k-1 (mod\ 4k) \\ B_1 \cup \{b_l\} & if\ l \equiv 1, 2, 3..., k (mod\ 4k). \end{cases}$$

If S is a CTATD-set of $((CL_l)^k)$ then

$$CTATD((CL_l)^k) \leq |S|$$

$$= \begin{cases} \left\lceil \frac{l}{2k} \right\rceil & if\ l \equiv 0, 2k, 2k+1, 2k+2, .., 3k+1\ (mod\ 4k) \\ \left\lceil \frac{l}{2k} \right\rceil - 1 & if\ l \equiv 2k, 2k+1, ..., 3k-1\ (mod\ 4k). \end{cases}$$

For any set D with a size of at most

$$f = \begin{cases} \left\lceil \frac{l}{2k} \right\rceil - 1 & if\ l \equiv 0, 2k, 2k+1, 2k+2, .., 3k+1 (mod\ 4k) \\ \left\lceil \frac{l}{2k} \right\rceil - 2 & if\ l \equiv 1, 2, 3..., 2k-1 (mod\ 4k), \end{cases}$$

the graph $< V - D >$ fails to be triple connected.Consequently,

$$|D| \geq f + 1 = \begin{cases} \left\lceil \frac{l}{2k} \right\rceil & if\ l \equiv 0, 2k, 2k+1, 2k+2, .., 3k+1 (mod\ 4k) \\ \left\lceil \frac{l}{2k} \right\rceil - 1 & if\ l \equiv 1, 2, 3..., 2k-1 (mod\ 4k). \end{cases}$$

Completing the proof.

Theorem 12: *If* $l \geq 3$, *then*

$$CTATD(ML_l)^k) = \begin{cases} \left\lceil \frac{l}{2k-1} \right\rceil & if\ l \equiv 0, 2, 3, .., 2k-2\ (mod\ 2k-1) \\ \left\lceil \frac{l}{2k-1} \right\rceil - 1 & if\ l \equiv 1\ (mod\ 2k-1). \end{cases}$$

Proof: Let $Y(M(L_l)^k) = \{p_i, b_i: 1 \leq i \leq l\}$ and $Z(M(L_l)^k) = \{p_i p_{i+1}, b_i b_{i+1}: 1 \leq i \leq l-1\} \cup \{p_i b_i; 1 \leq i \leq l\} \cup \{b_l p_1, p_l b_1\} \cup \{p_i p_j, b_i b_j, b_i p_j: d(p_i p_j) \leq k, d(b_i b_j) \leq k, d(b_i p_j) \leq k\}$. Let $B_1 = \{b_i: i = 2k - 1 + k\}$.

Assume $S = \begin{cases} B_1 & if\ l \equiv 0, 1, k, k+1, ..., 2k-2\ (mod\ 2k-1) \\ B_1 \cup \{b_l\} & if\ q \equiv 2, 3..., k-1\ (mod\ 2k-1). \end{cases}$

If S is a CTATD-set of $((ML_l)^k)$ then

$$CTATD((ML_l)^k) \leq |S|$$

$CTATD((ML_l)^k) \leq |S|$

$$= \begin{cases} \left\lceil \frac{l}{2k-1} \right\rceil & if\ l \equiv 0, 2, 3, .., 2k-2\ (mod\ 2k-1) \\ \left\lceil \frac{l}{2k-1} \right\rceil - 1 & if\ l \equiv 1\ (mod\ 2k-1). \end{cases}$$

For any set D with a size of at most

$$f = \begin{cases} \left\lceil \frac{l}{2k-1} \right\rceil - 1 & if\ l \equiv 0, 2, 3, .., 2k-2\ (mod\ 2k-1) \\ \left\lceil \frac{l}{2k-1} \right\rceil - 2 & if\ l \equiv 1\ (mod\ 2k-1), \end{cases}$$

the graph $< V - D >$ fails to be triple connected.Consequently,

$$|D| \geq f + 1 = \begin{cases} \left\lceil \frac{l}{2k-1} \right\rceil & if\ l \equiv 0, 2, 3, .., 2k-2 (mod\ 2k-1) \\ \left\lceil \frac{l}{2k-1} \right\rceil - 1 & if\ l \equiv 1 (mod\ 2k-1). \end{cases}$$

Completing the proof.

4. Conclusion

In this paper, we have generalized the outcomes for power graphs of some ladder graphs with distance $k \geq 2$, and future investigations will focus on applying the CTATD parameter to grid graphs.

References

[1] F. Harary, *Graph Theory*. Reading, MA, USA: Addison-Wesley (1972).

