

Exploring fuzzy rhotrix basic concepts and operational framework

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Abstract

This research paper presents a comprehensive analysis of the fuzzy rhotrix, encompassing its definition, basic properties, proofs, and future development. The paper situates fuzzy rhotrix within the broader context of fuzzy mathematics, highlighting its role as an extension of traditional rhotrix theory that incorporates fuzzy logic to manage uncertainty and imprecision. The fundamental properties of fuzzy rhotrix are systematically analyzed, including closure, associativity, and identity elements, with rigorous proofs provided to substantiate these characteristics. In addition to theoretical insights, the paper explores practical applications of fuzzy rhotrix in areas such as decision-making, optimization, and data analysis. It identifies key challenges and opportunities for future research, including algorithmic advancements and potential interdisciplinary applications.

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1. Introduction

A Fuzzy Rhotrix is one in which the elements are real numbers in $[0, 1]$. In our study, we explored the properties of fuzzy rhotrices, notably odd-dimensional rhotrices for $n > 3$ and $n \in 2N + 1$. We also investigated the theorem of odd-dimensional fuzzy rhotrices and their basic features. Thomason [1] suggested Fuzzy Matrix Theory in 1977 as a subdivision of Fuzzy Set Theory developed by Zadeh [13] 12 years earlier. The idea for investigating

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fuzzy sets stemmed from the fact that certain items in the actual world cannot be precisely classified according to membership criteria. Zadeh [13] gives the example of the class of all real numbers larger than 1. Such a set can be classified as a fuzzy set since it is impossible to define it precisely in a classical sense.

2. Literature Review

The knowledge about the fuzzy matrix was originally developed by Thomason [1]. Priya et al. [2] explore the definition, operations, and applications emphasizing their role in modeling uncertainty. Ajibade [3] initially conceived of rhotrices as an extension of matrix operations and the transformation between a rhotrix and a matrix was discussed. Sani [4] gave another method for multiplying rhotrices of dimension 3 using rows and columns. Isere [5] develops the concepts of Codeterminant functions and the index of a rhotrix. Isere [6] work on the diagonal function of natural rhotrix introduces and examines this new concept within the framework of natural rhotrices. Gilchrist [7] gave a detailed explanation of the basic properties and ideas of fuzzy matrices. Sani [8] discussed the coupled matrix as a combination of two matrices. Kumar and Singh [9] studied the applications of rhotrix structure and provide an efficient representation for certain algebraic operations. Sharma [10] developed methods to apply rhotrix algebra in solving optimization problems. Gupta and Verma [11] studied how rhotrix structures can be applied in statistical modelling and data analysis. Ali [12] discussed the dynamical properties of rhotrix systems. Zadeh [13] introduced fuzzy set theory by extending classical set theory to model uncertainty using membership values between 0 and 1.

3. Fuzzy Rhotrix

Definition 3.1: A matrix whose elements are fuzzy numbers representing degrees of truth or membership in a certain set is a fuzzy matrix [2].

Example 3.1:

$$A = \begin{bmatrix} 0.1 & 0.5 & 0 \\ 0.4 & 0.2 & 0.9 \\ 0.3 & 0.6 & 1 \end{bmatrix}$$

Definition 3.2: The rhotrix R_n of dimension $n \in 2Z^+ + 1$ is defined as

$$R_n = \left\langle \begin{array}{cccccccc} & & & & a_{11} & & & \\ & & & & c_{11} & & a_{12} & \\ & & & & \dots & & \dots & \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{t1} & \dots & \dots & \dots & \dots & \dots & \dots & a_{1t} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ & & & & a_{t-1} & c_{t-1} & a_{t-1t} & \end{array} \right\rangle$$

$a_{mn}, c_{pq} \in \mathbb{R}$, $1 \leq m, n \leq t$; $1 \leq p, q \leq t-1$ and $t = \frac{n+1}{2}$ [4].

Definition 3.3: Fuzzy Rhotrix (FR) is a rhotrix whose elements are in the interval $[0, 1]$.

Definition 3.4: The number of elements in a FR's major vertical or horizontal axis is known as the dimension of the FR.

