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# Employing the $\mathcal{L}$- spaces to study the interior operator and exterior operator

Zinah T. Abdulqader
*Department of Mathematics*
*College of Education for Women*
*Tikrit University*
*Tikrit*
*Iraq*

**Abstract**

In this work, we employ the $\mathcal{L}$ – space in order to study some notions such as $\mathcal{L}$ – interior operator and $\mathcal{L}$ – exterior operator. Many properties of these notions have been studied, and they are important theorems and propositions in topology. The applications of the present notions are in cryptography, particularly in schemes that leverage topological concepts to enhance security or to create new cryptographic primitives.

***Subject Classification:*** *05C07, 05C09.*

***Keywords:*** *Cryptography, Topology, Algebra, Interior operator, Security.*

## 1. Introduction

Recent studies in 2025 [1] have explored several notions in fuzzy set theory that play significant roles in cryptography and set theory [2].

There are many generalizations for the open sets, such as $\alpha$ −open, $\beta$ − open, semi-open, and $\mathcal{L}$ −open.

The $\alpha$ −open was first studied by Njastad [3], where the subsets $\mathrm{F}$ in the universal $\mathrm{V}$ is named the $\alpha$ − open iff $\mathrm{F} \subseteq (Int(cl(Int(\mathrm{F}))))$.

A subset $\mathrm{F}$ in the universal $\mathrm{V}$ will be named $\beta$ − open if $\mathrm{F} \subseteq (cl(int(cl(\mathrm{F}))))$, which was studied in 2014 [4]. The concept of semi-open sets was introduced in 1963 [5]. Then the idea of $\mathcal{L}$ −open as generalization for the open set, where the subsets $\mathrm{F}$ i the universal $\mathrm{V}$ is said to be $\mathcal{L}$ −o pen if $\mathrm{F} \subseteq \mathrm{K} \cup Int(\mathrm{F})$ $\forall \mathrm{K} \epsilon \mathfrak{F}$ & $\mathrm{K} \neq \Phi$, for any topology $\mathfrak{F}$ on $\mathrm{V}$ and we write $\mathcal{L}_{\mathrm{V}}$ to represent the set for every $\mathcal{L}$ − open and the $(\mathrm{V}, \mathcal{L}o_{\mathrm{V}})$ is called the $\mathcal{L}$ −space [6-7]. The exterior point is essential concept in topology, if $(\mathrm{V}, \mathfrak{F})$ is the topological space & $\mathrm{F} \subseteq \mathrm{V}$. A point $\mathfrak{b} \epsilon$ $\mathrm{V}$ is said and exterior point for the $\mathrm{F}$ if there is

*E-mail:* ztaha@tu.edu.iq

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$\mathcal{K}\epsilon\,\mathfrak{F}$ such that $\mathfrak{b}\epsilon\mathcal{K}\subseteq \mathcal{F}^{c}$. The set for every exterior points for the set $\mathcal{F}$ is known as the exterior for $\mathcal{F}$ and is denote as $ext\,(\mathcal{F})$ [8].

## 2. The $\mathcal{L}$ −interior operator

**Definition 2.1:** Assume $\mathcal{V}$ is the any nonempty set and $(\mathcal{V},\mathcal{L}o_{\mathcal{V}})$ is the $\mathcal{L}-space$. Then a $\mathcal{L}$ −interior operator is a map $\mathfrak{h}:(\mathcal{V},\mathcal{L}o_{\mathcal{V}})\to(\mathcal{V},\mathcal{L}o_{\mathcal{V}})$ which satisfies the following axioms:

1. $\mathfrak{h}(\mathcal{V})=\mathcal{V}$.
2. $\mathfrak{h}(\mathcal{F})\subseteq\mathcal{F}\ \forall\mathcal{F}\epsilon\mathcal{L}o_{\mathcal{V}}$.
3. If $\mathcal{F}_1,\mathcal{F}_2,\dots,\mathcal{F}_n\epsilon\mathcal{L}o_{\mathcal{V}}$, then $\mathfrak{h}(\bigcap_{i=1}^{n}\mathcal{F}_i)=\bigcap_{i=1}^{n}\mathfrak{h}(\mathcal{F}_i)$.
4. $\mathfrak{h}\big(\mathfrak{h}(\mathcal{F})\big)=\mathfrak{h}(\mathcal{F})\ \forall\mathcal{F}\epsilon\mathcal{L}o_{\mathcal{V}}$.

