*Journal of Discrete Mathematical Sciences & Cryptography*
ISSN 0972-0529 (Print), ISSN 2169-0065 (Online)
Vol. 29 (2026), No. 6, pp. 2299–2310
DOI : 10.47974/JDMSC-2574

# Domination of double graph of some regular graphs

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**Abstract**

A graph in which every vertex has the same valency or degree is known as a regular graph. The double graph D(G) of a regular graph is constructed by duplicating each vertex and connecting the corresponding duplicates if the original vertices are adjacent. This method creates a motivating and complex graph structure, raising questions about the domination properties of D(G). In this paper we focus on the domination of double graph of some regular graphs which is constructed using an algorithm. The domination number $\gamma$ of connected 2-regular graph on n nodes were found. Further the domination number $\gamma$ of regular graphs on even and odd orders are determined.

***Subject Classification (2020):*** *05C69.*

***Keywords:*** *Domination, Regular graph, Double graph, Graphs, Domination number.*

## 1. Introduction

Domination in graphs is a branch in graph theory. It has several applications in physical science, medical science, and more. Haynes, Hedetniemi, and Slater [2] argued about the centrality and domination for facilities location and independence. Allan and Laskar [1] have debated about the domination and independent domination number on middle graphs and induced subgraphs. The domination became familiar in graph theory. In 2017, Chelvam and Sivagami [8] was discussed domination in Cayley graphs and given a review about the results on dominating sets in Cayley graphs. In 2019, Domination on Cayley digraph and its complement discussed and concluded the results by Rajeswari, Udhayashree, and Nirmala [7]. The concept of dominator

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coloring in prism graph and labelling of Cayley digraph and its line digraphs was investigated in 2015 [3], [4]. Major concept on dominator coloring number of some classes of graph was concluded in the year 2018 [6]. Nema and Alwan [9] introduced the co-intersection graph of subsemimodules of a semimodule. The study presented mathematical models designed to reduce noise during image acquisition and transmission in IoT environments, improving processing accuracy [10]. Mathematical models are widely used to understand the dynamics of infectious diseases and evaluate preventive strategies. In particular, Jaafar, Mohammed, and Hussein [11] proposed a mathematical model to analyse how vaccination rates influence the spread of respiratory viruses, highlighting the importance of vaccination in controlling outbreaks. Recent research has explored the relationship between topology and game theory. In particular, Sadek and Karim [12] analysed different types of games using semi-generalized regular spaces and discussed their mathematical properties.

A graph is said to be a regular graph if every vertex in a graph G has the same number of neighbours (degree). A graph in which each vertex is of degree k is called a k-regular graph or regular graph with degree k.

The double graph D(G) of a given graph G is constructed by making two copies of G and adding edges $u_1v_2$ & $v_1u_2$ for every edge $u_1u_2$ of G.

A set of Vertices D in a graph G dominates G, if every vertex in G is either in D or adjacent to a vertex in D is known as domination. A cardinality of any smallest dominating set is called domination number γ of G denoted by γ(G).

Here, we emphasis on the domination of double graph of some regular graphs which is constructed using an algorithm (Llama New, 2016) [5]. The domination number γ of connected 2 - regular graph on $n$ nodes are determined. Further, the domination of regular graphs on odd and even orders are determined.

## 2. Algorithm for construction of regular graph (Llama New, 2016) [5]

Let a graph $G$ has $n$ nodes and let the degree of each node of $G$ be $k$, where $0 < k < n$. The procedure to construct $k$-regular graph is explained below.

First all the nodes are placed in a circle.

If $n$ is odd:

For $i$ in range $(1, k)$:

If $i$ is odd:

Make edge for each node $x$ steps away $(i)$

If $n$ is even:

If $k$ is even:

Count Even numbers = $\frac{n}{2}$

For $i$ in range (Count Even numbers):

Make edge for each node $x$ steps away$(i + 1)$

Else if $k$ is odd:

Count odd numbers = $\frac{n-1}{2} + 1$

For $i$ in range (Count odd numbers):

Make edge for each node $x$ steps away $\left(\frac{n}{2}\right) - i$

That is, a regular graph is constructed as explained below.

**Case 1**: $n$ is odd and $k$ must be even.

All the nodes are placed in a circle. Then the nodes are joined $x$ positions away where $x$ is all odd numbers between 1 to $k$.

