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Graph operations on Lanzhou index intuitionistic fuzzy graph

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Abstract

Intuitionistic fuzzy graphs *IFgs* are a generalization of same community which belongs to assets and liabilities fuzzy graphs which provide us with a broader context to address the uncertainty, hesitation, and ambiguity which naturally exists within real-world complex systems, such as social networks, communication systems and biological networks. With the intuitionistic fuzzy model, we can describe partial truths more accurately than we could with classical or fuzzy graphs. A more recent development within this real includes the Lanzhou index, a topological index that characterizes the molecular structure in chemical graph theory more closely than the current derivatives provide. In this paper, we explore the Lanzhou index's graph operations on *IFgs*. Similarly, these operations will reflect alterations in a network construction, such as combining systems, generating new links and expanding the network, and the Lanzhou index's behavior will be illustrative of the relative effect.

Subject Classification: *03E72, 03E75, 05C09.*

Keywords: *Degree, Distance, Graph operations, Intuitionistic fuzzy graph.*

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1. Introduction

The incorporation of classical fuzzy graph concepts, which inherently involve ambiguity and uncertainty, with club-based attributes assigned to nodes and their associated components has gained significant attention. Uncertainty or indistinctness often arises in relationships between entities for example, in social networks where user relationships may be vague or ambiguous. Similarly, decision-making processes may involve uncertain or equivocal choices, and membership in clubs or organizations may not always be clearly defined, leading to hesitation in classification. The principles of classical graph theory are extended to domains where uncertainty and hesitation are essential elements through various operations. In chemical graph theory, the complement of a graph may represent the absence of bonds (or weak interactions), which can be useful in designing molecules with desired properties and in identifying potential side effects in chemical reactions [13–15]. In social network analysis, the relationships among users and user groups are often uncertain; operations such as union and intersection can help identify communities or potential new connections. Gani and Ahamed [4] introduced new perspectives in this area for order and degree of an intuitionstic fuzzy graph. Mohamed and Ali [5] calculated the complement of max product of intuitionstic fuzzy graphs. Early researchers studied complements based on max–min concepts and examined key variations in intuitionistic fuzzy sets [7, 9]. Sahoo and Pal [10] compared fuzzy orderings with similarity relations. Rosenfeld [11] introduced fuzzy graphs and studied their properties. Binu, Mathew, and Mordeson [12] further investigated different product structures in intuitionistic fuzzy graphs. Okelo and Onyango [8] proposed concepts related to topological fuzzy Hausdorff spaces. The foundational idea of fuzzy set theory was introduced by Atanassov [6], who later developed intuitionistic fuzzy sets. Gani and Begum [3] defined degree, size, and order in intuitionistic fuzzy graphs. The Lanzhou index and its properties, which are related to physicochemical characteristics, were studied in [1, 2]. The development of fuzzy graphs, intuitionistic fuzzy graphs, and their applications is discussed in [16–18]. Cryptographic approaches in fuzzy graphs were extended in [21], while strong and regular fuzzy graphs were analyzed in [22, 23]. Furthermore, connectivity-based indices were correlated with fuzzy topological indices in [19, 20]. This study investigates various formulations of the Lanzhou index under different graph operations and examines optimal solutions based on membership criteria.

Throughout this paper, the basic notations related to $I\mathrm{F}\mathscr{g}$, if there exists a closed relation, it can be considered as a first neighborhood in the graph. The absence of any connections between two nodes can be treated as a complement relation. $\delta(\varepsilon) = (\delta\mu(\varepsilon), \delta\gamma(\varepsilon))$ as $\mathscr{g} = (\wp, \mathbb{Q})$. no connections between any two nodes as a complement $I\mathrm{F}\mathscr{g}s$, $\overline{\varepsilon} = (\wp, \mathbb{Q}, \bar{\sigma}, \bar{\mu})$, where $\sigma = \overline{\sigma}$,

$\overline{\mu_1}(\varepsilon_i, \varepsilon_j) = \sigma_1(\varepsilon_i) \wedge \sigma_1(\varepsilon_i) - \mu_1(\varepsilon_i, \varepsilon_j)$,

$\overline{\mu_2}(\varepsilon_i, \varepsilon_j) = \sigma_2(\varepsilon_i) \vee \sigma_2(\varepsilon_i) - \mu_2(\varepsilon_i, \varepsilon_j) \forall\, \varepsilon_i, \varepsilon_j \in \wp$

The degree-based index in $I\text{F}\mathcal{g}$, $M_1(\wp) = \sum_{u\in V(G)} \delta_u^2$. The Lanzhou index of graph $I\text{F}\mathcal{g}s$ is $Lz(\wp) = \sum_{u\in V(\varepsilon)} \delta_u^2 \overline{\delta_u}$.

