Journal of Discrete Mathematical Sciences & Cryptography
ISSN 0972-0529 (Print), ISSN 2169-0065 (Online)
Vol. 29 (2026), No. 5, pp. 1977–1983
DOI : 10.47974/JDMSC-2378

Maximal and minimal prime ideals in a near ring

Muna Sh. G. Shnain *
Ameer A. J. Al-Swidi †
Department of Mathematics
College of Education for Pure Science
University of Babylon
Babylon
Iraq

Abstract

This work introduces the important result related to a concepts for maximal prime and minimal prime ideals in a non-commutative near ring. We determined the relations among these notions with other kinds for the ideals. Also, we proved several propositions concerning pseudo-weakly completely prime ideals and prime ideals. In addition, we studied their properties in a near ring.

Subject Classification: *93E10, 94A13.*

Keywords: *Maximal ideal, Maximal prime ideal, Minimal ideal, Minimal prime ideal, Ps.w.com.p. ideal, Ps.w.com.2-p. ideal.*

1. Introduction

Foundational concepts and relationships in near ring theory have been extensively developed [1]. The properties of maximal and minimal ideals have also been defined and studied in detail [1] and [2], in those paper, it is shown that the intersection and union of maximal ideals in a non-commutative near ring are not maximal ideals. Additionally, the concepts of maximal prime (max.p.) ideals and minimal prime (min.p.) ideals are introduced, and the relationships between max.p. ideals and other types of ideals in symmetric, non-weakly commutative near rings are investigated. It is further proven that the union of non-maximal ps.w.com.p. ideals in a near ring forms a ps.w.com.p. ideal, provided their intersection is non-empty. Moreover, the relationship between min.p. ideals and completely prime ideals in non-zero symmetric near rings is established.

* *E-mail:* **m1u9n7a5@gmail.com (Corresponding Author)**
† *E-mail:* **ameer.alswidi@yahoo.com**

2. Preliminary

In this section, many important definitions for our work are presented.

A near rings (right near - rings) and ideals were studied in [1]. Where, Ṇ named weakly commutative near ring if $\mathrm{xyz} = \mathrm{xzy}$, $\forall$ xyzϵṆ.

(Groenewald [3]) the ideals Ϝ for Ṇ named com. prime ideals when $\mathrm{u.v}\epsilon$Ϝ so uϵϜ or vϵϜ.

(Anderson and Smith [4]) the proper ideals Ϝ for the Ṇ be the weakly prime (w.p.) ideals where K, M are ideal for the near rings Ṇ implies that $\emptyset \neq \mathrm{KM} \subseteq$ Ϝ then K $\subseteq$ Ϝ or M $\subseteq$ Ϝ.

The authors in [5], the ideal Ϝ for Ṇ named prime ideal (p.ideal) when α, β be ideal in any near - rings Ṇ with $\alpha.\beta \subseteq$ Ϝ , either $\alpha \subseteq$ Ϝ or $\beta \subseteq$ Ϝ.

The authors in [6], the non - trivials near - rings named symmetric where $\mathrm{x}.0 = 0$, for any x in Ṇ.

In [1] An element γ is called idempotent of Ṇ if $\gamma^2 = \gamma$, $for\ h\epsilon$of Ṇ, and the ideals Ϝ in Ṇ named radical when

$\sqrt{\text{Ϝ}} = \{\tau\epsilon$Ṇ: $\tau^{\mathrm{n}}\epsilon$Ϝ, $for\ some\ n\epsilon Z^+\}$, the authors in [2], the ideals Ϝ for the Ṇ named maximal (max.ideal) where Ϝ $\nsubseteq$ F $\subseteq$ Ṇ implies F = Ṇ.

A minimal ideal (min. ideal) is a non-zero ideals that minimal in sets for each non-zero ideals of Ṇ, that's: let Ϝ $\neq \{0\}$ be the ideals for the Ṇ if $\mathcal{H} \subseteq$ Ṇ be the ideals s.t $\{0\} \subseteq \mathcal{H} \subseteq$ Ϝ then either $\{0\} = \mathcal{H}$ or $\mathcal{H} =$ Ϝ.

And we can generalize it with other ideals [7-9] and [10-13].

3. Main Result

Proposition 1: The zero ideal Ϝ $= \{0\}$ of a non-symmetric integral domain Ṇ is a com.p. ideal.

