

Unveiling STFR : Single-valued trivalent logic based fuzzy rhotrices

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Abstract

This study introduces a new mathematical structure called a “single-valued trivalent logic-based fuzzy rhotrix” (STFR). An STFR combines a $r \times r$ matrix and a $(r-1) \times (r-1)$ matrix with a membership function. This membership function takes its values from a set based on trivalent logic (a logic system with three possible truth values). The paper explores the fundamental characteristics and properties of these newly defined STFRs, presenting the findings. Furthermore, the research initiates an investigation into the concepts of determinant functions, codeterminant functions, and an associated “index” within the context of STFRs.

Subject Classification: 47S40, 03E72, 08A72, 15B15.

Keywords: Fuzzy rhotrix, Single trivalent logic based fuzzy rhotrix, Codeterminant function, Index.

1. Introduction

Trivalent logic is an extension of binary logic that allows for more than two truth values. In trivalent logic, there are three truth values: true, false, and indeterminate. A trivalent logic-based fuzzy matrix can be represented as a matrix of three-valued logic. Each element in the matrix can have a truth value of true, false, or indeterminate. Trivalent logic-based fuzzy matrices have applications in artificial intelligence, control systems, decision-making etc. Trivalent logic is an extension of binary logic that allows for more than two truth values. In trivalent logic, there are three truth values: true, false, and

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indeterminate. It has unique properties of its own, akin to a rhomboidal array of numbers. It has unique properties of its own, akin to a rhomboidal array of numbers, and is reminiscent of matrices. Ajibade [3] initially conceived about rhotrices as an extension of matrix operations. It provides fascinating opportunities for mathematical investigation by bridging the gap between vectors and matrices.

2. Literature Review

The knowledge of fuzzy matrix was first pioneered by Thomason [1]. Sani [2] developed another method that uses rows and columns to multiply rhotrices of dimension 3. Rhotrices were initially established by Ajibade [3] as an extension of matrix operations. The fundamental characteristics and concepts of fuzzy matrices were thoroughly explained by Gilchrist [4]. The definition, functions, and uses of fuzzy matrices are examined by Priya et al. [5]. The idea of a coupled matrix, which is a combination of two matrices, was covered by Sani [6]. Isere [7] discussed the ideas of codeterminant functions and the index of a rhotrix. Isere [8] explored the diagonal functions of the natural rhotrix. Sidky and Emam [9] have significantly influenced the study of fuzzy matrix theory, particularly in operations, structural properties, and eigenvalue analysis. Saranya and Rani [10] provides a systematic exploration of fuzzy matrix operations, structural properties, and novel representations. Etieyibo [11] discussed the concepts of trivalent concepts in their study. Hemalatha and Dharinika [12] demonstrated that fuzzy logic is highly effective for weather prediction, as it adeptly manages the ambiguity and uncertainty inherent in meteorological data. Khan and Pal [13] developed the rhotrix structure and demonstrated its applications. Shyamal and Pal [14] proposed generalized fuzzy matrices, extending classical fuzzy matrix theory with additional algebraic operations. Khan and Pal [13] further investigated rhotrix multiplication and established several algebraic properties. Shyamal and Samanta [15] studied fundamental operations on fuzzy matrices, including addition and multiplication. Khan, Shyamal, and Pal [16] introduced the concept of fuzzy rhotrix, extending rhotrix theory to fuzzy environments.

3. Preliminaries

Definition 3.1: A **fuzzy matrix** is a mathematical construct where the elements of the matrix have values within $[0, 1]$.

Example 3.1:

$$A = \begin{bmatrix} 0.1 & 0.5 & 0 \\ 0.4 & 0.2 & 0.9 \\ 0.3 & 0.6 & 1 \end{bmatrix}$$

Definition 3.2: An expression for the n dimensional **rhotrix** R_n for $n \in 2\mathbb{Z}^+ + 1$ is described as

$$R_n = \left\langle \begin{array}{ccccccc} & & & & a_{11} & & \\ & & & & c_{11} & & \\ & & & a_{21} & & a_{12} & \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{t1} & \dots & \dots & \dots & \dots & \dots & a_{1t} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ & & & a_{tt-1} & c_{t-1t-1} & a_{t-1t} & \\ & & & & a_{tt} & & \end{array} \right\rangle$$

$$a_{mn}, c_{pq} \in \mathbb{R}, 1 \leq m, n \leq t; 1 \leq p, q \leq t-1 \text{ and } t = \frac{n+1}{2}.$$

Definition 3.3: A single valued trivalent logic based fuzzy rhotrix (*STFR*) is a rhotrix whose elements are in the interval $[-1, 1]$.