**Table 1**

**Procedure for construction of $k$ - regular graph of odd order where $k$ is even.**

| $n$ | $k$ | Procedure for construction of $k$ - regular graph |
|---|---|---|
| 11 | 2 | Produce edge for each node $x$ steps away 1 |
| 11 | 4 | Produce edge for each node $x$ steps away 1<br>Produce edge for each node $x$ steps away 3 |
| 11 | 6 | Produce edge for each node $x$ steps away 1<br>Produce edge for each node $x$ steps away 3<br>Produce edge for each node $x$ steps away 5 |
| 11 | 8 | Produce edge for each node $x$ steps away 1<br>Produce edge for each node $x$ steps away 3<br>Produce edge for each node $x$ steps away 5<br>Produce edge for each node $x$ steps away 7 |

**Case 2**: $n$ is even and $k$ is even

All the nodes are placed in a circle. Then the nodes are joined $x$ positions away where $x$ is a range of numbers from 1 to $\frac{n}{2}$ and the range of these numbers is limited by the number of even numbers from 1 to $k$ including $k$.

**Table 2**

**Procedure for construction of $k$ - regular graph of even order where $k$ is even.**

| $n$ | $k$ | Procedure for construction of $k$ - regular graph |
|---|---|---|
| 12 | 2 | Produce edge for each node $x$ steps away 1 |
| 12 | 4 | Produce edge for each node $x$ steps away 1<br>Produce edge for each node $x$ steps away 2 |
| 12 | 6 | Produce edge for each node $x$ steps away 1<br>Produce edge for each node $x$ steps away 2<br>Produce edge for each node $x$ steps away 3 |
| 12 | 8 | Produce edge for each node $x$ steps away 1<br>Produce edge for each node $x$ steps away 2<br>Produce edge for each node $x$ steps away 3<br>Produce edge for each node $x$ steps away 4 |

**Case 3**: $n$ is even and $k$ is odd

All the nodes are placed in a circle. Then the nodes are joined $x$ positions away where $x$ is a range of numbers from $\frac{n}{2}$ to 1 and the range of these numbers is limited by the number of odd numbers from 1 to $k$ including k.

**Table 3**

**Procedure for construction of $k$ - regular graph of even order where $k$ is odd.**

| $n$ | $k$ | Procedure for construction of $k$ - regular graph |
|---|---|---|
| 14 | 1 | Produce edge for each node $x$ steps away 7 |
| 14 | 3 | Produce edge for each node $x$ steps away 7<br>Produce edge for each node $x$ steps away 6 |
| 14 | 5 | Produce edge for each node $x$ steps away 7<br>Produce edge for each node $x$ steps away 6<br>Produce edge for each node $x$ steps away 5 |
| 14 | 7 | Produce edge for each node $x$ steps away 7<br>Produce edge for each node $x$ steps away 6<br>Produce edge for each node $x$ steps away 5<br>Produce edge for each node $x$ steps away 4 |

## 3. Domination number on regular graphs

3.1. Domination number of double graph of 2 – regular graph

A 2 - regular graph is a regular graph with all the degrees of a graph are 2. In this section, for a double graph of 2 - regular graph $G$ with $n$ nodes, the domination number is determined, when $G$ is connected.

**Proposition 3.1.1:** If $C_n$ is a cycle on $n$ nodes where $n \in Z_+$, $n \geq 3$ then its domination number γ of double graph is given by

