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Interpreting the corona domination number for the generalized Mycielskian graph of square and cube of some graphs

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Abstract

Corona domination number is defined as, if each vertex in the graph induced by a dominating set. $\mathcal{D}$ satisfies the condition that either $d(u) = 1$ or there exist $v \in \mathcal{D}$ such that $u \in N(v)$, and $d_{\langle \mathcal{D} \rangle}(v) = 1$, then such a dominating set is called a corona dominating set (*CDS*), and its smallest size is referred to as the corona domination number (*CD*-number), denoted by γ_{CD}. In this article, the *CD*-number for the generalized Mycielskian graph of square and cube of some graphs was computed. Also, the *CD*-numbers were given explicitly instead of giving their bounds. The γ_{CD} number for the graphs considered in this article is identified using some number theory techniques.

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Keywords: *Domination number, Corona domination number, Generalized Mycielskian graph, Power graph.*

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1. Introduction and Preliminaries

Let $G = (X, Y)$ be a finite, simple, and undirected graph with the vertex set X and the edge set Y, respectively. The complement of a graph G is represented as G^c. The symbols P_r, C_r, and K_r denote the path, cycle, and complete graph on r vertices, respectively. The lollipop graph $L(m, n)$ is obtained by attaching K_m and P_n by a bridge. If $S \subseteq X(G)$, then the subgraph induced by S is denoted by $\langle S \rangle$. For the basic definition refer [1-3]. The corona domination number (CD-number) in graph theory, as introduced by Mahadevan, Suganthi, and Sivagnanam [4], is defined as follows: a dominating set $\mathcal{D}$ is considered a corona dominating set (CDS) if every vertex in the subgraph induced by $\mathcal{D}$ satisfies one of two conditions: either $d(u) = 1$, or there exists a vertex $v \in \mathcal{D}$ such that $u \in N(v)$ and $d_{\langle \mathcal{D} \rangle}(v) = 1$. The minimum size of such a set is called the corona domination number (CD-number) and is represented as γ_{CD} and computed the CD-number for some standard graphs. In [5] and [6], the CD-number was computed by the authors for certain unique categories of graphs. Given a graph $G = (X, Y)$, its *Mycielskian*, denoted $\mu(G)$, is defined as a graph with a vertex set formed by the disjoint union $X \cup X' \cup \{u\}$, where $X' = \{x' : x \in X\}$. The edge set is described as $Y \cup \{x'y, xy' : xy \in Y\} \cup \{x'u : x' \in X'\}$. If G has r vertices and s edges, then $\mu(G)$ has $2r + 1$ vertices and $2s + r$ edges. For a graph $G = (X, Y)$, the *generalized Mycielskian*, symbolized by $\mu_k(G)$, refers to the graph with the vertex set given by the disjoint union $X \cup (\bigcup_{i=1}^{k} X^i) \cup \{u\}$, where $X^i = \{x^i : x \in X\}$ ia an independent set, $1 \leq i \leq k$, and edge set $Y(\mu_k(G)) = Y(\bigcup_{i=1}^{k} \{y^{i-1}x^i, x^{i-1}y^i : xy \in Y\}) \cup \{x^k u : x^k \in X^k\}$, where $x^0 = x$ and $y^0 = y$. If G has r vertices, then $\mu_k(G)$ has $kr + 1$ vertices and the number of edges depend on the edges in G and the additional connections between the new vertices and the vertex u. The *power graph* G^k of G with $X(G^k) = X(G)$ and two vertices $x_1, x_2 \in X(G^k)$ are adjacent if and only if $d(x_1, x_2) \leq k$. G^2 is called the square graph of G. Power domination and ring domination for the spider graph and some standard graphs were discussed in [7] and [8]. The Hamiltonicity, Determinant, Spectrum, Biclique Partition, Diameter, domination and packing of the Mycielskian graphs were discussed in [9]. In [10] the secure domination number for the generalized Mycielskian of path,cycles and some more standard graphs were discussed. In [11] the strong (weak), chromatic strong (weak), strong(weak) efficient open domination numbers were computed. Recent studies have extended domination theory to advanced parameters and interdisciplinary applications. Mehraban and Alikhani [12] introduced the independent domination bondage number, focusing on stability under edge removal. Mani, Ilyass, and Prasad [13] explored applications of connected certified domination in wireless sensor networks. Further, Madan et al. [14] and Muneshwar and Bondar [15] investigated graph structures arising from topological spaces, providing a foundation for studying domination in such frameworks. Having all the above as a motivation, here the CD-number of generalized Mycielskian graph for some graphs were computed and their γ_{CD} values are given explicitly. This article is mainly divided into three sections; the