**Proposition 2.2:** Suppose $\mathcal{V}$ is the any nonempty set and $\mathfrak{h}:(\mathcal{V},\mathcal{L}o_{\mathcal{V}})\to(\mathcal{V},\mathcal{L}o_{\mathcal{V}})$ is the $\mathcal{L}$ −interior operator. Assume that $\mathcal{F},\mathcal{Z}\epsilon\mathcal{L}o_{\mathcal{V}}$. Then

1. If $\mathcal{F}\subseteq\mathcal{Z}$, then $\mathfrak{h}(\mathcal{F})\subseteq\mathfrak{h}(\mathcal{Z})$.
2. $\mathfrak{h}(\mathcal{F})\cup\mathfrak{h}(\mathcal{Z})\subseteq\mathfrak{h}(\mathcal{F}\cup\mathcal{Z})$.
3. $\mathfrak{h}(\Phi)=\Phi$.
4. $\mathfrak{h}(\mathcal{F})\subseteq\mathcal{V}$.

***Proof:***

1. Suppose that $\mathcal{F}\subseteq\mathcal{Z}$, then $\mathcal{F}\cap\mathcal{Z}=\mathcal{F}$,
   hence $\mathfrak{h}(\mathcal{F})=\mathfrak{h}(\mathcal{F}\cap\mathcal{Z})$.
   Therefore, $\mathfrak{h}(\mathcal{F})=\mathfrak{h}(\mathcal{F})\cap\mathfrak{h}(\mathcal{Z})$ Since $\mathfrak{h}$ is a $\mathcal{L}$ −interior operator, then $\mathfrak{h}(\mathcal{F}\cap\mathcal{Z})=\mathfrak{h}(\mathcal{F})\cap\mathfrak{h}(\mathcal{Z})$.
   Therefore, $\mathfrak{h}(\mathcal{F})=\mathfrak{h}(\mathcal{F})\cap\mathfrak{h}(\mathcal{Z})\subseteq\mathfrak{h}(\mathcal{Z})$.
2. Since $\mathcal{F}\subseteq\mathcal{F}\cup\mathcal{Z}$ and $\mathcal{Z}\subseteq\mathcal{F}\cup\mathcal{Z}$, then from (1) we get,
   $\mathfrak{h}(\mathcal{F})\subseteq\mathfrak{h}(\mathcal{F}\cup\mathcal{Z})$ and $\mathfrak{h}(\mathcal{Z})\subseteq\mathfrak{h}(\mathcal{F}\cup\mathcal{Z})$.
   Hence $\mathfrak{h}(\mathcal{F})\cup\mathfrak{h}(\mathcal{Z})\subseteq\mathfrak{h}(\mathcal{F}\cup\mathcal{Z})$.
3. From the definition of $\mathcal{L}$ −interior operator we get:
   $\mathfrak{h}(\Phi)\subseteq\Phi$. But $\Phi\subseteq\mathfrak{h}(\Phi)$, then $\mathfrak{h}(\Phi)=\Phi$.
4. Since $\mathcal{F}\subseteq\mathcal{V}$, then by (1), we have:
   $\mathfrak{h}(\mathcal{F})\subseteq\mathfrak{h}(\mathcal{V})$. But $\mathfrak{h}(\mathcal{V})=\mathcal{V}$ becasuce $\mathfrak{h}$ is a $\mathcal{L}$ −interior operator, then $\mathfrak{h}(\mathcal{F})\subseteq\mathcal{V}$.