Example 3.2:

Consider FR, $P_{FR} = \begin{pmatrix} & & 0.3 \\ & 0.2 & 0 & 0.4 \\ 0.6 & 0.4 & 0.5 & 0.9 & 1 \\ & 0.5 & 0.4 & 0.3 \\ & & & & 0.1 \end{pmatrix}$ then $\dim(P_{FR}) = 5$.

Remark 3.1: For a n dimensional FR with $n \in 2z^+ + 1$ the cardinality is $t = \frac{n^2+1}{2}$.

Example 3.3: If $FR_3 = \begin{pmatrix} & & 1 \\ & 0.4 & 0.2 \\ 0 & & 0.5 \end{pmatrix}$ is of dimension 3, then the cardinality is $\frac{3^2+1}{2} = 5$.

Definition 3.5: Addition and Heart Based Multiplication of Rhotrix

If $P_{FR} = \begin{pmatrix} & r_1 & \\ r_2 & r_3 & r_4 \\ & r_5 & \end{pmatrix}$ and $Q_{FR} = \begin{pmatrix} & r_6 & \\ r_7 & r_8 & r_9 \\ & r_{10} & \end{pmatrix}$ then

$$P_{FR} + Q_{FR} = \begin{pmatrix} & r_1 + r_6 & \\ r_2 + r_7 & r_3 + r_8 & r_4 + r_9 \\ & r_5 + r_{10} & \end{pmatrix}$$

$$P_{FR} \circ Q_{FR} = \begin{pmatrix} & r_1 r_6 + r_6 r_3 & \\ r_2 r_8 + r_7 r_3 & r_3 r_8 & r_4 r_8 + r_9 r_3 \\ & r_5 r_8 + r_{10} r_3 & \end{pmatrix}$$

are the addition and multiplication of rhotrices [6].

Define fuzzy addition and fuzzy multiplication as $r + s = \text{mxm}(r, s)$ where mxm denotes the maximum and $r \cdot s = \text{mnm}(r, s)$ where mnm denotes the minimum.

Definition 3.6: Addition and Multiplication of two FR

$$\begin{aligned}
\text{Consider } P_{FR} + Q_{FR} &= \begin{pmatrix} p & & \\ q & t & w \\ & z & \end{pmatrix} + \begin{pmatrix} f & & \\ v & u & j \\ & k & \end{pmatrix} \\
&= \begin{pmatrix} \text{mxm}(p, f) & & \\ \text{mxm}(q, v) & \text{mxm}(t, u) & \text{mxm}(w, j) \\ & \text{mxm}(z, k) & \end{pmatrix} \\
P_{FR} \circ Q_{FR} &= \begin{pmatrix} p & & \\ q & t & w \\ & z & \end{pmatrix} \circ \begin{pmatrix} f & & \\ v & u & j \\ & k & \end{pmatrix} = \\
&\begin{pmatrix} \text{mxm}(\text{mnm}(p, u), \text{mnm}(f, t)) & & \\ \text{mxm}(\text{mnm}(q, u), \text{mnm}(v, t)) & \text{mnm}(t, u) & \text{mxm}(\text{mnm}(w, u), \text{mnm}(j, t)) \\ & \text{mxm}(\text{mnm}(z, u), \text{mnm}(t, k)) & \end{pmatrix}
\end{aligned}$$

Example 3.4: Find the Addition and Multiplication of the following FR

$$A = \begin{pmatrix} & 1 & \\ 0.5 & 1 & 0.2 \\ & 0.7 & \end{pmatrix} \& B = \begin{pmatrix} & 0.2 & \\ 0.4 & 1 & 0.4 \\ & 0.6 & \end{pmatrix}$$

Solution:

$$A + B = \begin{pmatrix} \text{mxm}(1, 0.2) & & \\ \text{mxm}(0.5, 0.4) & \text{mxm}(1, 1) & \text{mxm}(0.2, 0.4) \\ \text{mxm}(0.7, 0.6) & & \end{pmatrix} = \begin{pmatrix} & 1 & \\ 0.5 & 1 & 0.4 \\ & 0.7 & \end{pmatrix}$$