first section consists of all the preliminaries which are used for the upcoming sections, the second section will give the exact γ_{CD} for the $\mu_r(P_n^2)$ and $\mu_r(C_n^2)$, the third section discusses the CD- number of $\mu_r(P_n^3)$ and $\mu_r(C_n^3)$ in a detailed manner. We chose the square and cube graphs because they are common ways to make a graph more connected while still keeping its original shape. Studying these helps us understand how the CD-number changes when we allow each point in the graph to see more of its neighbors. This makes finding patterns easier and using number theory to determine the exact values.

2. Corona Domination number for the Generalized Mycielskian Graph of Square of Path and Cycle.

In this section, the corona domination number for the square of the Generalized Mycielskian graph of paths and cycles are discussed. The CD-number for the square of the Generalized Mycielskian graph of paths is explained in detail, while the case for cycles is not elaborated upon, as the CD-numbers for both graphs are identical.

The graph $\mu_r(P_n^2)$ is the r-th generalized Mycielskian of the square of the path graph P_n, with vertex set:

$$V(\mu_r(P_n^2)) = X \cup \left(\cup_{j=1}^{r} X^j\right) \cup \{x\},$$

where $X = \{x_1, x_2, \dots, x_n\}$ is the vertex set of P_n^2, X^j represents the j-th independent copy of X, and x is a vertex connected to all vertices in X^r. The edge set of $\mu_r(P_n^2)$, denoted as $Y(\mu_r(P_n^2))$, is given by:

$$Y(\mu_r(P_n^2)) = Y(P_n^2) \cup \left(\cup_{j=1}^{r} \{x^{j-1}y^j | xy \in Y(P_n^2)\}\right) \cup \{x^k x | x^k \in X^r\}.$$

Here, $Y(P_n^2)$ represents the edge set of the square of the path graph P_n^2, x^{j-1} and y^{j-1} correspond to vertices in the $(j-1)$-th distinct copy of X, x^j and y^j refer to vertices in the j-th distinct copy of X, and x is a vertex that is adjacent to all vertices in X^r.

Observation 2.1:

1. $\gamma_{CD}(\mu(P_r^2)) = \gamma_{CD}(P_r^2) + 1.$

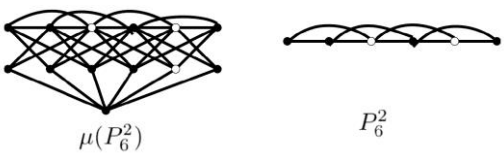

Figure 1
Square graph of path on 6 vertices and it's Mycielskian graph 1

Illustration: In Figure 1 the white vertices will gives us the CDS of both $\mu(P_6^2)$ and P_6^2.

2. $\gamma_{CD}(\mu_2(P_r^2)) = \gamma_{CD}(P_r^2) + 2.$
3. $\gamma_{CD}(\mu_3(P_r^2)) = 2\gamma_{CD}(P_r^2) + 1.$

Theorem 2.1: *Let P_n represent a path with n vertices. If $r \geq 4$ and $r = 4a$, then*

$$\gamma_{CD}(\mu_r(P_n^2)) = \begin{cases} 4\left\lceil \frac{r}{4} \right\rceil \left\lceil \frac{n}{7} \right\rceil + \frac{r}{2} + 2 & if \quad n = 7b + 1 or 7b + 2, \\ 4\left\lceil \frac{n}{7} \right\rceil \left\lceil \frac{r}{4} \right\rceil + 2 & otherwise. \end{cases}$$