**Theorem 2.3:** *If $\mathcal{V}$ is the any nonempty set and $\mathfrak{h}:(\mathcal{V},\mathcal{L}o_{\mathcal{V}})\to(\mathcal{V},\mathcal{L}o_{\mathcal{V}})$ is the any $\mathcal{L}$ −interior operator. Define a class $\mathfrak{F}$ as follows:*

$$\mathfrak{F}=\{\mathcal{F}\epsilon\mathcal{L}o_{\mathcal{V}}:\mathcal{F}=\mathfrak{h}(\mathcal{F})\}.\ \text{Then } \mathfrak{F} \text{ is a topology on } \mathcal{V} \text{ and}$$

$$\mathfrak{h}(\mathcal{F})=lnt\,(\mathcal{F}) \text{ for all } \mathcal{F}\epsilon\mathcal{V}.$$

***Proof:*** Since $\mathcal{V}\epsilon\mathcal{L}o_{\mathcal{V}}$ and $\mathfrak{h}$ is a $\mathcal{L}$ −interior operator, then $\mathfrak{h}(\mathcal{V})=\mathcal{V}$, hence $\mathcal{V}\epsilon\mathfrak{F}$. Now, $\Phi\epsilon\mathcal{L}o_{\mathcal{V}}$ and $\mathfrak{h}(\Phi)=\Phi$ by [Proposition (2.2) – 3], then $\Phi\epsilon\,\mathfrak{F}$.

Assume that $\{\mathrm{F}_i\}_{i\in\Delta}\in\mathfrak{F}$. Then $\mathrm{F}_i\in\mathcal{L}o_{\mathrm{V}}$ and $\mathrm{F}_i=\mathfrak{h}(\mathrm{F}_i)\ \forall i\in\Delta$. But $\mathcal{L}o_{\mathrm{V}}$ is satisfies the conditions of topology on V, then $\bigcup_{i\in\Delta}\mathrm{F}_i\in\mathcal{L}o_{\mathrm{V}}$.

Now, $\mathrm{F}_i\subseteq\bigcup_{i\in\Delta}\mathrm{F}_i\ \forall i\in\Delta$, then by [Proposition (2.2) – 1] we have:

$\mathfrak{h}(\mathrm{F}_i)\subseteq\mathfrak{h}(\bigcup_{i\in\Delta}\mathrm{F}_i)$, but $\mathfrak{h}(\mathrm{F}_i)=\mathrm{F}_i\forall i\in\Delta$, then $\mathrm{F}_i\subseteq\mathfrak{h}(\bigcup_{i\in\Delta}\mathrm{F}_i)\forall i\in\Delta$.

Hence $\bigcup_{i\in\Delta}\mathrm{F}_i\subseteq\mathfrak{h}(\bigcup_{i\in\Delta}\mathrm{F}_i)$.

From the definition of a $\mathcal{L}-$interior operator we have:

$$\mathfrak{h}(\bigcup_{i\in\Delta}\mathrm{F}_i)\subseteq\bigcup_{i\in\Delta}\mathrm{F}_i.$$

Therefore, $\mathfrak{h}(\bigcup_{i\in\Delta}\mathrm{F}_i)=\bigcup_{i\in\Delta}\mathrm{F}_i$ this implies that $\bigcup_{i\in\Delta}\mathrm{F}_i\in\mathfrak{F}$.

Let $\mathrm{F}_1,\mathrm{F}_2,\dots,\mathrm{F}_n\in\mathfrak{F}$. Then $\mathrm{F}_i\in\mathcal{L}o_{\mathrm{V}}$ and $\mathrm{F}_i=\mathfrak{h}(\mathrm{F}_i)\ \forall i=1,\dots,n$.

Hence, $\bigcap_{i=1}^{n}\mathrm{F}_i\in\mathcal{L}o_{\mathrm{V}}$.

To prove $\bigcap_{i=1}^{n}\mathrm{F}_i=\mathfrak{h}(\bigcap_{i=1}^{n}\mathrm{F}_i)$.

Now, $\bigcap_{i=1}^{n}\mathfrak{h}(\mathrm{F}_i)=\bigcap_{i=1}^{n}\mathrm{F}_i$ since $\mathrm{F}_i=\mathfrak{h}(\mathrm{F}_i)\ \forall i=1,2,\dots,n$.

By the definition of $\mathcal{L}-$interior operator we have

$$\mathfrak{h}(\bigcap_{i=1}^{n}\mathrm{F}_i)=\bigcap_{i=1}^{n}\mathfrak{h}(\mathrm{F}_i)=\bigcap_{i=1}^{n}\mathrm{F}_i.$$

Consequentially, $\bigcap_{i=1}^{n}\mathrm{F}_i\in\mathfrak{F}$.