$A \circ B =$

$$\begin{pmatrix} \text{mxm}(\text{mnm}(1, 1), \text{mnm}(0.2, 1)) & & \\ \text{mxm}(\text{mnm}(0.5, 1), \text{mnm}(0.4, 1)) & \text{mnm}(1, 1) & \text{mxm}(\text{mnm}(0.2, 1), \text{mnm}(0.4, 1)) \\ \text{mxm}(\text{mnm}(0.7, 1), \text{mnm}(0.6, 1)) & & \end{pmatrix}$$

$$\begin{aligned}
A \circ B &= \begin{pmatrix} \text{mxm}(1, 0.2) & & \\ \text{mxm}(0.5, 0.4) & \text{mnm}(1, 1) & \text{mxm}(0.2, 0.4) \\ \text{mxm}(0.7, 0.6) & & \end{pmatrix} \\
&= \begin{pmatrix} & 1 & \\ 0.5 & 1 & 0.2 \\ & 0.7 & \end{pmatrix}
\end{aligned}$$

Definition 3.7: The Inverse of given $P_{FR} = \begin{pmatrix} & a & \\ b & h(P) & d \\ & e & \end{pmatrix}$ is defined by

$$P_{FR}^{-1} = \frac{-1}{(h(P))^2} \begin{pmatrix} & a & \\ b & -h(P) & d \\ & e & \end{pmatrix}.$$

Definition 3.8: The determinant of $P_{FR} = \begin{pmatrix} a & b & c & d \\ e \end{pmatrix}$ is defined by

$\det(P_{FR}) = \text{mnm}(c, (\text{mnm}(a, e) + \text{mnm}(b, d)))$. If the determinant is zero then it is singular; otherwise, non-singular.

Example 3.5: If $P_{FR} = \begin{pmatrix} 1 & 0.5 & 0.2 \\ 0.7 \end{pmatrix}$ then the determinant is given by

$$\det(P_{FR}) = \text{mnm}(1, (\text{mnm}(1, 0.7) + \text{mnm}(0.5, 0.2))) = \text{mnm}(1, 0.9) = 0.9.$$

Definition 3.9: The Transpose is obtained by interchanging the row and column of the given FR and is denoted by FR^T .

Example 3.6:

$$\text{If } P_{FR} = \begin{pmatrix} 0.3 & 0.2 & 0 & 0.4 \\ 0.6 & 0.4 & 0.5 & 0.9 & 1 \\ 0.5 & 0.4 & 0.3 \\ 0.2 \end{pmatrix} \text{ then the}$$

$$P_{FR}^T = \begin{pmatrix} 0.3 & 0.4 & 0 & 0.2 \\ 1 & 0.9 & 0.5 & 0.4 & 0.6 \\ 0.3 & 0.4 & 0.5 \\ 0.2 \end{pmatrix}$$

Properties 3.1: For fuzzy rhotrices of dimension n then we can verify the following results

- i. $\det(A_{FR} + B_{FR}) = \det A_{FR} + \det B_{FR}$
- ii. $\det(A_{FR} B_{FR}) = \det A_{FR} \det B_{FR}$
- iii. $\det(kA_{FR}) = k \det A_{FR}$ where $k \in [0, 1]$
- iv. $\det(A_{FR}) = \det(A_{FR}^T)$

Examples 3.7:

1. Let $A_{FR} = \begin{pmatrix} 1 & 0.5 & 0.2 \\ 0.4 & 0.7 \end{pmatrix}$ & $B_{FR} = \begin{pmatrix} 0.2 & 0.4 & 0.4 \\ 1 & 0.6 \end{pmatrix}$ are the two fuzzy

$$\text{rhotrices then } A_{FR} B_{FR} = A_{FR} \circ B_{FR} = \begin{pmatrix} 1 & 0.5 & 0.4 \\ 0.4 & 0.7 \end{pmatrix}.$$

$$\det(A_{FR} B_{FR}) = \text{mnm}(0.4, 1.1) = 0.4.$$

$$\det A_{FR} = \text{mnm}(0.4, 0.7 + 0.2) = 0.4 \text{ \& } \det B_{FR} = \text{mnm}(1, 0.2 + 0.4) = 0.6.$$

$$\det A_{FR} \det B_{FR} = \text{mnm}(0.4, 0.6) = 0.4$$

$$\text{Hence } \det(A_{FR} B_{FR}) = \det A_{FR} \det B_{FR}$$

$$2. \quad \text{Let } A_{FR} = \begin{pmatrix} 1 & & \\ 0.5 & 0.4 & 0.2 \\ & 0.7 & \end{pmatrix} \text{ \& } B_{FR} = \begin{pmatrix} & 0.2 & \\ 0.4 & 1 & 0.4 \\ & 0.6 & \end{pmatrix}$$