Proof: Define the set S_1 as follows:

$$S_1 = \{x_i^j : i = 7a + 3 \text{ or } 7a + 5, j = 4a + 1 \text{ or } 4a + 2\} \cup \{x_1^r, x\}.$$

The structure of $\mu_r(P_n^2)$ varies with n. Thus, we define S by adding specific vertices to S_1 based on n (mod 7):

- If $n = 7b$, $7b + 5$, or $7b + 6$: Set $S = S_1$, as S_1 alone sufficient to dominate all vertices in $\mu_r(P_n^2)$.
- If $n = 7b + 1$ or $7b + 3$: Set $S = S_1 \cup \{x_{n-1}^j : j = 4a + 1 \text{or} 4a + 2\}$, adding vertices will dominate all vertices in $\mu_r(P_n^2)$.
- If $n = 7b + 2$: Set $S = S_1 \cup \{x_{n-2}^j : j = 4a + 1 \text{ or } 4a + 2\}$, ensuring coverage for the vertices, otherwise lack of adjacency in $\mu_r(P_n^2)$.
- If $n = 7b + 4$: Set $S = S_1 \cup \{x_n^j : j = 4a + 1 \text{ or } 4a + 2\}$, ensuring the last vertices are properly dominated in $\mu_r(P_n^2)$.

This case-based construction guarantees that every vertex in $\mu_r(P_n^2)$ is dominated by S and that S satisfies the corona domination condition, with every vertex in $\langle S \rangle$ having degree one or being adjacent to a vertex with degree one.

The size $|S|$ of S depends on the cases defined above:

$$|S| = \begin{cases} 4\left\lceil \frac{r}{4} \right\rceil \left\lfloor \frac{n}{7} \right\rfloor + \frac{r}{2} + 2 & \text{if } n = 7b + 1 \text{ or } 7b + 2, \\ 4\left\lceil \frac{n}{7} \right\rceil \left\lceil \frac{r}{4} \right\rceil + 2 & \text{otherwise.} \end{cases}$$

Let S be a dominating set of $\mu_r(P_n^2)$ satisfying $|S| \leq d$, where d represents the maximum bound and

$$d = \begin{cases} 4\left\lceil \frac{r}{4} \right\rceil \left\lfloor \frac{n}{7} \right\rfloor + \frac{r}{2} + 1 & \text{if } n = 7b + 1 \text{ or } 7b + 2, \\ 4\left\lceil \frac{n}{7} \right\rceil \left\lceil \frac{r}{4} \right\rceil + 1 & \text{otherwise.} \end{cases}$$

We will show that this assumption leads to a contradiction. Specifically, if $\mathcal{D}$ is a dominating set with size not greater than d, there must be some vertex $u \in \mathcal{D}$ such that u lacks adjacent vertices in the subgraph $\langle \mathcal{D} \rangle$. As a result, $\deg(u) = 0$, which contradicts with either the condition $\deg(u) = 1$ or the existence of a vertex $v \in \mathcal{D}$ such that $u \in N(v)$ and $\deg_{\langle \mathcal{D} \rangle}(v) = 1$. Thus, the size of any dominating set $\mathcal{D}$ must satisfy:

$$|\mathcal{D}| \geq d+1 = \begin{cases} 4\left\lceil\frac{r}{4}\right\rceil\left\lfloor\frac{n}{7}\right\rfloor + \frac{r}{2} + 2 & \text{if } n = 7b+1 or 7b+2, \\ 4\left\lceil\frac{n}{7}\right\rceil\left\lceil\frac{r}{4}\right\rceil + 2 & \text{otherwise.} \end{cases}$$

Hence, the $\gamma_{CD}(\mu_r(P_n^2))$ is at least $|S|$, and since we have already shown that $|S|$ is an upper bound for $\gamma_{CD}(\mu_r(P_n^2))$, we conclude that: $\gamma_{CD}(\mu_r(P_n^2)) = |S|$. This brings the proof to an end.