Proof: Let $\hbar, \jmath\epsilon$Ṇ $\ni$ h.$\jmath\epsilon$Ϝ, then 0= $\hbar.\jmath$ (as Ϝ $= \{0\}$) , since Ϝ no zero divisor so $\hbar$=0 or $\jmath$=0, therefore $\hbar\epsilon$Ϝ or $\jmath\epsilon$Ϝ.

Hence Ϝ is com.p. ideal.

Proposition 2: If Ϝ $\neq \{0\}$ is p. ideals for non-symmetric Ṇ so is com.p ideal.

Proof: Let $\varepsilon, \alpha\epsilon$Ṇ $\ni \varepsilon.\alpha\epsilon$Ϝ, and let $\mathrm{T}, \mathrm{H} \subseteq$ Ṇ $\ni$ T.H subset of Ϝ and $\varepsilon \in \mathrm{T}$ and $\alpha \in \mathrm{H}$.

So, $\varepsilon.\alpha\epsilon\mathrm{T.H} \subseteq$ Ϝ ,T $\subseteq$ Ϝ or H $\subseteq$ Ϝ (as Ϝ is p. ideal), then $\varepsilon\epsilon$Ϝ or $\alpha\epsilon$Ϝ, hence Ϝ com. p. ideal.

Remark 3: An opposite for proposition (2) be true if Ṇ be the non-symmetric.

Proposition 4: Let $\text{Ϝ}_1 \neq \{0\}$ and $\text{Ϝ}_2 \neq \{0\}$ are w. p. ideals of a non-symmetric integral domain Ṇ then $(\text{Ϝ}_1 \cap \text{Ϝ}_2)$ is com.p. ideal.

Proof: If $(\text{Ϝ}_1 \cap \text{Ϝ}_2) = \{0\}$ then proof is complete (Proposition 1.).

If $(\mathcal{F}_1 \cap \mathcal{F}_2) \neq \{0\}$, Let $\beta, \rho \epsilon \mathrm{N} \ni \beta.\rho \,\epsilon \mathcal{F}_1 \cap \mathcal{F}_2$, then $\beta.\rho \epsilon \mathcal{F}_1$ & $\beta.\rho \epsilon \mathcal{F}_2$, since $\mathcal{F}_1$ is w.p. ideal, hence $\forall\, \mathcal{K}, \mathcal{H} \subseteq \mathrm{N}$ & $\beta \epsilon \mathcal{K}, \rho \epsilon \mathcal{H} \ni \emptyset \neq \mathcal{K}.\mathcal{H} \subseteq \mathcal{F}_1$, then $\mathcal{K} \subseteq \mathcal{F}_1$ or $\mathcal{H} \subseteq \mathcal{F}_1$ and then $\beta \epsilon \mathcal{F}_1$ or $\rho \epsilon \mathcal{F}_1$.

In the same manner, we prove $\beta \epsilon \mathcal{F}_2$ or $\rho \epsilon \mathcal{F}_2$, so $\beta \epsilon (\mathcal{F}_1 \cap \mathcal{F}_2)$ or $\rho \,\epsilon (\mathcal{F}_1 \cap \mathcal{F}_2)$, hence $(\mathcal{F}_1 \cap \mathcal{F}_2)$ is com.p. ideal.

Example 5: Let N={0, s, t, k} is near rings with operations (+) & (.) define in Table 1. If $\mathcal{F}_1 = \{0, s\}$ and $\mathcal{F}_2 = \{0, t\}$ are ideals of N.

Table 1

Multiplications and additions table for N

+	0	s	t	k		.	0	S	t	k
0	0	s	t	k		0	0	0	0	0
s	s	0	k	t		s	s	S	s	s
t	t	k	0	s		t	t	T	t	t
k	k	t	s	0		k	k	K	k	k

We can see the w.p. ideals $\mathcal{F}_1 = \{0, s\}$ and $\mathcal{F}_2 = \{0, t\}$ such that $\mathcal{F}_1 \cap \mathcal{F}_2 = \{0\}$ is com.p. ideal.

Proposition 6: Let $\mathcal{F}_1 \& \mathcal{F}_2 \neq \{0\}$ two max. ideals of a non-commutative N where $\mathcal{F}_1 \nsubseteq \mathcal{F}_2$ and $\mathcal{F}_2 \nsubseteq \mathcal{F}_1$ so $(\mathcal{F}_1 \cap \mathcal{F}_2)$ is non-max. ideal.