$$\text{then } A_{FR} + B_{FR} = \begin{pmatrix} & \text{mxm}(1,0.2) & \\ \text{mxm}(0.5,0.4) & \text{mxm}(0.4,1) & \text{mxm}(0.2,0.4) \\ & \text{mxm}(0.7,0.6) & \end{pmatrix}$$

$$= \begin{pmatrix} 1 & & \\ 0.5 & 1 & 0.4 \\ & 0.7 & \end{pmatrix}$$

$$\det(A_{FR} + B_{FR}) = \text{mnm}(1, 1.1) = 1$$

$$\det A_{FR} = \text{mnm}(0.4, 0.2 + 0.7) = 0.4 \text{ \& } \det B_{FR} = \text{mnm}(1, 0.6) = 0.6$$

$$\det A_{FR} + \det B_{FR} = 0.4 + 0.6 = 1$$

$$\text{Hence } \det(A_{FR} + B_{FR}) = \det A_{FR} + \det B_{FR}$$

$$3. \quad \text{Let } A_{FR} = \begin{pmatrix} 1 & & \\ 0.5 & 0.4 & 0.2 \\ & 0.7 & \end{pmatrix} \text{ and } k = 0.5 \text{ then } k A_{FR} \text{ is}$$

$$\begin{pmatrix} & \text{mnm}(0.5, 1) & \\ \text{mnm}(0.5, 0.5) & \text{mnm}(0.5, 0.4) & \text{mnm}(0.5, 0.2) \\ & \text{mnm}(0.5, 0.7) & \end{pmatrix} =$$

$$\begin{pmatrix} 0.5 & & \\ 0.5 & 0.4 & 0.2 \\ & 0.5 & \end{pmatrix}$$

$$\det(k A_{FR}) = \text{mnm}(0.4, 0.7) = 0.4 \text{ \& } k \det A_{FR} = \text{mnm}(0.5, 0.4) = 0.4$$

$$\text{Hence } \det(k A_{FR}) = k \det A_{FR}$$

$$4. \quad \text{Let } A_{FR} = \begin{pmatrix} 1 & & \\ 0.5 & 0.4 & 0.2 \\ & 0.7 & \end{pmatrix} \text{ then } A_{FR}^T = \begin{pmatrix} & 1 & \\ 0.2 & 0.4 & 0.5 \\ & 0.7 & \end{pmatrix}$$

$$\det A_{FR} = \text{mnm}(0.4, 0.7 + 0.2) = 0.4$$

$$\det A_{FR}^T = \text{mnm}(0.4, 0.7 + 0.2) = 0.4$$

$$\text{Hence } \det(A_{FR}) = \det(A_{FR}^T)$$

Definition 3.10: The Trace of fuzzy rhotrix $P_{FR} = \begin{pmatrix} & a & & \\ e & b & c & d \\ & f & g & h \\ & i & m & n \\ & o & & \end{pmatrix}$ is

$$\text{given by } \text{Tr}(P_{FR}) = \text{mxm}(a, c, g, m, o)$$

Definition 3.11: The Coupled Fuzzy matrix is obtained by rotating the given FR through 45° (half transpose $\frac{T}{2}$). For a 5-dimensional FR, we have

$$P_{STFR_5}^{\frac{T}{2}} = \left\langle \begin{array}{ccccc} & & t & & \\ & s & z & u & \\ r & y & o & m & v \\ & q & x & w & \\ & & p & & \end{array} \right\rangle^{\frac{T}{2}} = \begin{pmatrix} r & s & t \\ y & z & u \\ q & o & m \\ x & w & v \\ p & & \end{pmatrix} = [BC]_5$$

Coupled matrix is made of two square matrices $B = \begin{pmatrix} r & s & t \\ q & o & u \\ p & w & v \end{pmatrix}$ and $C = \begin{pmatrix} y & z \\ x & m \end{pmatrix}$ with orders of t and $t - 1$ where $t = \frac{n+1}{2}$. The filled coupled matrix is obtained by filling '0' in the missing places of $[BC]_5$ and is $\begin{pmatrix} r & 0 & s & 0 & t \\ 0 & y & 0 & z & 0 \\ q & 0 & 0 & 0 & u \\ 0 & x & 0 & m & 0 \\ p & 0 & w & 0 & v \end{pmatrix}$.