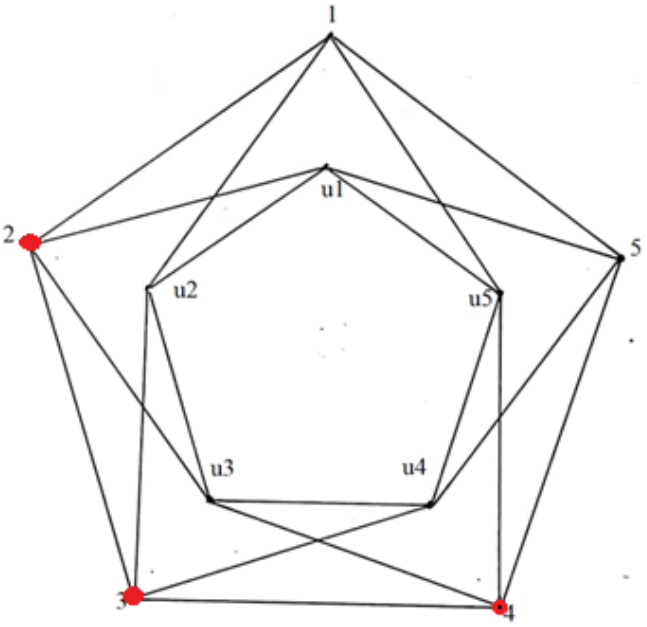
$$\gamma(D(C_n)) = \begin{cases} \frac{n}{2}, & n \equiv 0 \, mod \, 4 \\ \left\lfloor \frac{n}{2} \right\rfloor + 1, & n \not\equiv 0 mod \, 4 \end{cases}$$

**Theorem 3.1.2:** *If a double graph D ($G$) with n nodes ($n \geq 3$) is connected and 2 - regular graph then its domination number $\gamma$ is given by*

$$\gamma(D(G)) = \begin{cases} \frac{n}{2}, & n \equiv 0 \, mod \, 4 \\ \left\lfloor \frac{n}{2} \right\rfloor + 1, & n \not\equiv 0 mod \, 4 \end{cases}.$$

***Proof:*** Given $G$ is a connected 2 - regular graph on $n$ nodes. $u_1$, $u_2$, $u_3$, …. $u_n$ and $v_1$, $v_2$, $v_3$, …. $v_n$ be the nodes of the first and second copy of G respectively. Here double graph of G is constructed by connecting the nodes $u_1v_2$ & $v_1u_2$ for every edge $u_1u_2$ in G. Since $G$ is connected and each node has degree 2 exactly, there exists a Eulerian cycle Figure 1. Thus, all connected 2 - regular graphs with $n$ nodes are isomorphic to $n$ cycle. Hence by proposition 3.1.1 we have

$$\gamma(D(G)) = \begin{cases} \frac{n}{2}, & n \equiv 0 \, mod \, 4 \\ \left\lfloor \frac{n}{2} \right\rfloor + 1, & n \not\equiv 0 mod \, 4 \end{cases}.$$




**γ (D(G)) = 3**

**Figure 1**
**Domination number $\gamma$ (D(G)) of $n \not\equiv 0 mod\ 4$**

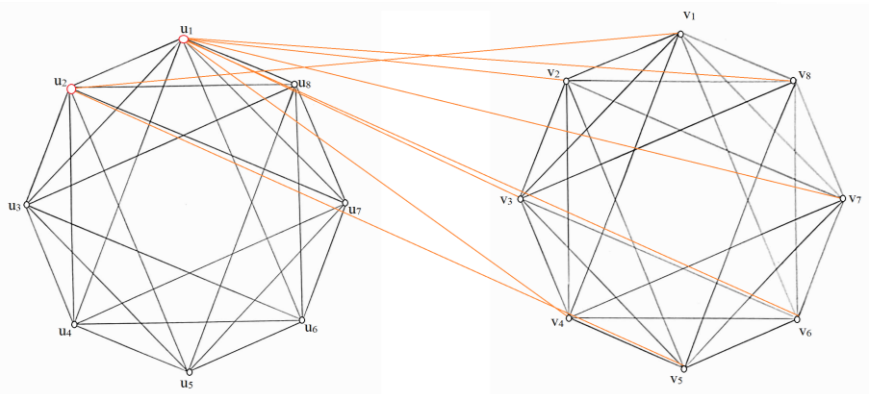
3.2 *Domination number of double graph of regular graph of even order*

A graph is said to be regular graph with even order, then all the nodes of the graph need to have the same degree and the degree needs to be even. This section determines γ(D) of some regular graph $G$ with even order.

**Theorem 3.2.1**: *If a graph $G$ with $n$ nodes where $n \geq 4$ and $n$ is even, is regular with degree $n-2$, then its domination number of double graph is $\gamma = 2$.*

**Proof:** Let $\{u_i / 1 \leq i \leq n\}$ be the collection of nodes of the regular graph G. The edges of the $n-2$ regular graph is constructed using algorithm (Table 2) for each copy. The double graph of G is created by taking another copy a regular graph whose nodes are represented as $\{v_i / 1 \leq i \leq n\}$ and connecting the nodes $u_1 v_2$ and $v_1 u_2$ for every edge $u_1 u_2$ of G.