2. Combination of IF$\mathcal{g}$

In this section, outline an intuitionistic fuzzy Lanzhou index. The Lanzhou index is described because the fabricated from the diploma of the vertex and the supplement diploma of the vertex. For the intuitionistic graph of the Lanzhou index, we outline each the diploma of the vertex and the supplement diploma of the vertex in phrases of reality club and fake club.

Definition 3.1: The Lanzhou index of $I\mathcal{F}\mathcal{g}$ is

$$Lz(\mathcal{g}) = \sum_{\varepsilon\in p^*}\left\{T_p(\varepsilon)^2\left(\alpha_{\mathcal{g}}^T(\varepsilon)\right)^2\overline{deg_{\mathcal{g}}^T(\varepsilon)}, F_p(a)^2\left(\alpha_{\mathcal{g}}^F(\varepsilon)\right)^2\overline{deg_{\mathcal{g}}^F(\varepsilon)}\right\}$$

Definition 3.2: Let $\mathcal{g}_1$ be a single valued $I\mathcal{F}\mathcal{g}$ and $\mathcal{g}_2$ be a single valued $I\mathcal{F}\mathcal{g}$, then $\mathcal{g}_1\otimes\mathcal{g}_2$ be a single valued $I\mathcal{F}\mathcal{g}$s.

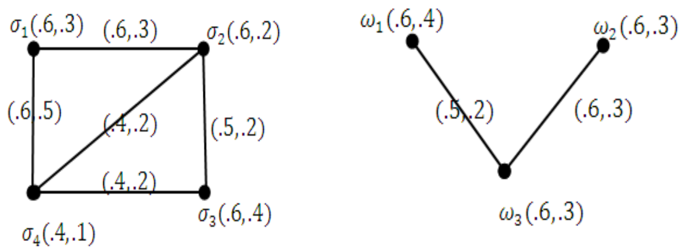

Figure 1
The $I\mathcal{F}$ $\mathcal{g}_1 = (\sigma,\tau)$ and $\mathcal{g}_2 = (\omega,\theta)$

Definition 3.3 Let $\mathcal{g}_1$ be a single valued $I\mathcal{F}\mathcal{g}$ and $\mathcal{g}_2$ be a single valued $I\mathcal{F}\mathcal{g}$, then $\mathcal{g}_1 \boxdot \mathcal{g}_2$ be a single valued $I\mathcal{F}\mathcal{g}$s.

$$\mathcal{g}_1 = (\sigma,\tau) \text{ and } \mathcal{g}_2 = (\omega,\theta)$$

The $I\mathcal{F}\mathcal{g}$, $\mathcal{g}_1 = (\sigma,\tau)$ and $\mathcal{g}_2 = (\omega,\theta)$ in Figure 1, by the definition of intuitionistic Lanzhou index in Figure 2, for tensor product of graph is,

$$\begin{aligned}
Lz[(\mathcal{g}_1\otimes\mathcal{g}_2)] &= [(0.6)^2(1.2)^2(5.4) + (0.6)^2(1.2)^2(5.4) + (0.6)^2(2.4)^2(4.2)\\
&+(0.6)^2(1.6)^2(3.8) + (0.6)^2(1.6)^2(5.0) + (0.6)^2(3.4)^2(2.6) + (0.6)^2(1.0)^2(5.6)\\
&+(0.6)^2(1.1)^2(6.0) + (0.6)^2(2.1)^2(4.6) + (0.6)^2(1.6)^2(0.5) + (0.6)^2(1.7)^2(4.9)\\
&+(0.6)^2(3.2)^2(3.9)], [(0.3)^2(0.4)^2(3.1) + (0.3)^2(0.6)^2(2.8) + (0.3)^2(1.0)^2(2.4)\\
&+(0.2)^2(0.6)^2(2.1) + (0.2)^2(0.7)^2(2.3) + (0.2)^2(1.3)^2(1.4) + (0.4)^2(0.6)^2(4.0)\\
&+(0.3)^2(0.4)^2(3.2) + (0.3)^2(0.8)^2(2.6) + (0.1)^2(0.5)^2(0.1) + (0.1)^2(0.6)^2(2.0)\\
&+(0.1)^2(1.3)^2(1.7)]
\end{aligned}$$