Proof: If $(\mathcal{F}_1 \cap \mathcal{F}_2) = \{0\}$ the proof is complete.

If $(\mathcal{F}_1 \cap \mathcal{F}_2) \neq \{0\}$, so $(\mathcal{F}_1 \cap \mathcal{F}_2)$ the ideals.

since $\mathcal{F}_1$ is max. ideal, so there is a sunset J_1 of the N an ideal $\ni \mathcal{F}_1$ proper subset of J_1 then $\mathcal{F}_1 = J_1$ or $J_1 = \mathrm{N}$, since $\mathcal{F}_2$ is max. ideal, then $\exists\, J_2 \subseteq \mathrm{N}$ an ideal $\ni \mathcal{F}_2 \subset J_2$ then $\mathcal{F}_2 = J_2$ or $J_2 = \mathrm{N}$,

$$(\mathcal{F}_1 \cap \mathcal{F}_2) \subset \mathcal{F}_1 \subset J_1 \text{ and } (\mathcal{F}_1 \cap \mathcal{F}_2) \subset \mathcal{F}_2 \subset J_2,$$
$$\text{if } \mathcal{F}_1 = J_1 \;\&\; \mathcal{F}_2 = J_2, \text{ so } (\mathcal{F}_1 \cap \mathcal{F}_2) \subset J_1 \;\&\; (\mathcal{F}_1 \cap \mathcal{F}_2) \subset J_2\,,$$
$$\text{so } (\mathcal{F}_1 \cap \mathcal{F}_2) \subset J_1 \cap J_2 \subseteq \mathrm{N} \;\&\; (\mathcal{F}_1 \cap \mathcal{F}_2) \neq J_1 \cap J_2,$$

if $J_1 = \mathrm{N}$ and $J_2 = \mathrm{N}$, then $(\mathcal{F}_1 \cap \mathcal{F}_2) \subset \mathcal{F}_1 \subset J_1 = \mathrm{N}$ and $(\mathcal{F}_1 \cap \mathcal{F}_2) \subset \mathcal{F}_2 \subset J_2 = \mathrm{N}$, so $(\mathcal{F}_1 \cap \mathcal{F}_2)$ is non-max. ideal.

Proposition 7: Let $\mathcal{F}_1 \neq \{0\}$ and $\mathcal{F}_2 \neq \{0\}$ are two max. ideals of a non-commutative N where $\mathcal{F}_1 \nsubseteq \mathcal{F}_2$ and $\mathcal{F}_2 \nsubseteq \mathcal{F}_1$ so $(\mathcal{F}_1 \cup \mathcal{F}_2)$ is non-max. ideal.

Proof: If $(\mathcal{F}_1 \cup \mathcal{F}_2) = \mathrm{N}$ the proof is complete.

If $(\mathcal{F}_1 \cup \mathcal{F}_2) \neq \mathrm{N}$, since the union for the ideals is not ideals, then $(\mathcal{F}_1 \cup \mathcal{F}_2)$ non-max. ideals.

$(\mathcal{F}_1 \cup \mathcal{F}_2) = \{0,1,2,4,5,6\}$ are non-max. ideals.

Definition 9: An ideal $\mathcal{F}$ of N is called ps.w-com.p. ideal if $\forall\, \gamma, \delta \in \mathrm{N}$, $0 = \gamma.\delta \epsilon \mathcal{F}$ then either $\gamma \in \mathcal{F}$ or $\delta \in \mathcal{F}$.

Proposition 10: Let $\digamma \neq \{0\}$ be an ideal of a non-symmetric $\dot{N}$ if $\digamma$ is p. ideal then is ps.w- com. p. ideal.

Proof: Let $\tau, \gamma\epsilon\dot{N} \ni 0 = \tau.\gamma\epsilon\digamma$, and let $\mathcal{R}, \mathcal{M} \subseteq \dot{N} \ni \mathcal{R}.\mathcal{M} \subseteq \digamma$ and $\tau\epsilon\mathcal{R}$, $\gamma\epsilon\mathcal{M}$, so $0 = \tau.\gamma\epsilon\mathcal{R}.\mathcal{M} \subseteq \digamma$ or $\mathcal{R} \subseteq \digamma$ (since $\digamma$ is p. ideal), then $\tau\epsilon\digamma$ or $\gamma\epsilon\digamma$, hence $\digamma$ ps.w- com. p. ideal.