As G is $n-2$ regular, the node $u_1$ dominates all the nodes of first and second copy except $\{u_{\frac{n}{2}+1}, v_{\frac{n}{2}+1}$, and $v_1\}$. To dominate those nodes, we choose $u_2$ which dominates all the above 3 undominated nodes. Therefore, all the nodes in the double graph are dominated by $u_1$ and $u_2$ Figure 2. Hence, domination number of double graph of $n-2$ regular graph on $n$ nodes, $n$ even and $n \geq 4$ is 2.




**γ (D(G)) = 2**

**Figure 2**

**Domination number $\gamma$ of double graph of regular graph of degree $n-2$**

**Theorem 3.2.2:** *If a graph G with n nodes where $n \geq 4$ and n is even, is regular with degree $\frac{n}{2}$ then its domination number of double graph is*

$$\gamma = \begin{cases} 2 & when\ n \equiv 2\ (mod\ 4)\ and\ n = 4 \\ 3 & when\ n \equiv 0\ (mod\ 4)\ and\ n \neq 4 \end{cases}$$

***Proof:*** Let $\{u_i / 1 \leq i \leq n\}$ be the collection of nodes of the regular graph. The edges of the $\frac{n}{2}$ regular graph is constructed using algorithm (Table 2) for even degree and algorithm (Table 3) for odd degree in each copy. The double graph of G is created by taking another copy a regular graph whose nodes are represented as $\{v_i / 1 \leq i \leq n\}$ and connecting the nodes $u_1v_2$ and $v_1u_2$ for every edge $u_1u_2$ of G.

**Case 1:** When $n \equiv 2\ (mod\ 4)$

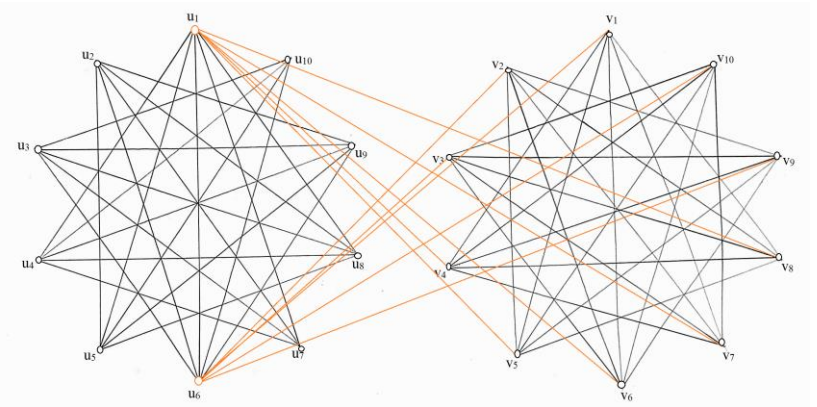
In this case, since it was a regular graph $u_1$ dominates all the adjacent nodes of the corresponding graph and its copy. In order to dominate the remaining nodes, we take $u_{\frac{n}{2}+1}$ which dominates all the nodes of the corresponding graph and its copy respectively Figure 3. Thus, the domination number γ of double graph of G with n$\equiv 2\ (mod\ 4)$ is always 2.

**Case 2:** When n = 4

In this case, $u_1$ dominates $u_2, u_4, v_2$ and $v_4$ but it will not dominate $u_3$, $v_1$ and $v_3$. So, we take $u_2$ as the dominating vertex to dominate the above 3 undominated nodes. Now all the nodes are dominated by the two vertices. Therefore, the domination number will be 2.