Example 3.2: Consider the 5 weather parameters temperature (wp_1), humidity (wp_2), wind speed (wp_3), precipitation (wp_4) and cloud cover (wp_5) and 5 weather conditions heat wave (wc_1), fog (wc_2), gale (wc_3), thunderstorm (wc_4) and overcast (wc_5) in which 3 parameters (wp_1, wp_3, wp_5) is associated with 3 weather conditions (wc_1, wc_3, wc_5) and the other two weather parameters (wp_2, wp_4) is associated with the other two weather conditions (wc_2, wc_4). Here wp_i wc_j represents the membership value of how the weather parameter wp_i contributes the occurrences of the weather condition wc_j . The weather parameter – weather condition single trivalent logic based fuzzy rhotrix is given by

$$\left\langle \begin{array}{ccccc} & & wp_1wc_1 & & \\ & & wp_3wc_1 & wp_2wc_2 & wp_1wc_3 \\ wp_5wc_1 & wp_3wc_1 & wp_4wc_2 & wp_3wc_3 & wp_2wc_4 & wp_1wc_5 \\ & wp_5wc_3 & wp_4wc_2 & wp_3wc_5 & \\ & & wp_5wc_5 & & \end{array} \right\rangle$$

where $wp_i wc_j \in [-1, 1]$

For numerical illustration,

$$\left\langle \begin{array}{ccccc} & & -0.5 & & \\ & & 0.3 & -1 & 0.9 \\ -0.2 & -0.4 & 1 & 0.4 & 0.2 \\ & 0.7 & -0.2 & -0.1 & \\ & & 0.5 & & \end{array} \right\rangle$$

Definition 3.4: The dimension of *STFR* is the number of elements presents in the major vertical or horizontal axis of *STFR*.

Example 3.3: If $P_{STFR} = \begin{pmatrix} & p \\ s & q & x & q \\ & u & y & b & r \\ & v & t & a \\ & & & w \end{pmatrix}$ then the dimension of P_{STFR} is

5.

Definition 3.5: The Cardinality of $STFR$ is $\frac{n^2+1}{2}$, where $n \in 2z^+ + 1$ and it is a dimension of $STFR$.

Example 3.4:

(i) Let $STFR_5 = \begin{pmatrix} & & 1 \\ 0.5 & 0.7 & -1 & 0.2 \\ & 0.3 & 0.4 & 0.4 & 0.3 \\ & -0.2 & 0.6 & -0.2 \\ & & & 0 \end{pmatrix}$ is dimension 5, then the

cardinality is 13.

Definition 3.6: Heart Based Addition and Multiplication of Rhotrix

Let P & Q be any two rhotrices then

$$P + Q = \begin{pmatrix} & a & \\ b & x & c \\ & d & \end{pmatrix} + \begin{pmatrix} & f & \\ h & y & j \\ & k & \end{pmatrix} = \begin{pmatrix} & a+f & \\ b+h & x+y & c+j \\ & d+k & \end{pmatrix}$$

$$P \circ Q = \begin{pmatrix} & a & \\ b & x & c \\ & d & \end{pmatrix} \circ \begin{pmatrix} & f & \\ h & y & j \\ & k & \end{pmatrix} = \begin{pmatrix} & ay+fx & \\ by+hx & xy & cy+jx \\ & dy+kx & \end{pmatrix}$$

We define fuzzy addition and fuzzy multiplication as $r + s = \text{mxm}(r, s)$ where mxm denotes the maximum $r \cdot s = \text{mnm}(r, s)$ where mnm denotes the minimum.

Definition 3.7: Addition and Multiplication of two $STFR$

$$P_{STFR} + Q_{STFR} = \begin{pmatrix} & p & \\ q & m & r \\ & e & \end{pmatrix} + \begin{pmatrix} & f & \\ g & y & j \\ & k & \end{pmatrix}$$

$$= \begin{pmatrix} & \text{mxm}(p, f) & \\ \text{mxm}(q, g) & \text{mxm}(m, y) & \text{mxm}(r, j) \\ & \text{mxm}(e, k) & \end{pmatrix}$$

$$P_{STFR} \circ Q_{STFR} = \begin{pmatrix} & a & \\ q & m & r \\ & e & \end{pmatrix} \circ \begin{pmatrix} & f & \\ g & y & j \\ & k & \end{pmatrix}$$

$$= \left\langle \begin{array}{ccc} & \text{mxm}(\text{mnm}(a, y), \text{mnm}(f, m)) & \\ \text{mxm}(\text{mnm}(q, y), & \text{mnm}(g, m)) \text{mnm}(m, y) & \text{mxm}(\text{mnm}(r, y), \text{mnm}(j, m)) \\ \text{mnm}(g, m)) & \text{mxm}(\text{mnm}(e, y), \text{mnm}(k, m)) & \text{mnm}(j, m)) \end{array} \right\rangle$$