Definition 3.12: The Codeterminant function of the higher dimensional FR is calculated by reducing into FR of dimension 3 and the determinant of each of them is calculated and summing up and is denoted by Codet.

Example 3.8: Find the Codeterminant function of FR,

$$P_{FR} = \left\langle \begin{array}{cccc} & & 0.1 & \\ & 0.5 & 0.2 & 0.1 \\ 0.7 & 0.3 & 0.4 & 0.2 \\ & 1 & 0 & 0.5 \\ & & 0.8 & \end{array} \right\rangle$$

Solution: Along the major column

$$\begin{aligned} \text{Codet}(P_{FR}) &= \det \begin{pmatrix} & 0.1 & \\ 0.5 & 0.2 & 0.1 \\ & 0.4 & \end{pmatrix} + \det \begin{pmatrix} & 0.4 & \\ 1 & 0 & 0.5 \\ & 0.7 & \end{pmatrix} \\ &= \text{mnm}(0.2, (\text{mnm}(0.1, 0.4) + \text{mnm}(0.5, 0.1))) + \\ &\quad \text{mnm}(0, (\text{mnm}(0.4, 0.7) + \text{mnm}(1, 0.5))) \\ &= \text{mnm}(0.2, (0.1 + 0.1)) + \text{mnm}(0, (0.4 + 1)) = 0.2. \end{aligned}$$

Along the major row

$$\begin{aligned} \text{Codet}(P_{FR}) &= \det \begin{pmatrix} & 0.5 & \\ 0.7 & 0.3 & 0.4 \\ & 1 & \end{pmatrix} + \det \begin{pmatrix} & 0.1 & \\ 0.4 & 0.2 & 1 \\ & 0.5 & \end{pmatrix} \\ &= \text{mnm}(0.3, (\text{mnm}(0.5, 1) + \text{mnm}(0.7, 0.4))) + \\ &\quad \text{mnm}(0.2, (\text{mnm}(0.1, 0.5) + \text{mnm}(0.4, 1))) \\ &= \text{mnm}(0.3, (0.5 + 0.4)) + \text{mnm}(0.2, (0.1 + 0.4)) = 0.5. \end{aligned}$$

For a natural rhotrix the Codet of a rhotrix is same for expanding through the major column and major row but this not true in the FR.

Definition 3.13: The number of three-dimensional FR than can be drawn either along the major row or major column is said to be index of FR which denoted by ρ . We can find that

$$\rho(FR_3) = 0, \rho(FR_3) = 1, \rho(FR_5) = 2, \rho(FR_7) = 3, \rho(FR_3) = 4.$$

4. Operators on fuzzy rhotrices.

Let $x_{FR}, y_{FR} \in [0, 1]$ then we defined the following operators for FR.

1. $x_{FR} \vee y_{FR} = \max(x_{FR}, y_{FR})$
2. $x_{FR} \wedge y_{FR} = \min(x_{FR}, y_{FR})$
3. $x_{FR}^{(\alpha)}$ (Upper α - cut) = $\begin{cases} 1 & \text{if } x_{FR} > \alpha \\ 0 & \text{if } x_{FR} \leq \alpha \end{cases}$
4. $x_{FR}^{(\alpha)}$ (Lower α - cut) = $\begin{cases} x & \text{if } x_{FR} > \alpha \\ 0 & \text{if } x_{FR} \leq \alpha \end{cases}$
5. x_{FR}^c (complement) = $1 - x_{FR}$
6. $x_{FR} \oplus y_{FR} = \max(x_{FR}, y_{FR}) - \min(x_{FR}, y_{FR})$
7. $x_{FR} \odot y_{FR} = x_{FR} \cdot y_{FR}$
8. $x_{FR} \ominus y_{FR} = \begin{cases} x_{FR} & \text{if } x_{FR} > y_{FR} \\ 0 & \text{if } x_{FR} \leq y_{FR} \end{cases}$