**Case 3:** *when* $n \equiv 0\ (mod\ 4)$ *and* $n \neq 4$

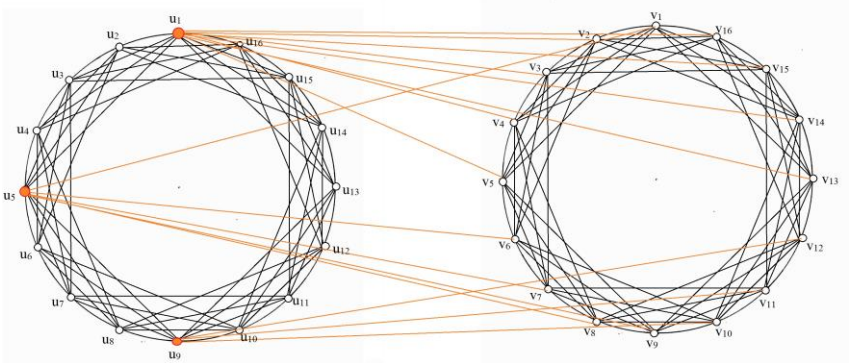
In this case, since it was a regular graph $u_1$ dominates all the adjacent nodes of the corresponding graph and its copy. In order to dominate the remaining nodes, we take, $u_{\frac{n+4}{4}}$ which dominates all the adjacent nodes of the corresponding graph and its copy respectively. Then for the remaining vertices we need to take $u_{\frac{n}{2}+1}$, which dominates all the remaining nodes of the corresponding graph and its copy Figure 4. Thus, the γ(D(G)) with n≡ $0\ (mod\ 4)$ is always 3.




**γ (D(G)) = 2**

**Figure 3**

**Domination number $\gamma$ of double graph of regular graph of degree $\frac{n}{2}$ and $n \equiv 2\ (mod\ 4)$**




**γ (D(G)) = 3**

**Figure 4**

**Domination number $\gamma$ of double graph of regular graph of degree $\frac{n}{2}$ and $n \equiv 0\ (mod\ 4)\ and\ n \neq 4$**

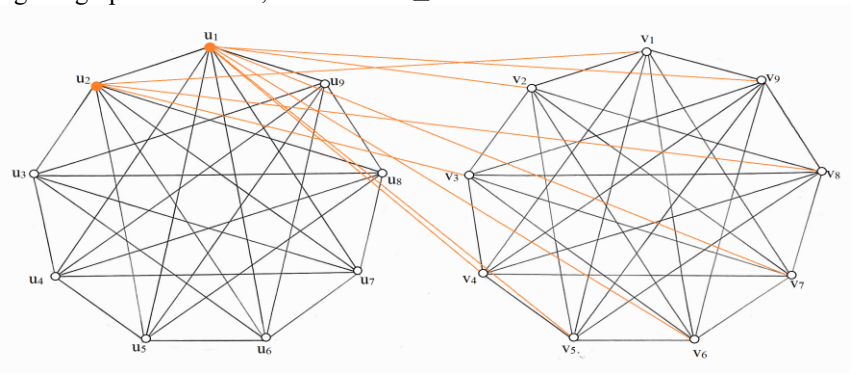
3.3 Domination number of regular graph of odd order

A graph is said to be regular graph with odd order, then all the nodes of the graph need to have the same degree and the degree needs to be odd. This section determines the domination number of double graph of some regular graph $G$ of odd order.

**Theorem 3.3.1:** *If a graph $G$ with $n$ nodes where $n \geq 7$ and $n$ is odd, is regular with degree $n-3$ then its domination number $\gamma$ of double graph is 2.*

***Proof:*** Let $\{u_i / 1 \leq i \leq n\}$ be the collection of nodes of the regular graph. The edges of the $n-3$ regular graph is constructed using algorithm (Table 1). The double graph of G is created by taking another copy a regular graph whose nodes are represented as $\{v_i / 1 \leq i \leq n\}$ and connecting the nodes $u_1 v_2$ and $v_1 u_2$ for every edge $u_1 u_2$ of G.

The node $u_1$ dominates all the nodes of the graph except $u_3$, $u_{n-1}$, $v_3$ and $v_{n-1}$. So, we take $u_4$ as the dominating vertex to dominate the above 4 undominated nodes. Therefore, all the nodes in the double graph are dominated by $u_1$ and $u_4$ Figure 5. Hence, domination number of double graph of $n-3$ regular graph on $n$ nodes, $n$ odd and $n \geq 7$ is 2.




