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Intuitionistic FBD-sub-algebra on BD-algebra

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Abstract

This paper establishes the concept of intuitionistic FBD-sub-algebra within BD-Algebras, along with proofs of multiple properties and theoretical results. We proved where f is an epimorphism on the BD-Algebras Q, Y correspondingly and if A be the intuitionistic FBD-sub-Algebra for Y, so A^f the intuitionistic FBD-sub-algebra for Q.

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1. Introduction

In recent years, many authors have studied new ideas in ring theory and topology using fuzzy sets and soft sets; see [1-4]. The authors studied the properties of RG-ideal Algebras with respect to homomorphism images and

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inverse images in the context of fuzzy structures. Another author introduced multipliers within intuitionistic [5] and [6]. Another author and his colleagues introduced intuitionistic to Bd-Algebras in the paper [7]. The authors investigated new concepts regarding P-intuitionistic fuzzy ideals of BCK-algebras. One of the authors, along with other authors, brought in multipliers as part of their research into intuitionistic of RG-Algebras [8]. The paper "Some Results on the anti-fuzzy Ideal for the RG-Algebra" presents the main result described in [9]. The following authors established intuitionistic theories for KU-Algebras [10]. The author pioneered research on intuitionistic theories for BCK-algebras in his paper [11]. This paper utilizes the concept of intuitionistic for BD-sub-Algebras alongside ideals and BD-ideals of BD-Algebras.

2. Results.

In this part, we present the main definition, which is BD-Algebra.

Definition 1: Assume that $A = \{(r, E_A(r), \nu_A\ (r)) \mid r \in Q\}$ is an intuitionistic fuzzy $\sqsubseteq$ of BD-Algebra $(Q;*\ ,0)$. A named intuitionistic FBD-sub-Algebra $for\ w\ where$

1) $E_A(r * s) \geq\ min\,\{E_A(r), E_A(s)\}$,
2) $\nu_A(r * s)\ \leq\ mar\,\{\ \nu_A(r), \nu_A(s)\}$.

E_A is an FBD-sub-Algebra and ν_A is an anti-fuzzy BD-Sub - Algebra

Proposition 2: Evers intuitionistic FBD-sub-Algebra

$A = \{(r, E_A(r), \nu_A\ (r)) \mid r \in Q\}$ of

BD-Algebra $(Q;*\ ,0)$, satisfies the inequalities $E_A(0) \geq E_A(r)$ and

$\nu_A(0) \leq \nu_A(r)$, for all $x\ \in Q$.

Proof: For any r $\in Q$, we have $E_A(0) =\ E_A(r * r) \geq \{E_A(r)\ , E_A(r)\} = E_A(r)$ and $\nu_A(0) = \nu_A(r * r) \leq \{\nu_A(r), \nu_A(r)\}\ = \nu_A(r)$.

Proposition 3: An intuitionistic fuzzy $\sqsubseteq A = \{(r, E_A(r), \nu_A\ (r)) \mid r \in Q\}$ named intuitionistic FBD-sub-algebra for the BD-Algebra $(Q;*\ ,0)$, then $\forall t, s \in$ [0,1], A sets $U(E_A, p)$ & $L(\nu_A, s)$ be the BD-sub-algebra for the Q.

Proof: Assume that $(E_A, p) \neq \emptyset \neq L(\nu_A, s)$. If it is the case for $r \in U(E_A, p)$, $s \in U(E_A, p)$, $\rightarrow\ E_A(r) \geq p, E_A(s) \geq p$ which follow $E_A(r * s) \geq min\{E_A(r)$, $E_A(s)\} \geq p$, So that $r * s \in U(E_A, p)$.

Hence $U(E_A, p)$ be the BD-sub-algebra for Q.

Now, we show $L(\nu_A, s)$ be the BD-sub-algebra for Q.

$$r\ \in L(\nu_A, s)\ \&\ s \in L(\nu_A, s)\ \&\ \nu_A(r\ ) \leq s\ \&\ \nu_A(s) \leq s$$

If follows that $\nu_A(r * s) \leq mar\{\nu_A(r\ ), \nu_A(s)\} \leq s$.So that $r * s \in L(\nu_A, s)$. Hence $L(\nu_A, s)$ is an BD-sub-algebra of Q.

Theorem 4: Assume that $A = \{(r, E_A(r), \nu_A\,(r)) \mid r \in Q\}$ is intuitionistic fuzzy sets in the BD - Algebra $(Q; *\ ,0)$, then A is the intuitionistic FBD-sub-algebra for Q.

$\underline{A}\ = \{(r, \underline{\nu}_A(r), \nu_A\,(r)) \mid r \in Q\}$ be the intuitionistic FBD-sub-algebra for Q.

Proof: Now, we have $A = \{(r, E_A(r), \nu_A\,(r)) \mid r \in Q\}$ be an intuitionistic fuzzy BD-sub-algebra.

Clearly E_A is the FBD-sub-algebra and ν_A be the AFBD-sub-algebra

Converse, assume that $E_A(r)$ is an FBD-sub-algebra of Q and $\nu_A\,(r)$ is a AFBD-sub-algebra of Q. For all $x, y \in Q$, we have:

$$\underline{\nu}_A(r * s) = 1 - \nu_A(r * s)$$
$$\geq 1 - mar\{\nu_A(r), \nu_A(s)\}$$
$$\geq \min\{1 - \nu_A(r), 1 - \nu_A(s)\ \}$$
$$\geq min\{\underline{\nu}_A(r), \underline{\nu}_A(s)\}$$

Hence $\underline{\nu}_A$ is a FBD-sub-algebra of Q.

$\underline{A} = \{r \in R\}$ is an intuitionistic FBD-sub-algebra of Q.