Example 4.1: Find the values of (i) $P_{FR} \vee Q_{FR}$, (ii) $P_{FR} \wedge Q_{FR}$, (iii) $P_{FR}^{(\alpha)}$, $P_{FR}^{(\alpha)}$, $\alpha = 0.5$, (iv) P_{FR}^c , (v) $P_{FR} \oplus Q_{FR}$ where

$$P_{FR} = \begin{pmatrix} & & 0.1 & & \\ & 0.3 & 0.5 & 0.4 & \\ 0.2 & 0.5 & 0.3 & 0.7 & 0.4 \\ & 0.1 & 0.3 & 0.5 & \\ & & & & 0.8 \end{pmatrix} \& Q_{FR} = \begin{pmatrix} & & 0.8 & & \\ & 0.2 & 0.3 & 0.5 & \\ 0.3 & 0.4 & 0.2 & 0.5 & 0.4 \\ & 0.3 & 0.3 & 0.3 & \\ & & & & 0.7 \end{pmatrix}$$

Solution:

$$\begin{aligned} \text{(i) } P_{FR} \vee Q_{FR} &= \begin{pmatrix} & & 0.8 & & \\ & 0.3 & 0.5 & 0.5 & \\ 0.3 & 0.5 & 0.3 & 0.7 & 0.4 \\ & 0.3 & 0.3 & 0.5 & \\ & & & & 0.8 \end{pmatrix} \text{ by using } x_{FR} \vee y_{FR} = \\ &\max(x_{FR}, y_{FR}) \\ \text{(ii) } P_{FR} \wedge Q_{FR} &= \begin{pmatrix} & & 0.1 & & \\ & 0.2 & 0.3 & 0.4 & \\ 0.2 & 0.4 & 0.2 & 0.5 & 0.4 \\ & 0.1 & 0.3 & 0.3 & \\ & & & & 0.7 \end{pmatrix} \text{ by using } x_{FR} \wedge y_{FR} = \min(x_{FR}, \\ &y_{FR}) \end{aligned}$$

$$(iii) \quad P_{FR}^{(\alpha)} = \begin{pmatrix} & & 0 \\ & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ & 0 & 0 & 1 \\ & & & 1 \end{pmatrix} \& P_{FR(\alpha)} = \begin{pmatrix} & & 0 \\ & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0.7 & 0 \\ & 0 & 0 & 0.5 \\ & & & 0.8 \end{pmatrix} \text{ by}$$

using

$$x_{FR}^{(\alpha)} (\text{Upper } \alpha - \text{cut}) = \begin{cases} 1 & \text{if } x_{FR} > \alpha \\ 0 & \text{if } x_{FR} \leq \alpha \end{cases}$$

$$x_{FR(\alpha)} (\text{Lower } \alpha - \text{cut}) = \begin{cases} x & \text{if } x_{FR} > \alpha \\ 0 & \text{if } x_{FR} \leq \alpha \end{cases}$$

$$(iv) \quad P_{FR}^c = \begin{pmatrix} & & 1 \\ & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ & 1 & 1 & 1 \\ & & & 1 \end{pmatrix} - \begin{pmatrix} & & 0.1 \\ & 0.3 & 0.5 & 0.4 \\ 0.2 & 0.5 & 0.3 & 0.7 & 0.4 \\ & 0.1 & 0.3 & 0.5 \\ & & & 0.8 \end{pmatrix}$$

$$P_{FR}^c = \begin{pmatrix} & & 0.9 \\ & 0.7 & 0.5 & 0.6 \\ 0.8 & 0.5 & 0.7 & 0.3 & 0.6 \\ & 0.9 & 0.7 & 0.5 \\ & & & 0.2 \end{pmatrix} \text{ by using } x_{FR}^c (\text{complement}) = 1 - x_{FR}$$

$$(v) \quad P_{FR} \oplus Q_{FR} = \begin{pmatrix} & & 0.7 \\ & 0.1 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.2 & 0 \\ & 0.2 & 0 & 0.2 \\ & & & 0.1 \end{pmatrix}$$

by using $x_{FR} \oplus y_{FR} = \max(x_{FR}, y_{FR}) - \min(x_{FR}, y_{FR})$

Conclusion

This paper gives a brief idea about Fuzzy Rhotrices and the basic definitions like determinant, transpose, Invertible, Codeterminant function and Index operators of FR. As research in this area progresses, fuzzy rhotrices are increasingly applied in fields such as weather forecasting, decision-making, control systems, and artificial intelligence, providing valuable tools for managing uncertainty in complex mathematical modelling.

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