**γ (D(G)) = 2**

**Figure 5**

**Domination number $\gamma$ of double graph of regular graph of degree $n-3$**

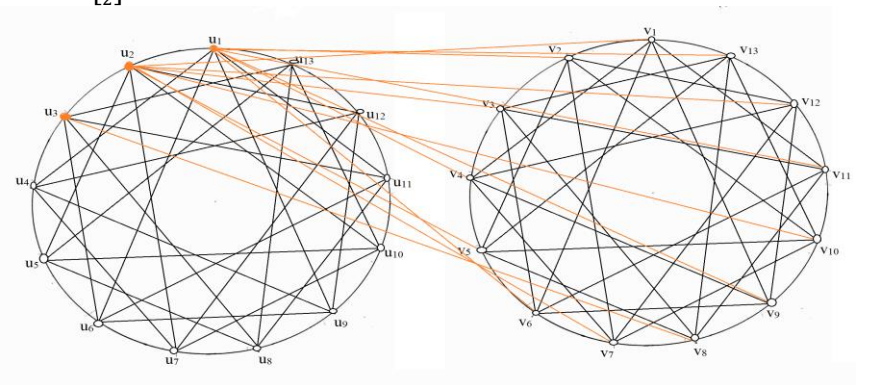
**Theorem 3.3.2:** *If a graph $G$ on $n$ nodes where $n > 5$, $n(mod\ 4) \equiv 1$ is regular with degree $\left\lfloor \frac{n}{2} \right\rfloor$, then its domination number is $\gamma = 3$.*

***Proof:*** Let $\{u_i / 1 \leq i \leq n\}$ be the collection of nodes of the regular graph G. The edges of the $\left\lfloor \frac{n}{2} \right\rfloor$ regular graph is constructed using algorithm (Table 1). The double graph of G is created by taking another copy a regular graph whose nodes are represented as $\{v_i / 1 \leq i \leq n\}$ and connecting the nodes $u_1 v_2$ and $v_1 u_2$ for every edge $u_1 u_2$ of G.

Let q be $\left\lceil \frac{n}{2} \right\rceil$. For every $1 \leq i \leq q$, the node $u_1$ dominates $u_i$ and $v_i$, whenever i(mod 2) ≡ 0 and for every q < i ≤ n, the node $u_1$ dominates $u_i$ and $v_i$, whenever i(mod 2) ≡ 1.

Then to dominate other undominated vertices $u_2$ was taken. For every $1 \leq i \leq q$, the node $u_2$ dominates $u_i$ and $v_i$, whenever i (mod 2) ≡ 1, and for every q+1 < i ≤ n, the node $u_2$ dominates $u_i$ and $v_i$, whenever i (mod 2) ≡ 0.

The only vertices which were not dominated are $u_{q+1}$ and $v_{q+1}$, i.e, $u_{\left\lceil \frac{n}{2} \right\rceil + 1}$ and $v_{\left\lceil \frac{n}{2} \right\rceil + 1}$. To dominate those vertices if we choose $u_3$, which dominates all the above 2 undominated nodes. Therefore, all the nodes in the double graph are dominated by $u_1$ , $u_2$ and $u_3$ Figure 6. Hence, domination number of double graph of $\left\lceil \frac{n}{2} \right\rceil$, regular graph on $n$ nodes, $n > 5$, $n(mod\ 4) \equiv 1$ is 3.




**γ (D(G)) = 3**

**Figure 6**

**Domination number $\gamma$ of double graph of regular graph of degree $\left\lceil \frac{n}{2} \right\rceil$**

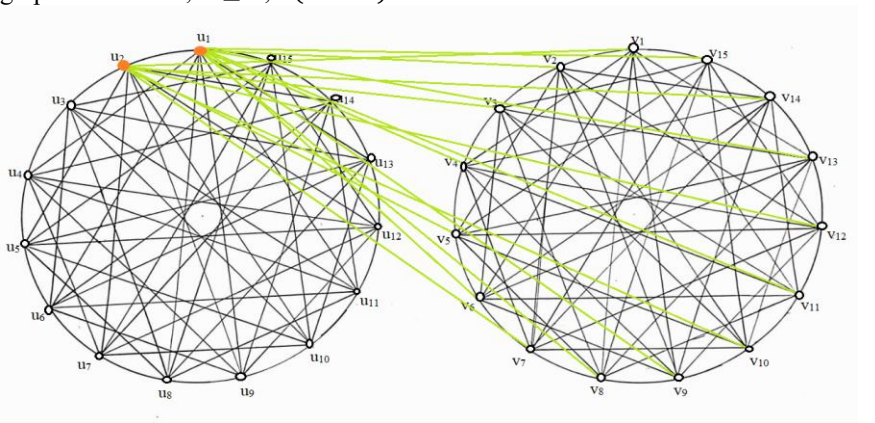
**Theorem 3.3.3:** *If a graph $G$ on $n$ nodes where $n \geq 5$, $n(mod\ 4) \equiv 3$ is regular with degree $\left\lceil \frac{n}{2} \right\rceil$, then its domination number $\gamma$ of a double graph is $\gamma = 2$.*