Example 3.5: Find the Addition and Multiplication of the following *STFR*

$$P_{STFR} = \left\langle \begin{array}{ccc} & 1 & \\ 0.5 & -1 & 0.2 \\ & 0.7 & \end{array} \right\rangle \& Q_{STFR} = \left\langle \begin{array}{ccc} & 0.2 & \\ 0.4 & 1 & 0.4 \\ & 0.6 & \end{array} \right\rangle$$

Solution:

$$\begin{aligned} P_{STFR} + Q_{STFR} &= \left\langle \begin{array}{ccc} & \text{mxm}(1, 0.2) & \\ \text{mxm}(0.5, 0.4) & \text{mxm}(-1, 1) & \text{mxm}(0.2, 0.4) \\ & \text{mxm}(0.7, 0.6) & \end{array} \right\rangle \\ &= \left\langle \begin{array}{ccc} & 1 & \\ 0.5 & 1 & 0.4 \\ & 0.7 & \end{array} \right\rangle \end{aligned}$$

$A \circ B =$

$$\begin{aligned} &\left\langle \begin{array}{ccc} & \text{mxm}(\text{mnm}(1, 1), \text{mnm}(0.2, -1)) & \\ \text{mxm}(\text{mnm}(0.5, 1), & \text{mnm}(0.4, -1)) \text{mnm}(-1, 1) & \text{mxm}(\text{mnm}(0.2, 1), \text{mnm}(0.4, -1)) \\ \text{mnm}(0.4, -1)) & \text{mxm}(\text{Mn}(0.7, 1), \text{mnm}(0.6, -1)) & \text{mnm}(0.4, -1)) \end{array} \right\rangle \\ A \circ B &= \left\langle \begin{array}{ccc} & \text{mxm}(1, -1) & \\ \text{mxm}(0.5, -1) & \text{mxm}(-1, 1) & \text{mxm}(0.2, -0.4) \\ & \text{mxm}(0.7, -1) & \end{array} \right\rangle \\ &= \left\langle \begin{array}{ccc} & 1 & \\ 0.5 & -1 & 0.2 \\ & 0.7 & \end{array} \right\rangle \end{aligned}$$

Definition 3.8: The Inverse of given *STFR*,

$$P_{STFR} = \left\langle \begin{array}{ccc} & a & \\ b & h(P) & d \\ & e & \end{array} \right\rangle \text{ is } P_{STFR}^{-1} = \frac{-1}{(h(P))^2} \left\langle \begin{array}{ccc} & a & \\ b & -h(P) & d \\ & e & \end{array} \right\rangle$$

If the determinant of any *STFR* is zero, it is said to be singular; otherwise, the *STFR* is non-singular. The *STFR* is said to be **invertible** if it has an inverse.

Definition 3.9: The determinant of $P_{STFR} = \left\langle \begin{array}{ccc} & a & \\ b & c & d \\ & e & \end{array} \right\rangle$ is defined by

$$\det(P_{STFR}) = \text{mnm}(c, (\text{mnm}(a, e) - \text{mnm}(b, d))).$$

Example 3. 6: If $P_{STFR} = \begin{pmatrix} & 1 & \\ 0.5 & -1 & 0.2 \\ & 0.7 & \end{pmatrix}$ then the determinant is given by $\det(P_{STFR}) = \text{mm}(-1, (\text{mm}(1, 0.7) - \text{mm}(0.5, 0.2))) = \text{mm}(-1, (0.7 - 0.2)) = -1$.

Definition 3.10: The **Transpose** is obtained by interchanging the row and column of the given *STFR* and is denoted by $STFR^T$.

Example 3.7: If

$$P_{STFR} = \begin{pmatrix} & p & & & \\ & q & x & m & \\ s & u & y & b & r \\ & v & t & a & \\ & & w & & \end{pmatrix} \text{ then } P_{STFR}^T = \begin{pmatrix} & m & x & q & \\ r & b & y & u & s \\ & a & t & v & \\ & & w & & \end{pmatrix} \text{ is the}$$

transpose of given *STFR*.