Therefore, $\mathfrak{F}$ is a topology on V.

To prove $\mathfrak{h}(\mathrm{F})=lnt\ (\mathrm{F})$ for all $\mathrm{F}\in\mathrm{V}$.

Let $\mathrm{F}\in\mathcal{L}o_{\mathrm{V}}$. Then $\mathfrak{h}(\mathrm{F})\subseteq\mathrm{F}$ and

$\mathfrak{h}\big(\mathfrak{h}(\mathrm{F})\big)=\mathfrak{h}(\mathrm{F})$ since $\mathfrak{h}$ is $\mathcal{L}-$interior operator. Hence by construction of $\mathfrak{F}$, we have:

$\mathfrak{h}(\mathrm{F})\in\mathfrak{F}$, that is, $\mathfrak{h}(\mathrm{F})$ is an open set in $\mathfrak{F}$.

But $\mathfrak{h}(\mathrm{F})\subseteq\mathrm{F}$, then $\mathfrak{h}(\mathrm{F})$ is an open set contained in F.

Now, let Z is an open set in $\mathfrak{F}$ such that $\mathrm{Z}\subseteq\mathrm{F}$. Then $\mathfrak{h}(\mathrm{Z})=\mathrm{Z}$.

Since $\mathrm{Z}\subseteq\mathrm{F}$, then $\mathfrak{h}(\mathrm{Z})\subseteq\mathfrak{h}(\mathrm{F})$, hence $\mathrm{Z}\subseteq\mathfrak{h}(\mathrm{F})$.

This implies that $\mathfrak{h}(\mathrm{F})$ is the largest open set in $\mathfrak{F}$ that contained in F, that is,

$$\mathfrak{h}(\mathrm{F})=lnt\ (\mathrm{F}).$$

Therefore, $\mathfrak{h}(\mathrm{F})=lnt\ (\mathrm{F})$ for all $\mathrm{F}\in\mathrm{V}$.

## 3. The $\mathcal{L}-$ exterior operator

**Definition 3.1:** Let us assume that V is any nonempty set and $(\mathrm{V},\mathcal{L}o_{\mathrm{V}})$ is the any $\mathcal{L}-space$. Then a $\mathcal{L}-$ exterior operator on a $(\mathrm{V},\mathcal{L}o_{\mathrm{V}})$ is a map $\mathfrak{t}:(\mathrm{V},\mathcal{L}o_{\mathrm{V}})\to(\mathrm{V},\mathcal{L}o_{\mathrm{V}})$ which satisfies the following axioms:

1. $\mathfrak{t}(\mathrm{V})=\Phi$.
2. $\mathfrak{t}(\Phi)=\mathrm{V}$.
3. $\mathfrak{t}(\mathrm{F})\subseteq\mathrm{F}^c\ \forall\mathrm{F}\in\mathcal{L}o_{\mathrm{V}}$.
4. If $\mathrm{F}_1,\mathrm{F}_2,\dots,\mathrm{F}_n\in\mathcal{L}o_{\mathrm{V}}$, then $\mathfrak{t}(\bigcup_{i=1}^{n}\mathrm{F}_i)=\bigcap_{i=1}^{n}\mathfrak{t}(\mathrm{F}_i)$.
5. $\mathfrak{t}((\mathfrak{t}(\mathrm{F}))^c)=\mathfrak{t}(\mathrm{F})\ \forall\mathrm{F}\in\mathcal{L}o_{\mathrm{V}}$.

**Proposition 3.2:** Suppose that V is the nonempty set and $\mathfrak{t}:(\mathrm{V},\mathcal{L}o_{\mathrm{V}})\to(\mathrm{V},\mathcal{L}o_{\mathrm{V}})$ is the $\mathcal{L}-$ exterior operator. If $\mathrm{F}\subseteq\mathrm{Z}$, then $\mathfrak{t}(\mathrm{Z})\subseteq\mathfrak{t}(\mathrm{F})\ \forall\mathrm{F},\mathrm{Z}\in\mathcal{L}o_{\mathrm{V}}$.