$$Lz(\mathcal{g}_1\otimes\mathcal{g}_2) = [45.261, 0.848]$$

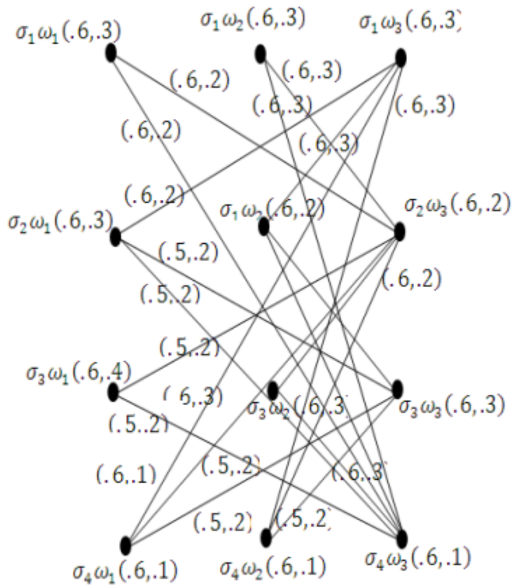

Figure 2
Tensor Product of $I\mathcal{F}$ $\alpha\otimes\beta$

Consider a $I\mathcal{F}g$ for $g_1$and g_2 in Figure 3,

$$
\begin{aligned}
Lz[L(\alpha \boxdot \beta)] = {} & [(0.6)^2(4.2)^2(2.4) + (0.6)^2(4.2)^2(2.4) \\
& +(0.6)^2(4.8)^2(1.8) + (0.6)^2(5.5)^2(1.1) + (0.6)^2(5.7)^2(0.9) \\
& +(0.6)^2(6.4)^2(0.2) + (0.6)^2(3.5)^2(3.1) + (0.6)^2(3.9)^2(2.7) \\
& +(0.6)^2(4.6)^2(2.6) + (0.6)^2(5.1)^2(1.5) + (0.6)^2(5.6)^2(1.6) \\
& +(0.6)^2(6.3)^2(0.3)] + [(0.3)^2(1.8)^2(1.7) + (0.3)^2(1.3)^2(2.1) \\
& +(0.3)^2(2.1)^2(1.3) + (0.2)^2(2.2)^2(0.7) + (0.2)^2(2.3)^2(0.4) \\
& +(0.2)^2(2.4)^2(0.4) + (0.4)^2(1.6)^2(2.8) + (0.2)^2(1.5)^2(1.6) \\
& +(0.3)^2(1.7)^2(1.5) + (0.1)^2(2.3)^2(0.4) + (0.1)^2(2.2)^2(0.5) \\
& +(0.1)^2(2.2)^2(0.6)]
\end{aligned}
$$

$$Lz(\alpha \boxdot \beta) = (155.52288, 3.59539).$$

Definition 3.4: Let g_1 be a single valued $I\mathcal{F}g$ and g_2 be a single valued $I\mathcal{F}g$, then $g_1 \times g_2$ be a single valued $I\mathcal{F}g$s.