Theorem 2.2: *Let P_n represent a path with n vertices. If $r \geq 4$ and $r = 4a + 1$, then*

$$\gamma_{CD}(\mu_r(P_n^2)) = \begin{cases} 4\left\lfloor\frac{r}{4}\right\rfloor\left\lfloor\frac{n}{7}\right\rfloor + 2(\left\lfloor\frac{r}{4}\right\rfloor + \left\lfloor\frac{n}{7}\right\rfloor + 1) & \text{if } n = 7b+1 \text{ or } 7b+2, \\ 4\left\lceil\frac{n}{7}\right\rceil\left\lfloor\frac{r}{4}\right\rfloor + 2\left\lceil\frac{n}{7}\right\rceil + 1 & \text{otherwise.} \end{cases}$$

Proof: Define the set S_1 as follows:

$$S_1 = \{x_i^j : i = 7a+3 \; or \; 7a+5, j = 4a+1 \; or \; 4a+2\} \cup \{x\}.$$

The structure of $\mu_r(P_n^2)$ varies with n. Thus, we define S by adding specific vertices to S_1 based on n (mod 7):

- If $n = 7b$, $7b+5$, or $7b+6$: Set $S = S_1$, as S_1 alone sufficient to dominate all vertices in $\mu_r(P_n^2)$.
- If $n = 7b+1$ or $7b+3$: Set $S = S_1 \cup \{x_{n-1}^j : j = 4a+1 \text{ or } 4a+2\}$, adding vertices will dominate all vertices in $\mu_r(P_n^2)$.
- If $n = 7b+2$: Set $S = S_1 \cup \{x_{n-2}^j : j = 4a+1 \text{ or } 4a+2\}$, ensuring coverage for the vertices, otherwise lack of adjacency in $\mu_r(P_n^2)$.
- If $n = 7b+4$: Set $S = S_1 \cup \{x_n^j : j = 4a+1 \text{ or } 4a+2\}$, ensuring the last vertices are properly dominated in $\mu_r(P_n^2)$.

This case-based construction guarantees that every vertex in $\mu_r(P_n^2)$ is dominated by S and that S satisfies the corona domination condition, with every vertex in $\langle S \rangle$ having degree one or being adjacent to a vertex with degree one.

The size $|S|$ of S depends on the cases defined above:

$$|S| = \begin{cases} 4\left\lfloor\frac{r}{4}\right\rfloor\left\lfloor\frac{n}{7}\right\rfloor + 2(\left\lfloor\frac{r}{4}\right\rfloor + \left\lfloor\frac{n}{7}\right\rfloor + 1) & \text{if } n = 7b+1 \text{ or } 7b+2, \\ 4\left\lceil\frac{n}{7}\right\rceil\left\lfloor\frac{r}{4}\right\rfloor + 2\left\lceil\frac{n}{7}\right\rceil + 1 & \text{otherwise.} \end{cases}$$

Let S be a dominating set of $\mu_r(P_n^2)$ satisfying $|S| \leq d$, where d represents the maximum bound and

$$d = \begin{cases} 4\left\lfloor\frac{r}{4}\right\rfloor\left\lfloor\frac{n}{7}\right\rfloor + 2(\left\lfloor\frac{r}{4}\right\rfloor + \left\lfloor\frac{n}{7}\right\rfloor + 1) - 1 & \text{if } n = 7b+1 \text{ or } 7b+2, \\ 4\left\lceil\frac{n}{7}\right\rceil\left\lfloor\frac{r}{4}\right\rfloor + 2\left\lceil\frac{n}{7}\right\rceil & \text{otherwise.} \end{cases}$$