Example 8: In example (5) the ideals $\digamma_1 = \{0, s\}$ and $\digamma_2 = \{0, t\}$ are p. ideals then they are ps.w.com.p. ideals of a non- symmetric of $\dot{N}$.

Proposition 12: If $\digamma$ is the max. ideals for the $\dot{N}$ then is ps.w.com.p. ideal.

Proof: Since every max. ideal is p. ideal, and from proposition (10), then $\digamma$ ideal is ps.w.com.p. ideal.

Remark 13: A reverse for the Proposition (12) not true.

Example 9: In the Example (8) as $\digamma_3 = \{0,2,4,6\}$ is ps.w.com.p. ideal but non-max. ideal.

Proposition 15: Let $\digamma \neq \{0\}$ be an ideals in non - zero symmetric for the near rings $\dot{N}$, when $0 \neq \delta\epsilon\digamma$ important element satisfying $\delta.\alpha = \delta$, $\forall$ $\alpha\epsilon\dot{N}$ then $\digamma$ be the weakly prime ideal.

Proof: Assume $\digamma$ not weakly prime ideals & K, B are ideals for the near rings $\dot{N}$ s.t $\emptyset \neq \mathrm{K}.\mathrm{B} \subseteq \digamma$ then $\mathrm{K} \nsubseteq \digamma$ and $\mathcal{B} \nsubseteq \digamma$,

Let $\delta\epsilon\mathrm{K}$ and $\alpha\epsilon\mathcal{B}$ s.t $\delta.\alpha\epsilon\mathrm{K}.\mathcal{B}$ so we get $0 \neq \delta.\alpha\epsilon\digamma$, so $\delta = \delta.\alpha\epsilon\digamma$ as $\delta.\alpha = \delta$, $\forall$ $\alpha\epsilon$ $\dot{N}$ then $\delta\epsilon\digamma$ but $\delta\epsilon\mathrm{K} \nsubseteq \digamma$,so $\delta \notin \digamma$, contradictions.

So $\digamma$ be the weakly prime ideals.

Example 10: In example (5) as $\digamma_1 = \{0, s\}$ be an ideal containing an idempotent element satisfying $s.x = s$, $\forall$ $x\epsilon\dot{N}$ so $\digamma_1$ be the weakly prime ideal

Proposition 17: If $\digamma_1, \digamma_2 \neq \{0\}$ are max. ideals of a symmetric weakly commutative $\dot{N}$ containing nilpotent elements such that $\emptyset \neq (\digamma_1 \cap \digamma_2)$ is ps.w.com.p. ideal, if

i) $(\digamma_1.\digamma_2) = (\digamma_1 \cap \digamma_2)$ then $\digamma_1.\digamma_2$ is ps.w.com.p. ideal.
ii) $(\digamma_1.\digamma_2) \subset (\digamma_1 \cap \digamma_2)$ then $\digamma_1.\digamma_2$ is non-ps.w.com.p. ideal.

Proof:

i) Directly as $(\digamma_1 \cap \digamma_2)$ is ps.w.com.p. ideal then $\digamma_1.\digamma_2$ is ps.w.com.p. ideal.
ii) Let $\rho, \mu\epsilon\dot{N} \ni 0 = \rho.\mu\epsilon(\digamma_1.\digamma_2)$ so $0 = \rho.\mu\epsilon(\digamma_1 \cap \digamma_2)$, and $\rho\epsilon(\digamma_1 \cap \digamma_2)$ or $\mu\epsilon(\digamma_1 \cap \digamma_2)$ $\forall$ $\rho, \mu\epsilon\dot{N}$, then $(\digamma_1 \cap \digamma_2) = ((\digamma_1.\digamma_2) \cup (\digamma_1 \cap \digamma_2))$, since $\digamma_1$ and $\digamma_2$ are max. ideals containing nilpotent elements, then $\exists$ $\rho\epsilon(\digamma_1 \cap \digamma_2)$ $\ni$ $\rho \notin (\digamma_1.\digamma_2)$ and $\rho^2 = 0\epsilon(\digamma_1.\digamma_2)$, and similarly for $\mu \notin (\digamma_1.\digamma_2)$.

Hence $(\mathrm{F}_1.\mathrm{F}_2)$ is non ps.w.com.p. ideal.

