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# Some analysis on the geodesic polynomial of graphs and its vertex

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**Abstract**

A graph geodesic polynomial is an algebraic expression linked with its polynomial that gathers information about geodesics, which means shortest paths in a graph connecting two vertices. Examining these polynomials' characteristics and behaviors, and how they interact with the underlying network, graph structure is necessary for researching the geodesic polynomials of graphs and their vertices. In this paper, we present geodesic polynomials for fan graphs, merging of two star graphs, hericyclic graphs formed by attaching $n$ edges at one or all vertices of $C_3$, Helm graph and geodesic polynomial at a vertex for star graphs and the extended star graphs.

***Subject Classification:*** *05C10, 05C38, 05C78.*

***Keywords:*** *Geodesic polynomial, Geodesic polynomial at a vertex, Geodesic path, Eccentricity, Diameter.*

## 1. Introduction

The theory of graphs illustrates real-world issues own unique way by examining the relationship between vertices and edges. We refer to the Harary [9] textbook for standard terminology and ideas in graph theory. Graph difficulties are an attraction for fresh academics these days. Graph theory is used in a significant variety of fields, such as computer, biology, chemistry, physics, sociology and others. Algorithms are used to find the shortest path between vertices in a graph, and applications in various fields such as road networks, logistics, computer networks, and mapping

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software. The relationship between the geodesic polynomial of the graph and its vertices lies in the definition of the geodesic polynomial. This is a graph invariant defined by counting the number of geodetic sets that have a particular cardinality, which illustrates a finite, simple, connected, and undirected graph $G=(V,E)$. In $G$ the shortest path between $x$ and $y$ can be referred to as a $x-y$ geodesic. The diameter of $G$, represented as diam $(G)$, is the distance along the longest geodesic in $G$. The greatest length of $v$ from any other vertex in $G$ is called its eccentricity. For a pair vertices $(u,y)$ beginning with $u$ and ending with $y$, $f_G$ or simply $f$ indicates the number of geodesics in $G$, while $f_i$ illustrates the quantity of a distance $i$ geodesics in $G$. It is clear that for a network with $n$ vertices $f_1=|E|$

$$f=\sum_{i=1}^{n-1} f_i=\sum_{i=1}^{d} f_i \tag{1}$$

Where $d$ is the diameter of $G$. Graph equations are algebraic properties of the graph. Different types of graph polynomials, such as Martin polynomials, matching polynomials, sign polynomials, and chromatic polynomials, can be found in research. [1, 5, 3, and 4] have discussed different types of polynomial of a graph. [2, 6, 7, and 8] and [10] have identified two graph polynomials. The polynomials of a graph and its vertex identifies the location in a network that is connected to its geodesics by fixing one end vertex. In addition, some intricate graphs are examined in this study. [11, 15] and [12, 13, and 14] for additional information about polynomials of graphs and their uses.

## 2. Our Contribution

According to the existing literature, there exist geodesic polynomials for a few graphs, such as complete graph, complete bipartite graph, path, and cycle graph, as well as their vertices. This study [15] explores the use of geodesic polynomials as graph properties and their relationship to a graph's structure. The Paper [15] already describes some geodesic polynomials of graphs and their vertex-like properties. This paper covers a variety of geodesic polynomials of graphs and their vertices for graphs like

$$F_n(C_3),(K^n{}_{1,2}\oplus K^n{}_{1,2}),C_3\oplus K_{1,n},C_3\oplus 3K_{1,n},H_n,S_n,K_{1,n,n}.$$

## 3. Preliminaries

**Description 2.1.1:** According to [13], A structure known as $G$ consists of $n$ vertices, and the geodesic sequences related to that graph are the sequences $f_{1,}f_2, f_3.....f_d$ from the vertices where $f_i$ denotes the whole number of geodesic paths in G with length $1 \le i \le d$ and $d = diam(G)$.

**Description 2.1.2:** According to [13], the geodesic polynomial of a graph node is represented by $gG(x)$ is the function that is represented by a polynomial whose coefficients are in the geodesic sequence $f_{1,}f_2, f_3.....f_d$ of $G$ which corresponds to

$$g_G(x) = f_1 + f_2x + f_3x^2 + .....f_dx^{d-1} = \sum_{i-1}^{d} f_ix^{i-1}$$

**Description 2.1.3:** According to [13], Let $v$ is a vertex in a graph $G$. The sequence $f_{1v}, f_{2v}...f_{iv}.....f_{e(v)v}$ is the geodesic sequence at $v$ where $f_{iv}$ is the quantity of the length of geodesic structures where $v$ is an end-vertex in $G$, and $1 \le i \le e(v)$

**Description 2.1.4:** We refer to [13] for the Geodesic polynomial at a vertex $v$ in a graph $G$ is a generating function (polynomial) for the geodesic sequence, and is represented as

$$gGv(x) = f_{1v} + f_{2v}x + f_{3v}x^2 + ..... + f_{e(v)v}x^{e(v)-1} = \sum_{i=1}^{e(v)} f_{iv}x^{i-1}$$

## 4. Results Formulation

**Proposition 4.1:** Let $(F_n(C_3))$ be a graph with $n$ number of $C_3$ (cycle of length 3) coinciding at one vertex. Then the geodesic polynomial is represented as $g(F_n(C_3)\ (x)) = f_1 + f_2x$ where $f_1 = 3n,\ f_2 = 2n(n-1)$ and $f_n$ is the number of $n$ geodesic paths in $G$.