Consider a $I\mathcal{F}g$ for g_1 and g_2 as in Figure 1, and Figure 2, the $I\mathcal{F}gs$ for cartesian product are in Figure 4,

$$
\begin{aligned}
Lz(g_1 \times g_2) = {} & (0.6)^2(1.9)^2(4.8) + (0.3)^2(0.8)^2(2.3) + (0.6)^2(1.8)^2(4.8) \\
& +(0.3)^2(0.9)^2(2.5) + (0.6)^2(2.4)^2(4.2) + (0.3)^2(1.1)^2(2.3)
\end{aligned}
$$

$$+(0.6)^2(2.3)^2(4.2)+(0.2)^2(0.9)^2(2.0)+(0.6)^2(2.3)^2(4.2)$$
$$+(0.2)^2(0.9)^2(2.0)+(0.6)^2(3.0)^2(3.6)$$
$$+(0.2)^2(1.1)^2(1.9),(0.6)^2(1.8)^2(4.2)+(0.4)^2(0.6)^2(3.8)+(0.6)^2(2.4)^2(4.2)$$
$$+(0.3)^2(0.9)^2(2.9)+(0.6)^2(2.4)^2(4.2)+(0.3)^2(0.9)^2(2.5)$$
$$+(0.6)^2(2.4)^2(4.3)+(0.1)^2(0.9)^2(1.9)+(0.6)^2(2.4)^2(4.2)$$
$$+(0.1)^2(0.8)^2(1.9)+(0.6)^2(2.4)^2(3.7)+(0.1)^2(0.9)^2(1.8)$$

Definition 3.5: Let g_1 be a single valued $I\mathcal{F}g$ and g_2 be a single valued $I\mathcal{F}g$, then $g_1 * g_2$ be a single valued $I\mathcal{F}g$s.

Let $g_1 * g_2$ be two single valued $I\mathcal{F}g$s, the strong product of $g_1 * g_2$ is a single valued $I\mathcal{F}g$.

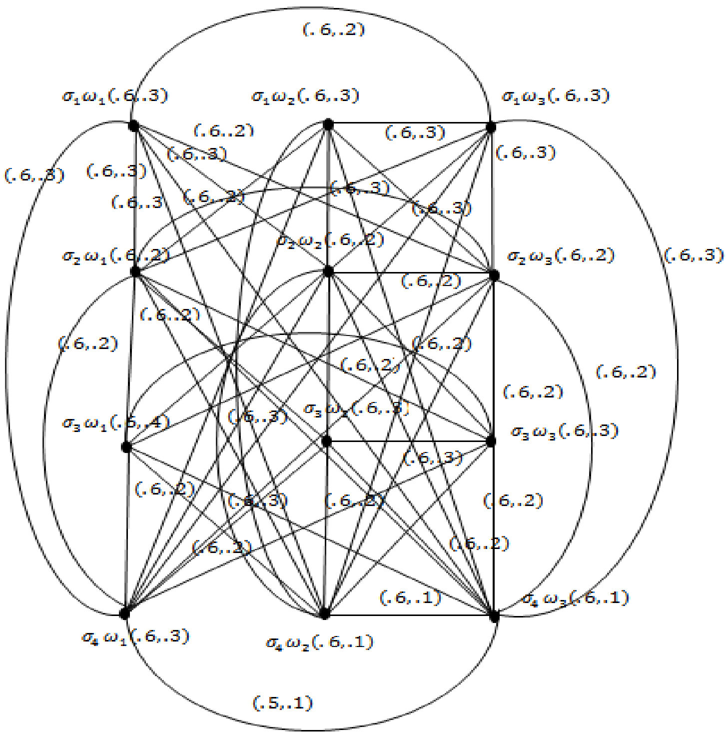

Figure 3
Lexicographic Product of $I\mathcal{F}g$ $\alpha \boxdot \beta$

$$Lz(g_1 \times g_2) = (42.42778, 39.475)$$

3. Structural representations

Numerical descriptors, which have shape and meaningful connectivity, are useful in analyzing and defining their properties. The connecting and non-connecting relations are included in the indices. For example, the intuitionistic fuzzy Hosoya index incorporates the uncertainty associated with bond strength or presence within molecular configurations. In a similar vein, the intuitionistic fuzzy clustering coefficient facilitates the analysis of local cohesion in networks that are based on imprecise interaction data, while the intuitionistic fuzzy possible relationships quantifies the significance of connectivity, the absence of interest, the relevance of connectivity, the drawbacks associated with non-neighborhood activities, and their intended applications, which are contingent upon specific criteria and requirements.