We will show that this assumption leads to a contradiction. Specifically, if $\mathcal{D}$ is a dominating set with size not greater than d, there must be some vertex $u \in \mathcal{D}$ such that u lacks adjacent vertices in the subgraph $\langle D \rangle$. As a result, $\deg(u) = 0$, which contradicts with either the condition $\deg(u) = 1$ or the existence of a vertex $v \in \mathcal{D}$ such that $u \in N(v)$ and $\deg_{\langle \mathcal{D} \rangle}(v) = 1$. Thus, the size of any dominating set $\mathcal{D}$ must satisfy:

$$|\mathcal{D}| \geq d + 1 = \begin{cases} 4\left\lfloor \frac{r}{4} \right\rfloor \left\lfloor \frac{n}{7} \right\rfloor + 2\left(\left\lfloor \frac{r}{4} \right\rfloor + \left\lfloor \frac{n}{7} \right\rfloor + 1\right) & \text{if } n = 7b+1 \text{ or } 7b+2, \\ 4\left\lfloor \frac{n}{7} \right\rfloor \left\lfloor \frac{r}{4} \right\rfloor + 2\left\lfloor \frac{n}{7} \right\rfloor + 1 & \text{otherwise.} \end{cases}$$

Hence, the $\gamma_{CD}(\mu_r(P_n^2))$ is at least $|S|$, and since we have already shown that $|S|$ is an upper bound for $\gamma_{CD}(\mu_r(P_n^2))$, we conclude that: $\gamma_{CD}(\mu_r(P_n^2)) = |S|$. This brings the proof to an end.

Theorem 2.3: *Let P_n represent a path with n vertices. If $r \geq 4$ and $r = 4a + 2$, then*

$$\gamma_{CD}(\mu_r(P_n^2)) = \begin{cases} 4\left\lfloor \frac{n}{7} \right\rfloor \left\lceil \frac{r}{4} \right\rceil + 2\left\lceil \frac{r}{4} \right\rceil & \text{if } n = 7b+1 \text{ or } 7b+2, \\ 4\left\lceil \frac{n}{7} \right\rceil \left\lceil \frac{r}{4} \right\rceil & \text{otherwise.} \end{cases}$$

Proof: Define the set S_1 as follows:

$$S_1 = \{x_i^j : i = 7a+3 \text{ or } 7a+5, j = 4a+1 \text{ or } 4a+2\}.$$

The structure of $\mu_r(P_n^2)$ depends on the residue of n modulo 7. Accordingly, we define S by adding specific vertices to S_1, and the remaining proof proceeds analogously to the previous theorem.

Theorem 2.4: *Let P_n represent a path with n vertices. If $r \geq 4$ and $r = 4a + 3$, then*

$$\gamma_{CD}(\mu_r(P_n^2)) = \begin{cases} 4\left\lfloor \frac{n}{7} \right\rfloor \left\lceil \frac{r}{4} \right\rceil + 2\left\lceil \frac{r}{4} \right\rceil + 1 & \text{if } n = 7b+1 \text{ or } 7b+2, \\ 4\left\lceil \frac{n}{7} \right\rceil \left\lceil \frac{r}{4} \right\rceil & \text{otherwise.} \end{cases}$$

Proof: Define the set S_1 as follows:

$$S_1 = \{x_i^j : i = 7a+3 \text{ or } 7a+5, j = 4a+1 \text{ or } 4a+2\} \cup \{x_1^r\}.$$

The structure of $\mu_r(P_n^2)$ depends on the residue of n modulo 7. Accordingly, we define S by adding specific vertices to S_1, and the remaining proof proceeds analogously to the previous theorem.

3. Corona Domination number for the Generalized Mycielskian Graph of Cube of Path and Cycle.

In this section, the corona domination number (CD-number) of the cube of the Generalized Mycielskian graph for paths and cycles is discussed. Since the

CD-numbers for paths and cycles are identical, the detailed analysis focuses on paths, with results applying equally to cycles.

The graph $\mu_r(P_n^3)$, the r-th Generalized Mycielskian of the cube of the path graph P_n, has vertex set:

$$V(\mu_r(P_n^3)) = X \cup \left(\cup_{j=1}^{r} X^j\right) \cup \{x\},$$

where $X = \{x_1, x_2, \dots, x_n\}$ is the vertex set of P_n^3, X^j is the j-th independent copy of X, and x is adjacent to all vertices in X^r.