***Proof:*** Let $\mathrm{F}, \mathrm{Z} \in \mathcal{L}o_{\mathrm{V}}$ and $\mathrm{F} \subseteq \mathrm{Z}$. Then $\mathrm{F} \cup \mathrm{Z} = \mathrm{Z}$.

Hence $\mathfrak{t}(\mathrm{Z}) = \mathfrak{t}(\mathrm{F} \cup \mathrm{Z})$.

Since $\mathfrak{t}$ is a $\mathcal{L}-$ exterior operator, then $\mathfrak{t}(\mathrm{F} \cup \mathrm{Z}) = \mathfrak{t}(\mathrm{F}) \cap \mathfrak{t}(\mathrm{Z})$.

Therefore, $\mathfrak{t}(\mathrm{Z}) = \mathfrak{t}(\mathrm{F}) \cap \mathfrak{t}(\mathrm{Z}) \subseteq \mathfrak{t}(\mathrm{F})$.

**Theorem 3.3:** *Let $\mathfrak{t}$ be a $\mathcal{L}-$ exterior operator on $(\mathrm{V}, \mathcal{L}o_{\mathrm{V}})$ such that $\mathcal{L}o_{\mathrm{V}}$ is closed under complements. Define a class $\mathfrak{F}$ as follows:*

$$\mathfrak{F} = \{\mathrm{F} \in \mathcal{L}o_{\mathrm{V}} : \mathrm{F} = \mathfrak{t}(\mathrm{F}^{c})\}.$$

Then $\mathfrak{F}$ is a topology on V and $\mathfrak{t}(\mathrm{F}) = ext(\mathrm{F})\ \forall \mathrm{F} \in \mathfrak{F}$.

***Proof:*** Since $\Phi, \mathrm{V} \in \mathcal{L}o_{\mathrm{V}}$ and $\mathfrak{t}$ is a $\mathcal{L}-$ exterior operator, then

$$\mathfrak{t}(\mathrm{V}) = \Phi \text{ and } \mathfrak{t}(\Phi) = \mathrm{V} \text{ then } \Phi, \mathrm{V} \in \mathfrak{F}.$$

Let $\{\mathrm{F}_i\}_{i \in \Delta} \in \mathfrak{F}$. Then $\mathrm{F}_i \in \mathcal{L}o_{\mathrm{V}}$ and $\mathrm{F}_i = \mathfrak{t}(\mathrm{F}_i^{c})\ \forall i \in \Delta$.

But $\mathcal{L}o_{\mathrm{V}}$ is satisfies the conditions of topology on V, then $\bigcup_{i \in \Delta} \mathrm{F}_i \in \mathcal{L}o_{\mathrm{V}}$.

Now, $\mathrm{F}_i \subseteq \bigcup_{i \in \Delta} \mathrm{F}_i\ \forall i \in \Delta$, then $(\bigcup_{i \in \Delta} \mathrm{F}_i)^{c} \subseteq \mathrm{F}_i^{c}\ \forall i \in \Delta$,
hence by Proposition (3.2) we have $\mathfrak{t}(\mathrm{F}_i^{c}) \subseteq \mathfrak{t}((\bigcup_{i \in \Delta} \mathrm{F}_i)^{c})\ \forall i \in \Delta$,
thus $\mathrm{F}_i \subseteq \mathfrak{t}((\bigcup_{i \in \Delta} \mathrm{F}_i)^{c})\ \forall i \in \Delta$,
therefore $\bigcup_{i \in \Delta} \mathrm{F}_i \subseteq \mathfrak{t}((\bigcup_{i \in \Delta} \mathrm{F}_i)^{c})$.

Since $\mathfrak{t}$ is a $\mathcal{L}-$ exterior operator, then

$$\mathfrak{t}((\bigcup_{i \in \Delta} \mathrm{F}_i)^{c}) \subseteq ((\bigcup_{i \in \Delta} \mathrm{F}_i)^{c})^{c} = \bigcup_{i \in \Delta} \mathrm{F}_i.$$

Hence $\bigcup_{i \in \Delta} \mathrm{F}_i = \mathfrak{t}((\bigcup_{i \in \Delta} \mathrm{F}_i)^{c})$.

Therefore, $\bigcup_{i \in \Delta} \mathrm{F}_i \in \mathfrak{F}$.