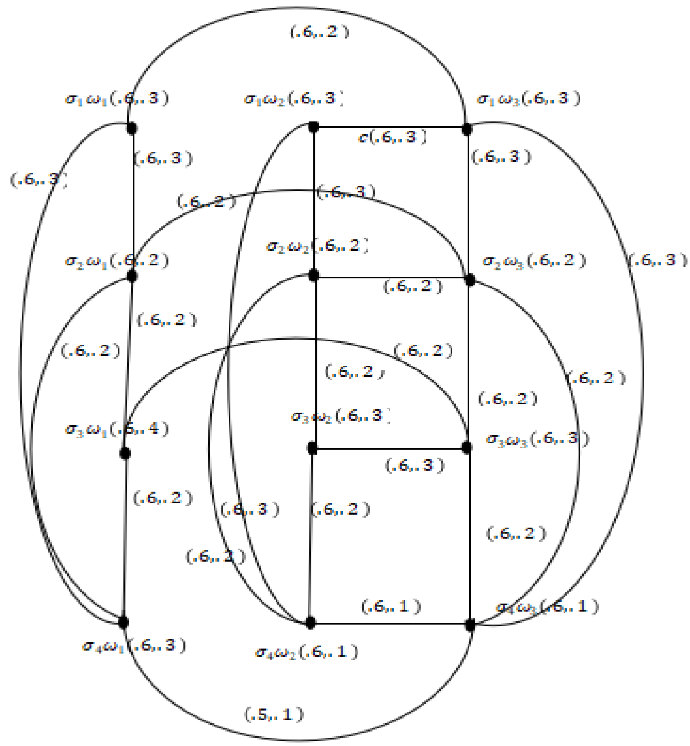

Figure 4
Cartesian Product of $I\mathcal{F}g$ $\alpha \times \beta$

4. Conclusion

The fuzzy graphs are associating both failure and winners for relations ships connected mathematically for each edge. It provides a good representation of uncertainty and hesitation that we typically see in real-world systems. In particular, $I\mathrm{F}\mathcal{g}s$s is useful for topological indices because they extend classical descriptors based on graphs to environments where the information is imprecise, vague, or incomplete. The topological indices for $I\mathrm{F}\mathcal{g}s$ include aspects that are structural, or uncertain - providing measures that are both more accurate and meaningful. This is especially relevant in Chemistry. $I\mathrm{F}\mathcal{g}s$ allow us to model molecular features that may not be definitively known regarding the strength of a bond or interaction of an atom. This is useful in drug design or molecular property predictions. In biology, $I\mathrm{F}\mathcal{g}s$ allow us to model biological networks, such as looking at protein-protein interactions when the existence of a connection or it's strength may be uncertain. The Social Networks allow us to model a range of trust or levels of influence between individuals, in a much clearer way than traditional methods which focus on the arithmetic connection and not the complexity of human relationships. For transportation can allow us to model systems for both reliability and uncertain travel times or connections. This is useful as it represents the world we are living in and can still allow us to optimize a route based on an ambiguous situation. In recent days the decision-making allows us to model uncertain or vague preferences of a multi-criteria decision analysis. This is especially beneficial when the stakeholders' preferences are not crisply defined.

References

[1] D. Vukičević, Q. Li, J. Sedlar, and T. Došlić, "Lanzhou index," *MATCH Communications in Mathematical and in Computer Chemistry*, vol. 80, no. 3, pp. 863–876 (2018).

[2] F. Falahati-Nezhad and M. Azari, "Some inequalities for Lanzhou index of graphs," *Combinatorics, Cryptography, Computer Science and Computing,* vol. 20, pp. 1–5 (2019).

[3] A. Nagoor Gani and S. Shajitha Begum, "Degree, Order and Size in Intuitionistic Fuzzy Graphs," *International Journal of Algorithms, Computing and Mathematics,* vol. 3, no. 3, pp. 11-16 (2010).

[4] A. Nagoor Gani and M. Basheer Ahamed, "Order and Size in Fuzzy Graphs," *Bulletin of Pure and Applied Science,* vol. 22E, no. 1, pp. 145-148 (2003).

[5] S. Yahya Mohamed and A. Mohamed Ali, "Complement of max product of intuitionistic fuzzy graphs," *Complex & Intelligent Systems,* vol. 7, pp. 2895-2905 (2021).

[6] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets Systems*, vol 20, no. 1, pp. 87-96 (1986).