***Proof:*** Let $\{u_i / 1 \leq i \leq n\}$ be the collection of nodes of the regular graph G. The edges of the $\left\lceil \frac{n}{2} \right\rceil$ regular graph is constructed using algorithm (Table 1). The double graph of G is created by taking another copy a regular graph whose nodes are represented as $\{v_i / 1 \leq i \leq n\}$ and connecting the nodes $u_1 v_2$ and $v_1 u_2$ for every edge $u_1 u_2$ of G.

Let q be $\left\lceil \frac{n}{2} \right\rceil$ . For every $1 \leq i \leq q$, the node $u_1$ dominates $u_i$ and $v_i$, whenever i(mod 2) ≡ 0 and for every q < i ≤ n, the node $u_1$ dominates $u_i$ and $v_i$, whenever i(mod 2) ≡ 1

Then to dominate other undominated vertices $u_2$ was taken. For every $1 \leq i \leq q+1$, the node $u_2$ dominates $u_i$ and $v_i$, whenever i (mod 2) ≡ 1, and for every $q+1 < i \leq n$, the node $u_2$ dominates $u_i$ and $v_i$, whenever i (mod 2) ≡ 0.

Therefore, all the nodes in the double graph are dominated by $u_1$ and $u_2$ Figure 7. Hence, domination number of double graph of $\left\lceil \frac{n}{2} \right\rceil$ regular graph on $n$ nodes, $n \geq 5, n(mod\ 4) \equiv 3$ is 2.




**γ (D(G)) = 2**

**Figure 7**

**Domination number $\gamma$ of double graph of regular graph of degree $\left\lceil \frac{n}{2} \right\rceil$**

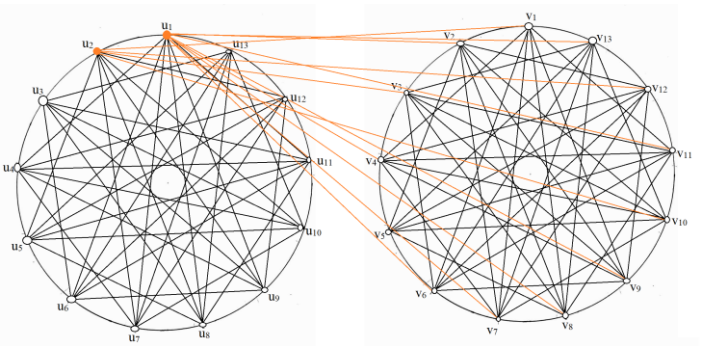
**Theorem 3.3.4:** *If a graph G on n nodes where* $n \geq 5$, $n(mod\ 4) \equiv 1$ *is regular with degree* $\left\lceil \frac{n}{2} \right\rceil + 1$, *then its domination number γ of a double graph is given by* $\gamma = 2$.

***Proof:*** Let $\{u_i / 1 \leq i \leq n\}$ be the collection of nodes of the regular graph G. The edges of the $\left\lceil \frac{n}{2} \right\rceil + 1$ regular graph is constructed using algorithm (Table 1). The double graph of G is created by taking another copy a regular graph whose nodes are represented as $\{v_i / 1 \leq i \leq n\}$ and connecting the nodes $u_1v_2$ and $v_1u_2$ for every edge $u_1u_2$ of G.

Let q be $\left\lceil \frac{n}{2} \right\rceil$ . For every $1 \leq i \leq q$, the node $u_1$ dominates $u_i$ and $v_i$, whenever i(mod 2) ≡ 0 and for every $q+1 < i \leq n$, the node $u_1$ dominates $u_i$ and $v_i$, whenever i(mod 2) ≡ 1.