***Proof:*** Since the graph is of dimeter 2, $f_n = 0, n > 2$. Here $f_n, n$ in each $(F_n(C_3))$, $n \le 2$ is generalized by using the recurrence relation as follows.

1. For $n = 2, f_1 = 6, f_2 = 4$
2. For $n = 3, f_1 = 9, f_2 = 12$
3. For $n = 4, f_1 = 12, f_2 = 24$ and so on.

Following the sequences of $f_1$ and $f_2$
we get $f_1 = 3n, f_2 = 2n(n-1)$

$$g(F_n(C_3))(x)) = 3n + 2n(n-1)x \tag{4.1}$$

Here proposition 4.1 is explained by Figure 1.

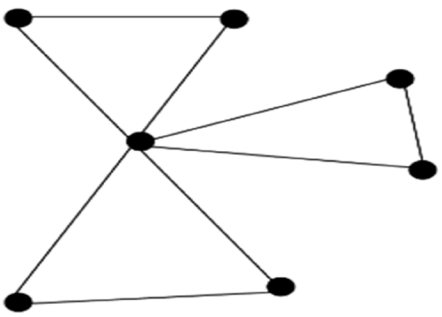


**Figure 1**
$F_n(C_3)$

**Proposition 4.2:** Let $K^n{}_{1,2} \oplus K^n{}_{1,2}$ be a graph which is formed by merging $n$ number of two different sets of $K_{1,2}$ graphs at two vertices. $g(K^n{}_{1,2} \oplus K^n{}_{1,2})\ (x) = f_1 + f_2 x,$ where $f_1 = 4n,\ f_2 = 4n^2$

***Proof:*** Since the diameter of the graph is 2.
So $f_i = 0$ for $i > 2$ and $f_1 = 4n,\ f_2 = 4n^2$ (similar to proposition 4.1)
Hence

$$g(K^n{}_{1,2} \oplus K^n{}_{1,2})(x) = 4n + 4n^2 x \tag{4.2}$$

Here proposition 4.2 is explained by Figure 2.

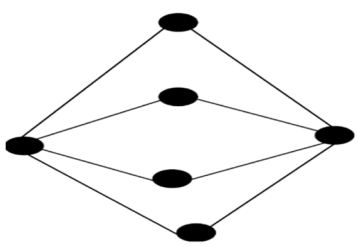


**Figure 2**
$K^n{}_{1,2} \oplus K^n{}_{1,2}$

**Proposition 4.3:** Let $C_3 \oplus K_{1,n}$ be a graph that is formed by attaching $n$ pendant edges at one vertex of $C_3$. Then $g(C_3 \oplus K_{1,n})(x) = (n+3) + \frac{n(n+3)}{2}x$

***Proof:*** $f_i = 0, i > 2$ and $f_1 = n+3$, $f_2 = \frac{n(n+3)}{2}$ (Similar to Proposition 4.1)

$$g(C_3 \oplus K_{1,n})(x) = (n+3) + \frac{n(n+3)}{2}x \tag{4.3}$$

Here proposition 4.3 is explained by Figure 3.

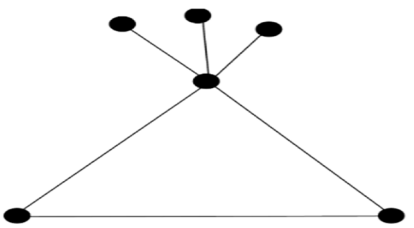


**Figure 3**

$C_3 \oplus K_{1,n}$

**Proposition 4.4:** Let $C_3 \odot K_{1,n}$ be a graph that is formed by joining $n$ pendant edges at each vertex of $C_3$. Then

$$g(C_3 \odot 3K_{1,n})(x) = (3n+3) + [\frac{3}{2}\{(n-1)(n+2)\} - 2]x + (3n^2)x^2$$

***Proof:*** $g(C_3 \odot K_{1,n})(x) = f_1 + f_2 x + f_3 x^3$ where $f_1 = 3n+3$ $f_2 = [\frac{3}{2}\{(n-1)(n+2)\} - 2]$, $f_3 = 3n\text{^}2$ $f_i = 0, i > 3$ (Similar to Proposition 4.1).

$$g(C_3 \odot 3K_{1,n})(x) = (3n+3) + [\frac{3}{2}\{(n-1)(n+2)\} - 2]x + (3n^2)x^2 \tag{4.4}$$

Here proposition 4.4 is explained by Figure 4.