Example 11: In example (8) as $\mathrm{F}_1 = \{0,1,2,4,6\}, \mathrm{F}_2 = \{0,2,4,5,6\}$ and $\mathrm{F}_3 = \{0,2,4,6,7\}$ are max. ideals of N s.t $(\mathrm{F}_1 \cap \mathrm{F}_2) = \{0,2,4,6\}$ & $(\mathrm{F}_3 \cap \mathrm{F}_1) = \{0,2,4,6\}$ are ps.w.com.p.ideals. And $(\mathrm{F}_1.\mathrm{F}_2) = \{0,2,4\}$ is a non-ps.w.com.p. ideal as $(\mathrm{F}_1.\mathrm{F}_2) \subset (\mathrm{F}_1 \cap \mathrm{F}_2)$, also $(\mathrm{F}_3.\mathrm{F}_1) = \{0,2,4,6\}$ is ps.w.com.p. ideal as $(\mathrm{F}_3.\mathrm{F}_1) = (\mathrm{F}_3 \cap \mathrm{F}_1)$ is ps.w.com.p. ideal.

Definition 19: A prime ideal F is a max.p. ideals for the near rings N if there is prime ideals J s.t $\mathrm{F} \subseteq \mathrm{J}$ then $\mathrm{F} = \mathrm{J}$, so that: Ideal → prime → maximal.

Proposition 20: Let $\mathrm{F} \neq \{0\}$ a max.p. ideal of a symmetric non-weakly commutative N then $\mathrm{F} = \mathrm{rad}(\mathrm{F})$.

Proof: Let $a \epsilon \mathrm{rad}(\mathrm{F})$, then $a^n \epsilon \mathrm{F}$, since F is max.p. ideal then is prime, therefore $a \epsilon \mathrm{F}$ and then $\mathrm{rad}(\mathrm{F}) \subseteq \mathrm{F}$. Now, let $a \epsilon \mathrm{F}$, so $a^n \epsilon \mathrm{F}$ for some $n \epsilon Z^+$, then $a \epsilon \mathrm{rad}(\mathrm{F})$, hence $\mathrm{F} \subseteq \mathrm{rad}(\mathrm{F})$, and then $\mathrm{rad}(\mathrm{F}) = \mathrm{F}$.

Remark 22: The intersection (the union) of min. ideals for near rings not min. ideal.

Example 23: In example (21) as $\mathrm{F}_1=\{0,\alpha\}$ and $\mathrm{F}_2=\{0,\gamma\}$ are min. ideals then $(\mathrm{F}_1 \cap \mathrm{F}_2) = \{0\}$ is not min. ideal , also $(\mathrm{F}_1 \cup \mathrm{F}_2) = \{0, \alpha, \gamma\}$ is not min. ideal.

Proposition 24: Suppose $\mathrm{F} \neq \{0\}$ is ideals for non-symmetric N if F is min. ideal such that $\mathrm{F} = \mathrm{F}^2$ then is p. ideal.

Proof: Let $\mathrm{T}, \mathrm{S} \subseteq \mathrm{N} \ni \mathrm{T}.\mathrm{S} \subseteq \mathrm{F}$, since F is min. ideal then either $\{0\} = \mathrm{T}.\mathrm{S} \subseteq \mathrm{F}$ or $\{0\} \subseteq \mathrm{T}.\mathrm{S} = \mathrm{F}$,

Case (1): If $\mathrm{T} = \mathrm{S} = \{0\}$ then $\mathrm{T} \subseteq \mathrm{F}$ or $\mathrm{S} \subseteq \mathrm{F}$,

Case (2): If $\mathrm{T} = \{0\}, \mathrm{S} \neq \{0\}$ then $\mathrm{T} \subseteq \mathrm{F}$,

Case (3): If $\mathrm{T} \neq \{0\}, \mathrm{S} \neq \{0\} \ni \{0\} = \mathrm{T}.\mathrm{S} \subseteq \mathrm{F} = \mathrm{F}^2$ then $\mathrm{T} \subseteq \mathrm{F}$ and $\mathrm{S} \subseteq \mathrm{F}$, as $\{0\} \subseteq \mathrm{T}.\mathrm{S} = \mathrm{F}$, so $\mathrm{T}.\mathrm{S} = \mathrm{F}.\mathrm{F}$ (as $\mathrm{F} = \mathrm{F}^2$), so $\mathrm{T} = \mathrm{S} = \mathrm{F}$, then $\mathrm{T} \subseteq \mathrm{F}$ or $\mathrm{S} \subseteq \mathrm{F}$, hence F is p. ideal.