Converses, suppose $\underline{A} = \{r \in R\}$ be the intuitionistic FBD-sub-algebra for Q. So $\forall x, y \in Q$, implies that

$$E_A(r * s) \geq \{E_A(r), E_A(s)\}\ \&\ 1 - \nu_A(r * s) = \underline{\nu}_A(r * s) \geq min\{\underline{\nu}_A(r), \underline{\nu}_A(s)\}$$
$$= min\{1 - \nu_A(r), 1 - \nu_A(s)\}$$
$$=1 - mar\{\nu_A(r), \nu_A(s)\}$$

That is $\nu_A(r * s) \leq\ mar\{\nu_A(r), \nu_A(s)$.

Hence $A = \{(r, E_A(r), \nu_A\,(r)) \mid r \in Q\}$ an intuitionistic FBD-sub-algebra of Q.

Theorem 5: *Assume that $f\colon (Q;\ ,0) \to\ (Y;\ `, 0`)$ is epimorphism on the BD-Algebras Q, Y correspondingly If $A\ = \{a \in S\}$ the intuitionistic FBD-sub-algebra for Y, so $A^{\,f}\ = \{r \in R\}$ the intuitionistic FBD-sub-algebra for Q.*

Theorem 6: *Assume that $f\colon (R;\ ,0) \to\ (S;\ `, 0`)$ is the homomorphism on the BD-Algebras Q, Y correspondingly and $A^{\,f} = \{a \in R\}$ be the intuitionistic FBD-sub-algebra of Q, thus $A\ = \{r \in S\}$ is an intuitionistic FBD-sub-algebra for Y.*

Proof: Now, $\forall\ x, y \in Y$, also f (a)=r, f (b)=s, & $f\ (a * b) = x *'\ y$,

let a, b $\in R$, $E_A(r) = \mu_A(f(a))$, $E_A(s) = \mu_A(f(b))$, &

$$E_A(r * 's) = \mu_A^f\ (a * b)$$
$$\geq\ min\{\mu_A^f(a\ ), \mu_A^f(b)\}$$
$$= \{E_A(f(a\ )), E_A(f(s))\}$$
$$=\{E_A(x), E_A(y)\}$$

And

$$\nu_A(r *' s) = v_A^f\ (a * b)$$
$$\leq\ \{v_A^f(a\ ), v_A^f(b)\}$$
$$= \{\nu_A(f(a\ )), \nu_A(f(b))\}$$
$$=\{\nu_A(x), \nu_A(y)\}$$

Hence $A\ = \{r \in S\}$ be the intuitionistic fuzzy sets BD-sub-algebra for Y.

Definition 7: Assume that $A_i = \{(r, \mu_{A_i}(r), \nu_{A_i}\ (r)) \mid r \in R\}$ be the intuitionistic fuzzy ⊑for BD-Algebra $(Q;* ,0)$ wherever $i \in \Lambda$, thus

1. **An R- intersections**: $(\cap\ \mu_{A_i})(r) = inf\ inf\ \mu_{A_i}(r)\ , \left(\cup\ \nu_{A_i}\right)(r) = sup\ \nu_{A_i}(r)$.
2. **The P- intersection**: $(\cap\ \mu_{A_i})(r) = inf\ inf\ \mu_{A_i}(r)\ , \left(\cap\ \nu_{A_i}\right)(r) = inf\ \nu_{A_i}(r)$.
3. **The P-union**: $(\cup\ \mu_{A_i})(r) =\ \mu_{A_i}(r)\ , \left(\cup\ \nu_{A_i}\right)(r) = sup\ \nu_{A_i}(r)$.
4. **The R-union**: $(\cup\ \mu_{A_i})(r) =\ \mu_{A_i}(r)\ , \left(\cap\ \nu_{A_i}\right)(r) =inf\ inf\ \ \nu_{A_i}(r)$.

Theorem 8: *Assume that $\{A_i\}$ is collection for intuitionistic FBD-sub-algebra for BD-Algebra $(Q;* ,0)$, so an R- intersections in intuitionistic fuzzy A_i be the intuitionistic FBD-sub-algebra for Q, where*

$$\cap\ A_i = (inf\ inf\ \mu_{A_i}(r)\ , sup\ \nu_{A_i}(r)).$$

Proof: Assume $r, s \in \cap\ A_i$,then $r, s\ \in A_i$ for all $i \in \Lambda$ implies

$$\cap\ \mu_{A_i}(r * s) = inf\{\mu_{A_i}\ (r*s)\}$$
$$\geq inf\ inf\ \left\{\{\mu_{A_i}(r), \mu_{A_i}\ (s)\}\right\}$$
$$= min\{inf\ \mu_{A_i}(r), inf\ \mu_{A_i}\ (s)\}$$
$$\geq min\{\cap\ \mu_{A_i}(r),\cap\ \mu_{A_i}\ (s)\} \text{ and}$$
$$\cup\ \nu_{A_i}(r * s) = sup\{\nu_{A_i}\ (r*s)\}$$
$$\leq sup\ sup\ \left\{\{\nu_{A_i}(r), \nu_{A_I}\ (s)\}\right\}$$
$$= \left\{sup\ \nu_{A_i}(r), sup\ \nu_{A_i}(s)\right\}$$
$$\leq \left\{\cup\ \nu_{A_i}(r),\cup\ \nu_{A_i}\ (s)\right\}.$$

So, an R- intersections for the intuitionistic fuzzy A_i be the intuitionistic fuzzy BD-sub-algebra in the Q.

3. Conclusions

This article establishes a concept for intuitionistic FBD-sub-algebras within BD-Algebras and we prove many properties of this idea as well as we give some important results related to this concept.

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