Definition 3.11: The coupled STF matrix is obtained by rotating the given *STFR* through 45° (represented by half transpose $\frac{T}{2}$). For a 5-dimensional STF, we have

$$P_{STFR_5}^{\frac{T}{2}} = \begin{pmatrix} & p & & & \\ & q & x & m & \\ s & u & y & b & r \\ & v & t & a & \\ & & w & & \end{pmatrix} = \begin{pmatrix} p & & m & & r \\ & x & & b & \\ q & & y & & a \\ & u & & t & \\ s & & v & & w \end{pmatrix} = [BC]_5.$$

A coupled matrix is made of two square matrices with orders of t and $t-1$ where $t = \frac{n+1}{2}$.

$$B = \begin{pmatrix} p & m & r \\ q & y & a \\ s & v & w \end{pmatrix} \text{ and } C = \begin{pmatrix} x & b \\ u & t \end{pmatrix}$$

The resulting $n \times n$ coupled matrix is formed by filling zeros in the missing places of above and is given by

$$\begin{pmatrix} p & 0 & m & 0 & r \\ 0 & x & 0 & b & 0 \\ q & 0 & y & 0 & a \\ 0 & u & 0 & t & 0 \\ s & 0 & v & 0 & w \end{pmatrix}$$

Definition 3.12: The Codeterminant function of the higher dimensional *STFR* is calculated by reducing into *STFR* of dimension 3 and the determinant of each of them is calculated and summing up and is denoted by Codet.

Example 3.8: Find the Codeterminant function of the following STFR

$$P_{STFR} = \begin{pmatrix} & 0.1 & & & \\ & 0.5 & 0.3 & -0.1 & \\ 0.7 & 0.3 & 0.4 & 0.2 & 1 \\ & -1 & 0 & 0.5 & \\ & & 0.8 & & \end{pmatrix}$$

Solution: Along the major column, Codet (A) = $\det \begin{pmatrix} & 0.1 & \\ & 0.5 & 0.3 & -0.1 \\ & & 0.4 \end{pmatrix} +$

$$\det \begin{pmatrix} & 0.4 & \\ -1 & 0 & 0.5 \\ & 0.7 \end{pmatrix}$$

$$= \text{mnm} (0.3, (\text{mnm} (0.1, 0.4) - \text{mnm} (0.5, -0.1))) + \text{mnm} (0, (\text{mnm} (0.4, 0.7) - \text{mnm} (-1, 0.5)))$$

$$= \text{mnm} (0.3, (0.1 + 0.1)) + \text{mnm} (0, (0.4+1)) = 0.2$$

Along the major row, Codet (A) = $\det \begin{pmatrix} & 0.5 & \\ 0.7 & 0.3 & 0.4 \\ & -1 \end{pmatrix} +$

$$\det \begin{pmatrix} & -0.1 & \\ 0.4 & 0.2 & 1 \\ & 0.5 \end{pmatrix}$$

$$= \text{mnm} (0.3, (\text{mnm} (0.5, -1) - \text{mnm} (0.7, 0.4))) + \text{mnm} (0.2, (\text{mnm} (-0.1, 0.5) - \text{mnm} (0.4, 1)))$$

$$= \text{mnm} (0.3, (-1 - 0.4)) + \text{mnm} (0.2, (-0.1-0.4)) = -1.9$$

For a natural rhotrix the sum is same either expanding through the major column and major row but this not true in the *STFR*.

Definition 3.13: The number of three dimensional *STFR* than can be drawn either along the major row or major column is said to be index of *STFR* which denoted by ρ .

Remarks 3.1:

$$(i) \quad \rho(STFR_1) = 0$$

$$(ii) \quad \rho(STFR_3) = 1$$

$$(iii) \quad \rho(STFR_5) = 2$$

$$(iv) \quad \rho(STFR_7) = 3$$

$$(v) \quad \rho(STFR_9) = 4.$$

Example 3. 9: Find the index of the following

$$P_{STFR} = \begin{pmatrix} & & 0.1 \\ & 0.5 & 0.3 & -0.1 \\ 0.7 & 0.3 & 0.4 & 0.2 & 1 \\ & -1 & 0 & 0.5 \\ & & 0.8 \end{pmatrix}.$$

Solution: The 3 - dimensional rhotrices that can be derived along major column are $\begin{pmatrix} & 0.1 \\ 0.5 & 0.3 & -0.1 \\ & 0.4 \end{pmatrix}$ and $\begin{pmatrix} & 0.4 \\ -1 & 0 & 0.5 \\ & 0.7 \end{pmatrix}$. Hence the index of given *STFR* is 2. Likewise, the 3 - dimensional rhotrices that can be derived along major rows are $\begin{pmatrix} & 0.5 \\ 0.7 & 0.3 & 0.4 \\ -1 & & \end{pmatrix}$ and $\begin{pmatrix} & -0.1 \\ 0.4 & 0.2 & 1 \\ & 0.5 \end{pmatrix}$. Hence the index of given *STFR* is 2.