The edge set $Y(\mu_r(P_n^3))$ is:

$$Y(\mu_r(P_n^3)) = Y(P_n^3) \cup \left(\cup_{j=1}^{r} \{x^{j-1}y^j | xy \in Y(P_n^3)\}\right) \cup \{x^k x | x^k \in X^r\},$$

where $Y(P_n^3)$ is the edge set of the cube of the path graph P_n^3, and $x^{j-1}, y^{j-1}, x^j, y^j$ are corresponding vertices in different copies.

Observation 3.1:

1. $\gamma_{CD}(\mu(P_r^3)) = \gamma_{CD}(P_r^3) + 1$.

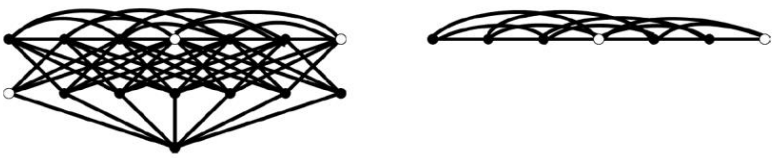

Figure 2
Cube graph of path on 7 vertices and it's Mycielskian graph 2

Illustration: In Figure 2 the white vertices will gives us the CDS of both $\mu(P_7^3)$ and P_7^3.

2. $\gamma_{CD}(\mu_2(P_r^3)) = \gamma_{CD}(P_r^3) + 2$.
3. $\gamma_{CD}(\mu_3(P_r^3)) = 2\gamma_{CD}(P_r^3) + 1$.
4. For $k \geq 1$, $\gamma_{CD}(\mu_k(C_r^3)) = \gamma_{CD}(\mu_k(P_r^3))$.

Theorem 3.1: *Let P_n represent a path with n vertices. If $r \geq 4$ and $r = 4a$, then*

$$\gamma_{CD}(\mu_r(P_n^3)) = \begin{cases} 4\left\lceil \frac{r}{4} \right\rceil \left\lfloor \frac{n}{10} \right\rfloor + 2\left\lceil \frac{r}{4} \right\rceil + 2 & \text{if } n = 10b+1 \text{ or } 10b+2 \text{ or } 10b1+3, \\ 4\left\lceil \frac{n}{10} \right\rceil \left\lceil \frac{r}{4} \right\rceil + 2 & \text{otherwise.} \end{cases}$$

Proof: Define the set S_1 as follows:

$$S_1 = \{x_i^j : i = 10a+4 \text{ or } 10a+7, j = 4a+1 \text{ or } 4a+2\} \cup \{x_1^r, x\}.$$

The structure of $\mu_r(P_n^3)$ varies with n. Thus, we define S by adding specific vertices to S_1 based on n (mod 10):

- If $n = 10b$ or $10b+7$ or $10b+8$ or $10b+9$: Set $S = S_1$, as S_1 alone sufficient to dominate all vertices in $\mu_r(P_n^3)$.

- If $n = n = 10b + 1$ or $10b + 4$: Set $S = S_1 \cup \{x_{n-1}^j : j = 4a + 1$ or $4a + 2\}$, adding vertices will dominate all vertices in $\mu_r(P_n^3)$.
- If $n = 10b + 2$ or $10b + 5$: Set $S = S_1 \cup \{x_{n-2}^j : j = 4a + 1$ or $4a + 2\}$, ensuring coverage for the vertices, otherwise lack of adjacency in $\mu_r(P_n^3)$.
- If $n = 10b + 3$ or $10b + 6$: Set $S = S_1 \cup \{x_n^j : j = 4a + 1$ or $4a + 2\}$, ensuring the last vertices are properly dominated in $\mu_r(P_n^3)$.

This case-based construction guarantees that every vertex in $\mu_r(P_n^3)$ is dominated by S and that S satisfies the corona domination condition, with every vertex in $\langle S \rangle$ having degree one or being adjacent to a vertex with degree one.