Then to dominate other undominated vertices $u_2$ was taken. For every $1 \leq i \leq q$, the node $u_2$ dominates $u_i$ and $v_i$, whenever i (mod 2) ≡ 1, and for every $q+1 \leq i \leq n$, the node $u_2$ dominates $u_i$ and $v_i$, whenever i (mod 2) ≡ 0.

Therefore, all the nodes in the double graph are dominated by $u_1$ and $u_2$ Figure 8. Hence, domination number of double graph of $\left\lceil \frac{n}{2} \right\rceil + 1$ regular graph on $n$ nodes, $n \geq 5, n(mod\ 4) \equiv 3$ is 2.




**γ (D(G)) = 2**

**Figure 8**

**Domination number $\gamma$ of double graph of regular graph of degree $\left\lceil \frac{n}{2} \right\rceil + 1$**

## 4. Conclusion

The domination number γ of 2-regular graph on n nodes were found and the domination number γ of regular graphs on even order and odd order are determined. In Further research we have planned to study different type of dominations for the above graphs and relation between them.

## References

[1] R. B. Allan and R. Laskar, "On domination and independent domination numbers of a graph," *Discrete Mathematics*, vol. 23, no. 2, pp. 73–76 (1978), doi:10.1016/0012-365X(78)90105-X.

[2] T. W. Haynes, S. T. Hedetniemi, and P. J. Slater, *Fundamentals of Domination in Graphs*, 1st ed. Boca Raton, FL, USA: CRC Press, (1998), doi: 10.1201/9781482246582.

[3] R. Thamizharasi and R. Rajeswari, "Labelling of Cayley digraphs and its line digraphs," *International Journal of Pure and Applied Mathematics*, vol. 9, no. 19, pp. 947–954 (2015).

[4] T. Manjula and R. Rajeswari, "Dominator coloring of prism graph," *Journal of Applied Mathematical Sciences*, vol. 9, no. 38, pp. 1889–1894 (2015), doi:10.12988/AMS.2015.5176.

[5] Llama.new, "How to construct a k-regular graph?," *Mathematics Stack Exchange, answer to question*, (Sep. 2, 2016). [Online]. Available: https://math.stackexchange.com/a/1911863. [Accessed: Jun. 19, 2025].

[6] R. Rajeswari, T. Manjula, K. Deepika, and A. Dey, "Dominator coloring number of some class of graphs," *International Journal of Engineering and Advanced Technology,* vol. 8, no. 2S, pp. 269–274 (2018).

[7] R. Rajeswari, R. Udhayashree, and M. Nirmala, "Domination of Cayley digraph and its complement" *International Journal of Recent Technology and Engineering,* vol. 8, no. 2S11, pp. 4005–4008 (2019).

[8] T. Tamizh Chelvam and M. Sivagami, "Domination in Cayley graphs: A survey," *AKCE International Journal of Graphs and Combinatorics*, vol. 16, no. 1, pp. 27–40 (2019), doi:10.1016/j.akcej.2017.11.005.

[9] Z. A. Nema and A. H. Alwan, "Co-intersection graph of subsemimodules of a semimodule," *Journal of Interdisciplinary Mathematics,* vol. 28, no. 7, pp. 2377–2384 (2025), doi:10.47974/JIM-1500.

[10] V. Dankan Gowda, P. Gite, H. M. Nethravathi, P. Kumar Lakineni, K. D. V. Prasad, and S. Chitta, "Mathematical models for noise reduction in IoT image processing applications," *Journal of Interdisciplinary Mathematics,* vol. 28, no. 6, pp. 2289–2299 (2025), doi:10.47974/JIM-2371.

[11] M. N. Jaafar, A. Salah Mohammed, and R. Waleed Hussein, "Mathematical model studying the effect of vaccination rate on the spreading of respiratory system virus," *Journal of Interdisciplinary Mathematics,* vol. 26, no. 7, pp. 1563–1569 (2023), doi:10.47974/JIM-1581.

[12] A. R. Sadek and M. A. Abdel Karim, "Some games via semi-generalized regular spaces," *Journal of Interdisciplinary Mathematics*, vol. 26, no. 5, pp 863–872 (2023), doi:10.47974/JIM-1581.

*Received May, 2025*