Definition 25: A prime ideal $\mathrm{F} \neq \{0\}$ be the min. p. ideals for near rings N if there exist prime ideals $\mathcal{M}$ s.t $\{0\} \subseteq \mathcal{M} \subseteq \mathrm{F}$ then $\mathcal{M} = \mathrm{F}$, so that: Ideal → prime → minimal.

Proposition 26: The min. p. ideal $\mathrm{F} \neq \{0\}$ of a non-symmetric N is com.p. ideal.

Proof: Since F is min. p. ideal, so F is p. ideal (according to proposition (2)), then F is com.p. ideal.

Proposition 27: The intersection of min. p. ideals in N is not ps.w.com.p. ideal.

Proof: Let $K = \{\bigcap \digamma | \digamma \text{ min. p. ideal}\}$, suppose that K is ps.w.com.p. ideal, $\forall$ $\tau, \mu \epsilon Ṇ, 0 = \tau, \mu \epsilon K, \tau \epsilon K$ or $\mu \epsilon K$, if $\tau \epsilon K$, so $\tau \epsilon \{\bigcap \digamma | \digamma \text{ min. p. ideal}\}$, then $\tau \epsilon \digamma$, $\forall$ min. p. ideal $\digamma$, so $K \subseteq \digamma$, contradiction (since $\digamma$ min. p. ideal).

In the same manner, we proved that $\mu \epsilon K$. Hence, K is not ps.w.com.p. ideal.

Example 12: In example (21) as $\digamma_1=\{0,\alpha\}$ and $\digamma_2=\{0,\gamma\}$ are min.p. ideals of symmetric non-weakly commutative Ṇ then $(\digamma_1 \bigcap \digamma_2) = \{0\}$ is not ps.w.com.p.ideal.

Definition 29: An ideal $\digamma$ is said ps.w-com.2-p. ideal if 0= p.n $\epsilon \digamma$ so either $p^2 \epsilon \digamma$ or $n^2 \epsilon \digamma$, $\forall$ p, nϵṆ .

Proposition 30: Every min. p. ideal $\digamma$ in a near ring Ṇ is ps.w.com.2-p. ideal whenever $\digamma = \digamma^2$.

Proof: Let $\digamma$ min. p. ideal, and let $\gamma, \omega \epsilon Ṇ \ni 0 = \gamma.\omega \epsilon \digamma$, since $\digamma$ is p. ideal, then $\mathcal{H}.\mathcal{B} \subseteq \digamma$, $\mathcal{H} \subseteq \digamma$ or $\mathcal{B} \subseteq \digamma\ \forall\ \mathcal{H}, \mathcal{B} \subseteq Ṇ$ and $\gamma \epsilon \mathcal{H}, \omega \epsilon \mathcal{B}$, so $0 = \gamma.\omega \epsilon \mathcal{H}.\mathcal{B} \subseteq \digamma$,since $\digamma$ min. p. ideal, so we have $\{0\} = \mathcal{H}.\mathcal{B} \subseteq \digamma$ or we have $\{0\} \subseteq \mathcal{H}.\mathcal{B} = \digamma$.

Now, when $\{0\}\mathcal{H}.\mathcal{B} \subseteq \digamma$

Case (1): If $\mathcal{H} = \mathcal{B} = \{0\}$, as $\gamma = \omega = 0$, then $\gamma^2 = 0^2 = 0 = \gamma \epsilon \digamma$, $\omega^2 = 0^2 = 0 = \omega \epsilon \digamma$,

Case (2): If $\mathcal{H} = \{0\}, \mathcal{B} \neq \{0\}$, as $\gamma = 0, \omega \neq 0$, then $\gamma^2 = 0^2 = 0 = \gamma \epsilon \digamma$, similarly if $\mathcal{H} \neq \{0\}, \mathcal{B} = \{0\}$,