The size $|S|$ of S depends on the cases defined above:

$$|S| = \begin{cases} 4\left\lceil \frac{r}{4} \right\rceil \left\lfloor \frac{n}{10} \right\rfloor + 2\left\lceil \frac{r}{4} \right\rceil + 2 & \text{if } n = 10b + 1 \text{ or } 10b + 2 \text{ or } 10b + 3, \\ 4\left\lceil \frac{n}{10} \right\rceil \left\lceil \frac{r}{4} \right\rceil + 2 & \text{otherwise.} \end{cases}$$

Let S be a dominating set of $\mu_r(P_n^2)$ satisfying $|S| \leq d$, where d represents the maximum bound and

$$d = \begin{cases} 4\left\lceil \frac{r}{4} \right\rceil \left\lfloor \frac{n}{10} \right\rfloor + 2\left\lceil \frac{r}{4} \right\rceil + 1 & \text{if } n = 10b + 1 \text{ or } 10b + 2 \text{ or } 10b + 3, \\ 4\left\lceil \frac{n}{10} \right\rceil \left\lceil \frac{r}{4} \right\rceil + 1 & \text{otherwise.} \end{cases}$$

We will show that this assumption leads to a contradiction. Specifically, if $\mathcal{D}$ is a dominating set with size not greater than d, there must be some vertex $u \in \mathcal{D}$ such that u lacks adjacent vertices in the subgraph $\langle \mathcal{D} \rangle$. As a result, $\deg(u) = 0$, which contradicts with either the condition $\deg(u) = 1$ or the existence of a vertex $v \in \mathcal{D}$ such that $u \in N(v)$ and $\deg_{\langle \mathcal{D} \rangle}(v) = 1$. Thus, the size of any dominating set $\mathcal{D}$ must satisfy:

$$|\mathcal{D}| \geq d + 1 = \begin{cases} 4\left\lceil \frac{r}{4} \right\rceil \left\lfloor \frac{n}{10} \right\rfloor + 2\left\lceil \frac{r}{4} \right\rceil + 2 & \text{if } n = 10b + 1 \text{ or } 10b + 2 \text{ or } 10b + 3, \\ 4\left\lceil \frac{n}{10} \right\rceil \left\lceil \frac{r}{4} \right\rceil + 2 & \text{otherwise.} \end{cases}$$

Hence, the $\gamma_{CD}(\mu_r(P_n^3))$ is at least $|S|$, and since we have already shown that $|S|$ is an upper bound for $\gamma_{CD}(\mu_r(P_n^3))$, we conclude that: $\gamma_{CD}(\mu_r(P_n^3)) = |S|$. This brings the proof to an end.

Theorem 3.2: *Let P_n represent a path with n vertices. If $r \geq 4$ and $r = 4a + 1$, then*

$$\gamma_{CD}(\mu_r(P_n^3)) = \begin{cases} 4\left\lceil \frac{r}{4} \right\rceil \left\lfloor \frac{n}{10} \right\rfloor + 2\left(\left\lfloor \frac{n}{10} \right\rfloor + \left\lceil \frac{r}{4} \right\rceil + 1\right) & \text{if } n = 10b + 1 \text{ or } 10b + 2 \text{ or } 10b + 3, \\ 4\left\lceil \frac{n}{10} \right\rceil \left\lceil \frac{r}{4} \right\rceil + 2\left\lceil \frac{n}{10} \right\rceil + 1 & \text{otherwise.} \end{cases}$$

Proof: Define the set S_1 as follows:

$$S_1 = \{x_i^j : i = 10a + 4 \text{ or } 10a + 7, j = 4a + 1 \text{ or } 4a + 2\} \cup \{x\}.$$

The structure of $\mu_r(P_n^3)$ depends on the residue of n modulo 10. Accordingly, we define S by adding specific vertices to S_1, and the remaining proof proceeds analogously to the previous theorem.