[7] E. Szmidt E and J. Kacpzyk, "Distance between intuitionistic fuzzy sets," *Fuzzy Sets Systems*, vol. 114, no. 3, pp. 505-518 (2000).

[8] B. Okelo and A. Onyango, "Characterization of topological fuzzy sets in Hausdorff spaces," *Transactions on Fuzzy Sets and Systems*, vol. 1, no. 2, pp. 32–36 (2022), doi: 10.30495/TFSS.2022.1955757.1020.

[9] M. Akram and R. Akmal, "Operations on intuitionistic fuzzy graph structures," *Fuzzy Information and Engineering*, vol. 8, no. 4, pp. 389–410 (2016), doi: 10.1016/j.fiae.2017.01.001.

[10] S. Sahoo and M. Pal, "Different types of products on intuitionistic fuzzy graphs," *Pacific Science Review A: Natural Science and Engineering,* vol. 17, no. 3, pp. 87–96 (2015), doi: 10.1016/j.psra.2015.12.007.

[11] A. Rosenfeld, "Fuzzy graphs," *Fuzzy sets and their applications to cognitive and decision processes,* vol. 63, pp. 77-95 (1975).

[12] M. Binu, S. Mathew, and J. N. Mordeson, "Wiener index of a fuzzy graph and application to illegal immigration networks," *Fuzzy Sets and Systems*, vol. 384, pp. 132–147 (2020).

[13] J. Dinar, Z. Hussain, S. Zaman, and S. U. Rehman, "Wiener index for an intuitionistic fuzzy graph and its application in water pipeline network," *Ain Shams Engineering Journal,* vol. 14, no. 1, Art. no. 101826 (2023), doi: 10.1016/j.asej.2022.101826.

[14] M. Akram and R. Akmal, "Operations on intuitionistic fuzzy graph structures," *Fuzzy Information and Engineering,* vol. 8, no. 4, pp. 389–410 (Dec. 2016), doi: 10.1016/j.fiae.2017.01.001.

[15] M. Akram and B. Davvaz, "Strong intuitionistic fuzzy graphs," *Filomat*, vol. 26, no. 1, pp. 177–196 (2012), doi: 10.2298/FIL1201177A.

[16] M. Akram and W. A. Dudek, "Intuitionistic fuzzy hypergraphs with applications," *Information Sciences*, vol. 218, pp. 182–193 (Jan. 2013), doi: 10.1016/j.ins.2012.06.060.

[17] R. Parvathi and M. G. Karunambigai, "Intuitionistic fuzzy graphs," *Journal of Computational Intelligence: Theory and Applications*, vol. 1, no. 2, pp. 139–150 (2006).

[18] R. Parvathi, M. G. Karunambigai, and K.T. Atanassov, "Operations on Intuitionistic Fuzzy graphs, fuzzy systems," *IEEE International Conference,* pp. 1396-1401 (2009).

[19] S. Ghobadi, M. Ghods, and M. Ghorbaninejad, "Hyper and multiple f-index of nano star dendrimers," *Journal of Discrete Mathematical Sciences and Cryptography,* vol. 26, no. 6, pp. 1623–1638 (2023).

[20] A. Karimi, F. Movahedi and S. M. Taheri, "Computing new degree-based topological indices associated with the zero-divisor graphs of finite commutative rings," *Journal of Discrete Mathematical Sciences and Cryptography,* vol. 29, no. 1, pp. 1-13 (2026).

[21] T. Bharathi, S. Paulin, and S. Leo, "Cryptographic approach on edge regular power fuzzy graph," *Journal of Discrete Mathematical Sciences and Cryptography,* vol. 28, no. 3, pp. 945-958 (2025).

[22] H. J. Yousif, A. Ameer, and H. K. Sulaiman, "Cutoff fuzzy dominating set of strong fuzzy graph," *Journal of Discrete Mathematical Sciences and Cryptography,* vol. 28, no. 2, pp. 577–585 (2025).

[23] T. A. Al-Hawary, "Density results for perfectly regular and perfectly edge-regular fuzzy graphs," *Journal of Discrete Mathematical Sciences and Cryptography,* vol. 26, no. 2, pp. 521–528 (2022).

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