Case (3): If $\mathcal{H} \neq \{0\}$, $\mathcal{B} \neq \{0\} \ni \mathcal{H}.\mathcal{B} = \{0\}$, then $\mathcal{H} \nsubseteq \digamma, \mathcal{B} \nsubseteq \digamma$ (since $\digamma$ min.p. ideal containing only the zero ideal) and if $\{0\} \subseteq \mathcal{H}.\mathcal{B} = \digamma$, as $\mathcal{H} \subseteq \digamma$ or $\mathcal{B} \subseteq \digamma$ so $\gamma \epsilon \digamma$ or $\omega \epsilon \digamma$, therefore, $\gamma.\gamma \epsilon \digamma.\digamma = \digamma^2 = \digamma$ or $\omega.\omega \epsilon \digamma.\digamma = \digamma^2 = \digamma$ (since $\digamma = \digamma^2$) hence $\gamma^2 \epsilon \digamma$ or $\omega^2 \epsilon \digamma$, therefore $\digamma$ is ps.w.com.2-p. ideal.□

Example 13: In example (5) as $\digamma_1 = \{0, s\}$ & $\digamma_2 = \{0, t\}$ are min. p. ideals s.t $(\digamma_1)^2 = \digamma_1$ & $(\digamma_2)^2 = \digamma_2$ then $\digamma_1$ and $\digamma_2$ are ps.w.com.2-p. ideals.

4. Conclusion

We concluded that every ideals for non - zero symmetric near rings with zero divisors be the ps.w-com.p. ideal as well as every ps. w-com. p. ideal is ps.w.com.2-p while the converse is not. Moreover, in zero - symmetric weakly commutative near rings, the non-trivial nilpotent elements are zero divisors between them. Also, in a symmetric non-weakly commutative near ring, every maximal prime ideals implies to prime ideals, but not conversely. In addition, a completely prime ideals are contained in a symmetric non- weakly commutative near ring, while they are not contained in a symmetric weakly commutative near ring. Furthermore, the maximal ideals in a zero-symmetric weakly commutative near ring are neither prime nor completely prime ideals.

References

[1] G. Pilz, *Near Rings: The Theory and Its Applications*, North-Holland Mathematics Studies, vol. 23. Amsterdam, The Netherlands; New York, USA: North-Holland Publishing Co./Elsevier Science, (1983).

[2] F. Kasch, *Modules and Rings*, London Mathematical Society Monographs, vol. 17. London, U.K.; New York, USA: Academic Press, (1982).

[3] N. J. Groenewald, "The completely prime radical in near rings," *Acta Mathematica Hungarica*, vol. 51, no. 3–4, pp. 301–305 (1988).

[4] D. D. Anderson and E. Smith, "Weakly prime ideals," *Houston Journal of Mathematics*, vol. 29, no. 4, p. 831 (1997).

[5] G. Birkenmeier, H. Heatherly, and E. Lee, "Prime ideals in near rings," *Results Math.*, vol. 24, no. 1, pp. 27–48 (1993).

[6] H. E. Bell, "Near rings in which each element is a power of itself," *Bull. Aust. Math. Soc.*, vol. 2, no. 3, pp. 363–368 (1970).

[7] A. A. J. Al-Swidi and A. A. Omran, "Uniquely colorable completely equiprime graph of a near ring," *AIP Conference Proceedings*, vol. 2845, no. 1, p. 050045 (Sep. 2023), doi: 10.1063/5.0157185.

[8] M. E. Hellman, "The mathematics of public-key cryptography," *Sci. Am.*, vol. 241, no. 2, pp. 146–157 (1979).

[9] A. A. J. Al-Swidi and A. A. Omran, "Completely equiprime graph of a near ring," *AIP Conference Proceedings*, vol. 2475, no. 1, p. 080011 (Mar. 2023), doi: 10.1063/5.0102632.

[10] A. A. Al-Alawani and A. A. J. Al-Swidi, "Union of subgraphs with respect to some ideals on near rings," *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 28, no. 2, pp. 1903–1910 (2025).

[11] M. Bhargava, "Groups as unions of proper subgroups," *The American Mathematical Monthly*, vol. 116, no. 5, pp. 413–422 (1959).

[12] A. M. Almamoori, D. Z. Al-Swefee, and M. M. Alzuhery, "Biodegradation of Two Textile Dyes by Bacillus Subtilis," *Asian Journal of Water, Environment and Pollution*, vol. 21, no. 3, pp. 71–76 (2024).

[13] H. Q. Raheem, T. S. Al-meamar, and A. M. Almamoori, "Antibiotic profile and antibacterial activity of copper oxide nanoparticles against Pseudomonas fluorescens isolated from wound infection," *Journal of Drug Delivery Technology*, vol. 39, no. 3, pp. 383–385 (2019).

Received May, 2025