Theorem 3.3: *Let P_n represent a path with n vertices. If $r \geq 4$ and $r = 4a + 2$, then*

$$\gamma_{CD}(\mu_r(P_n^3)) = \begin{cases} 4\left\lceil \frac{r}{4} \right\rceil \left\lfloor \frac{n}{10} \right\rfloor + 2\left\lceil \frac{r}{4} \right\rceil & \text{if } n = 10b + 1 \text{ or } 10b + 2 \text{ or } 10b + 3, \\ 4\left\lceil \frac{n}{10} \right\rceil \left\lceil \frac{r}{4} \right\rceil & \text{otherwise.} \end{cases}$$

Proof: Define the set S_1 as follows:

$$S_1 = \{x_i^j : i = 10a + 4 \text{ or } 10a + 7, j = 4a + 1 \text{ or } 4a + 2\}.$$

The structure of $\mu_r(P_n^3)$ depends on the residue of n modulo 10. Accordingly, we define S by adding specific vertices to S_1, and the remaining proof proceeds analogously to the theorem 3.1.

Theorem 3.4: *Let P_n represent a path with n vertices. If $r \geq 4$ and $r = 4a + 3$, then*

$$\gamma_{CD}(\mu_r(P_n^3)) = \begin{cases} 4\left\lceil \frac{r}{4} \right\rceil \left\lfloor \frac{n}{10} \right\rfloor + 2\left\lceil \frac{r}{4} \right\rceil + 1 & \text{if } n = 10b + 1 \text{ or } 10b + 2 \text{ or } 10b + 3, \\ 4\left\lceil \frac{n}{10} \right\rceil \left\lceil \frac{r}{4} \right\rceil + 1 & \text{otherwise.} \end{cases}$$

Proof: Define the set S_1 as follows:

$$S_1 = \{x_i^j : i = 10a + 4 \text{ or } 10a + 7, j = 4a + 1 \text{ or } 4a + 2\} \cup \{x_1^r\}.$$

The structure of $\mu_r(P_n^3)$ depends on the residue of n modulo 10. Accordingly, we define S by adding specific vertices to S_1, and the remaining proof proceeds analogously to the theorem 3.1.

Example 3.1:

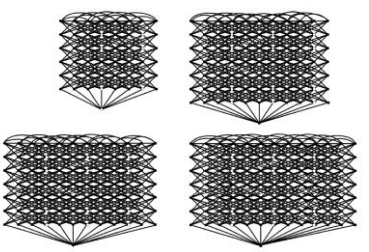

Figure 3
$\boldsymbol{\mu_4(P_8^3), \mu_6(P_{11}^3), \mu_6(P_{12}^3), \mu_6(P_{13}^3)}$

Illustration: The graphs given in Figure 3 are $\mu_4(P_8^3), \mu_6(P_{11}^3), \mu_6(P_{12}^3), \mu_6(P_{13}^3)$ respectively and the CDS is given in with the white vertices.

Applications to Real-World Problems

The concept of corona domination, especially when applied within generalized Mycielskian graphs, offers valuable insights for addressing challenges in various real-world domains such as network design, fault-tolerant systems, and social network analysis. This property supports the efficient functioning of complex systems where cost, energy, or communication constraints are significant.

Sensor Networks

One key challenge in wireless sensor networks (WSNs) is selecting a minimal set of sensor nodes that can effectively monitor an area (i.e., dominate the network) while reducing communication overhead to preserve battery life. A CDS provides a way to identify such sets by ensuring each sensor is either minimally connected or directly connected to one, achieving both redundancy and energy efficiency. The hierarchical nature of the generalized Mycielskian model is beneficial in representing multi-tiered deployments of sensors, where each level introduces redundancy or extended coverage.

Case Study: In an innovative agriculture system, sensors are deployed across farmland to monitor soil moisture, temperature, and other environmental factors. Organizing these sensors using a generalized Mycielskian graph allows the creation of layered communication structures. A CDS helps identify a minimal yet robust subset of sensors for direct data collection, ensuring full area coverage while reducing inter-sensor communication. This structure extends the overall lifespan of the network and enhances fault tolerance.

4. Conclusion

This article studied the CD-number of square and cube versions of generalized Mycielskian graphs. We used number theory methods to find the exact values of these numbers. Instead of just giving estimates or limits, we provided the actual CD numbers. This finding shows a clearer understanding of how the CD-number behaves when the graph becomes more connected. Our results add helpful information to graph theory and domination in graphs. This work can help future studies involving graph powers and domination parameters.

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