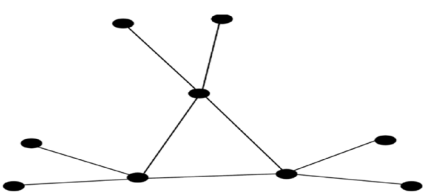


**Figure 4**

$C_3 \odot 3K_{1,n}$

**Proposition 4.5:** Let $H_n$ be a helm graph which is formed by attaching pendant edges to $n$ vertices of the wheel graph. Then

$$g(H_n)(x) = 3n + \frac{(n^2+3n)}{2}x + n(n-2)x^2 + \frac{n(n-3)}{2}x^3, n \geq 3$$

***Proof:***

$$g(H_n)(x) = 3n + \frac{(n^2+3n)}{2}x + n(n-2)x^2 + \frac{n(n-3)}{2}x^3, n \geq 3 \quad (4.5)$$

Similar to Proposition 4.1.

Here proposition 4.5 is explained by Figure 5.

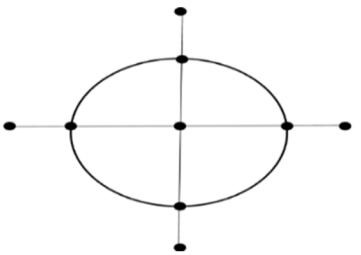


**Figure 5**

$H_4$

**Proposition 4.6:** For a vertex v in the star graph $S_n$

$$G(S_n v(x)) = \begin{cases} n & \text{if } v \text{ is the central vertex} \\ 1+(n-1)x, & \text{if } v \text{ is the end vertex} \end{cases} \quad (4.6)$$

***Proof:*** Let $S_n$ be a star graph with one central vertex is of degree $n$ and the rest of the vertices are of degree 1 (pendant vertices). By using a recurrence relations,

**Case 1:** When $v$ is a central vertex. Let $f_{1,v} = \text{number of } 1$ distant paths(number of edges) from $v$. observe $f_{1,v} = n$ and $f_{i,v} = 0 \text{ for } i > 1$. So $g(S_n v(x)) = n$

**Case 2:** When $v$ is a pendant vertex.

Let $f_{1,v} = 1$, $f_{2,v} = \text{number of } 2$ distant paths $= n-1$.

Hence $g(S_n v(x)) = 1 + (n-1)x.$

Here proposition 4.6 is explained by Figure 6.

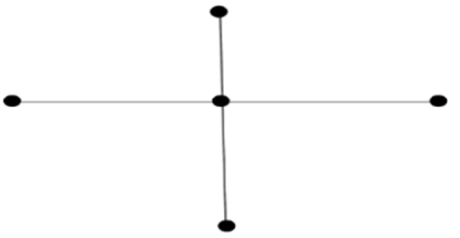


**Figure 6**

$S_4$

**Proposition 4.7:** For a vertex v in the extended star graph $K_{1,n,n}$

$$g(K_{1,n,n} v(x)) = \begin{cases} n + nx & \text{, if } v \text{ is the top vertex} \\ 2 + (n-1)x + (n-1)x^2 & \text{, if } v \text{ is any fixed middle vertex} \\ 1 + x + (n-1)x^2 + (n-1)x^3 & \text{, if } v \text{ is any fixed bottom vertex} \end{cases}$$

***Proof:*** Let $K_{1,n,n}$ is a graph obtained by merging $n$ paths of length two at a point, as shown in Figure 7. $f_{i,v} =$ number of $i$ distant paths from a fixed vertex $v$.

**Case 1:** When $v$ is the top vertex, $f_{1,v} = n, f_{2,v} = n$ by using a recurrence relation, by considering a sequence of one and two distant paths for different values $n$.

**Case 2:** When $v$ is any fixed middle vertex

$f_{1,v} = 2,\ f_{2,v} = n-1,\ f_{3,v} = n-1$ (similar to case 1)

**Case 3:** When $v$ is any fixed end vertex $f_{1,v} = 1,\ f_{2,v} = 1,\ f_{3,v} = n-1,$ $f_{4,v} = n-1$ (similar as case 1 and 2).

Here proposition 4.7 is explained by Figure 7.

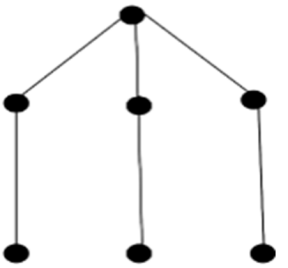


**Figure 7**
$K_{1,3,3}$

## 5. Conclusion

In this paper, the geodesic polynomials of fan graphs, the merging of two star graphs, hericyclic graphs formed by attaching $n$ edges at one or all vertices of $C_3$, Helm graph and geodesic polynomial at a vertex for star graph and the extended star graph are discussed in a brief manner. By referring to the above paper, one can easily measure the shortest distance between two vertices in a graph. For this purpose, one has to consider the coefficients of the Geodesic polynomials in graphs. Observations into the distribution of shortest paths inside graphs are obtained from geodesic polynomials, which are useful for examining the shortest distance. One may find more results on geodesic polynomial of graphs and its vertex by considering graphs of different variation.

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