[2] T. W. Haynes, S. T. Hedetniemi, and P. J. Slater, *Fundamentals of Domination in Graphs*. New York, USA: Marcel Dekker, Inc., (1998).

[3] M. Chellali, T. W. Haynes, S. T. Hedetniemi, and A. A. McRae, "[1,2]-sets in graphs," *Discrete Applied Mathematics*, vol. 161, no. 18, pp. 2885–2893 (Dec. 2013).

[4] G. Mahadevan, K. Priya, and C. Sivagnanam, "Complementary triple connected at most twin domination number of a graph," *Advances and Applications in Mathematical Sciences*, vol. 22, no. 8, pp. 1867–1878 (Jun. 2023).

[5] J. Paulraj Joseph, M. K. Angel Jebitha, P. Chithra Devi, and G. Sudhana, "Triple connected graphs," *Indian Journal of Mathematics and Mathematical Sciences*, vol. 8, no. 1, pp. 61–75 (2012).

[6] X. Yang and B. Wu, "[1, 2]-domination in graphs," *Discrete Applied Mathematics*, vol. 175, pp. 79–86 (Oct. 2014), doi: 10.1016/j.dam.2014.05.035.

[7] K. Priya, G. Mahadevan, C. Sivagnanam, and Sanjay Kumar Tyagi, "CTATD Number for Power Graph of Some Special Graphs and Tadpole Graph," *Discrete Mathematics and Mathematical Modelling,* Springer Proceedings in Mathematics and Statistics, vol. 458, pp. 231-239 (2024).

[8] G. Priscilla Pacifica and K. Christy Rani, "Total Restrained detour number of family of ladder graphs," vol. 20, no. 3A, pp. 34-37 (2022).

[9] M. Joice Punitha and S. Rajakumari, "Skew chromatic index of comb, ladder and Mobius ladder graphs," *International Journal of Pure and Applied Mathematics,* vol. 101, no. 6, pp. 1003–1011 (2015), doi: 10.12732/ijpam.v101i6.18.

[10] Doha Adel Abbass, Manal N. Al-Harere, and Emad Bakr Al-Zangana, "On planar domination in graphs," *Journal of Discrete Mathematical Sciences & Cryptography,* vol. 28, no. 4-B, pp. 1355-1360 (2025).

[11] T. V. Sateesh Kumar and S. Meenakshi, "Family of Ladder Graphs are Properly Lucky," *Journal of Pharmaceutical Negative Results*, vol. 13, Special Issue 3, pp. 1579–1583 (Oct. 2022), doi: 10.47750/pnr.2022.13.S03.243.

[12] S. Meena and G. Gajalakshmi, "Odd Prime Labeling of Graphs Related to Circular Ladder," *Communications in Mathematics and Applications,* vol. 13, no. 4, pp. 1307-1315 (2022).

[13] A. Vijayalekshmi and S. Abisha, "Total dominator color class total dominating sets in ladder and Mobius ladder graph," *Malaya Journal of Matematik,* vol. 9, no. 1, pp. 1233–1236 (Mar. 2021), doi: 10.26637/MJM0901/0214.

[14] C. Berge, "Theory of graphs and applications," Methuen, London (1962).

[15] K. Priya, G. Mahadevan, C. Sivagnanam, and S.K. Tyagi, "CTATD Number for Power Graph of Some Special Graphs and Tadpole

Graph," In *International Conference on Discrete Mathematics and Mathematical Modelling in Digital Era*, pp. 231-239 (2023).

[16] B. Jaya Prasad, T. Tamizh Chelvam, and S. Robinson Chellathurai, "Private domination number of a graph," *Journal of Discrete Mathematical Sciences & cryptography*, vol. 10, no. 5, pp. 661-666 (2007).

[17] Adel P. Kazemi, "On the domination number of some graphs," *Journal of Discrete Mathematical Sciences & Cryptography,* vol. 12 , no. 4, pp. 489-493 (2009).

[18] Z. Shao, S. M. Sheikholeslami, S. Nazari-Moghaddam, and S. Wang, "Global double Roman domination in graphs," *Journal of Discrete Mathematical Sciences & Cryptography*, vol. 22, no. 1, pp. 31-44 (2019).

[19] Ravindra Kumar, Ashutosh Singh, and Om Prakash, "Double Roman domination number of the zero-divisor graphs of commutative rings," *Journal of Discrete Mathematical Sciences & Cryptography,* vol. 28, no. 3, pp. 765-781 (2025).

Received May, 2025