4. Application of STFR Techniques in Meteorology for Weather Forecasting

Consider the 5 weather teams $wt_1, wt_2, wt_3, wt_4, wt_5$ and 5 weather regions $wr_1, wr_2, wr_3, wr_4, wr_5$. In which 3 weather stations wt_1, wt_3, wt_5 is associated with 3 weather regions wr_1, wr_3, wr_5 and another 2 teams wt_2 and wt_4 is associated with other 2 regions wr_2 and wr_4 . Here $wt_i wr_j$ represents the membership value of how the weather team wt_i contributes to the weather station wr_j to handle to weather situations. The weather team – weather region fuzzy rhotrix is given by

$$\begin{pmatrix} & \mathbf{wt_1 wr_1} \\ & \mathbf{wt_3 wr_1} & \mathbf{wt_2 wr_2} & \mathbf{wt_1 wr_3} \\ \mathbf{wt_5 wr_1} & \mathbf{wt_4 wr_2} & \mathbf{wt_3 wr_3} & \mathbf{wt_2 wr_4} & \mathbf{wt_1 wr_5} \\ & \mathbf{wt_5 wr_3} & \mathbf{wt_4 wr_4} & \mathbf{wt_3 wr_5} \\ & & \mathbf{wt_5 wr_5} \end{pmatrix}$$

where $wt_i wr_j \in [-1, 1]$.

Consider the following 5-dimensional weather team – weather region single trivalent fuzzy rhotrix.

$$\begin{pmatrix} & & 0.4 \\ & 0.6 & 0.5 & -0.3 \\ 0.7 & 0.9 & -0.2 & 0.8 & 0.6 \\ & -0.2 & 0.5 & 0.4 \\ & & 1 \end{pmatrix}$$

The associated couple matrix is given by

$$\begin{pmatrix} 0.4 & 0 & -0.3 & 0 & 0.6 \\ 0 & 0.5 & 0 & 0.8 & 0 \\ 0.6 & 0 & -0.2 & 0 & 0.4 \\ 0 & 0.9 & 0 & 0.5 & 0 \\ 0.7 & 0 & -0.2 & 0 & 1 \end{pmatrix}$$

Convert this coupled matrix with positive entries by taking maximum of each entry with absolute value of most negative entry. The new coupled matrix is

$$\begin{pmatrix} 0.4 & 0.3 & 0.3 & 0.3 & 0.6 \\ 0.3 & 0.5 & 0.3 & 0.8 & 0.3 \\ 0.6 & 0.3 & 0.3 & 0.3 & 0.4 \\ 0.3 & 0.9 & 0.3 & 0.5 & 0.3 \\ 0.7 & 0.3 & 0.3 & 0.3 & 1 \end{pmatrix}$$

Using Hungarian method, we can reduce the matrix to

$$\begin{pmatrix} 0.1 & 0 & 0.2 & 0 & 0.3 \\ 0 & 0.2 & 0 & 0.5 & 0 \\ 0.3 & 0 & 0 & 0 & 0.1 \\ 0 & 0.6 & 0 & 0.2 & 0 \\ 0.4 & 0 & 0.2 & 0 & 0.7 \end{pmatrix}$$

$$\text{The corresponding rhotrix is } \begin{pmatrix} & & 0.1 & & \\ & 0.3 & 0.2 & 0.2 & \\ 0.4 & 0.6 & 0 & 0.5 & 0.3 \\ & 0.2 & 0.2 & 0.1 & \\ & & 0.7 & & \end{pmatrix}$$

We can assign that, the team wt_1 to the region wr_3 , wt_2 to wr_4 , wt_3 to wr_1 , wt_4 to wr_2 and wt_5 to wr_5 , by finding maximum of each row this assignment minimizes the total cost and time.

5. Conclusion

This paper gives a brief idea about single trivalent logic based fuzzy rhotrices and the basic definitions like determinant, transpose, invertible, codeterminant function and Index of *STFR*. We could extend and draw a conclusion with the operations and properties that are available in rhotrices and fuzzy matrices.

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Received May, 2025