Let $\mathrm{F}_1, \mathrm{F}_2, \ldots, \mathrm{F}_n \in \mathfrak{F}$. Then by the definition of $\mathfrak{F}$, we get
$\mathrm{F}_i \in \mathcal{L}o_{\mathrm{V}}$ and $\mathrm{F}_i = \mathfrak{t}(\mathrm{F}_i^{c})\ \forall i = 1,2 \ldots, n$. Hence, $\bigcap_{i=1}^{n} \mathrm{F}_i \in \mathcal{L}o_{\mathrm{V}}$.

To prove $\bigcap_{i=1}^{n} \mathrm{F}_i = \mathfrak{t}((\bigcap_{i=1}^{n} \mathrm{F}_i)^{c})$.

From De- Morgan Laws, we get $(\bigcap_{i=1}^{n} \mathrm{F}_i)^{c} = \bigcup_{i \in \Delta} \mathrm{F}_i^{c}$.

So, we have

$$\mathfrak{t}((\bigcap_{i=1}^{n} \mathrm{F}_i)^{c}) = \mathfrak{t}(\bigcup_{i \in \Delta} \mathrm{F}_i^{c})$$

$$= \bigcap_{i=1}^{n} \mathfrak{t}(\mathrm{F}_i^{c}) \text{ Since } \mathfrak{t} \text{ is a } \mathcal{L}-\text{ exterior operator}$$

$$= \bigcap_{i=1}^{n} \mathrm{F}_i \text{ since } \mathrm{F}_i = \mathfrak{t}(\mathrm{F}_i^{c})\ \forall i = 1, 2 \ldots, n$$

Consequentially, $\bigcap_{i=1}^{n} \mathrm{F}_i \in \mathfrak{F}$.

Therefore, $\mathfrak{F}$ is a topology on V.

To prove $\mathfrak{t}(\mathrm{F}) = ext(\mathrm{F})$ for all $\mathrm{F} \in \mathfrak{F}$.

Let $\mathrm{F} \in \mathfrak{F}$. Then $\mathrm{F} \in \mathcal{L}o_{\mathrm{V}}$ and $\mathrm{F} = \mathfrak{t}(\mathrm{F}^{c})$.

Now, $\mathrm{F} \in \mathcal{L}o_{\mathrm{V}}$ and $\mathfrak{t}$ is $\mathcal{L}-$ exterior operator, then we get
$\mathfrak{t}((\mathfrak{t}(\mathrm{F}))^{c}) = \mathfrak{t}(\mathrm{F})$ and $\mathfrak{t}(\mathrm{F}) \subseteq \mathrm{F}^{c}$. So, we have

$$\mathfrak{t}(\mathrm{F}) \in \mathfrak{F} \text{ and } \mathfrak{t}(\mathrm{F}) \subseteq \mathrm{F}^{c}.$$

Hence $\mathfrak{t}(\mathrm{F})$ is an open set contained in $\mathrm{F}^{c}$.

We claim $\mathfrak{t}(\mathrm{F}) = int(\mathrm{F}^{c})$.

Let $\mathrm{Z} \in \mathfrak{F}$ and $\mathrm{Z} \subseteq \mathrm{F}^{c}$. Then $\mathrm{Z} \in \mathcal{L}o_{\mathrm{V}}$ and $\mathrm{Z} = \mathfrak{t}(\mathrm{Z}^{c})$.

Since $\mathrm{Z} \subseteq \mathrm{F}^{c}$, then $\mathrm{F} \subseteq \mathrm{Z}^{c}$.

Hence by Proposition (3.2) we have, $\mathfrak{t}(\mathrm{Z}^c) \subseteq \mathfrak{t}(\mathrm{F})$.
But $\mathrm{Z} = \mathfrak{t}(\mathrm{Z}^c)$, then $\mathrm{Z} \subseteq \mathfrak{t}(\mathrm{F})$.
That is $\mathfrak{t}(\mathrm{F})$ is the largest open set in $\mathfrak{F}$ that contained in $\mathrm{F}^c$.
Thus $\mathfrak{t}(\mathrm{F}) = int(\mathrm{F}^c)$.
But $int(\mathrm{F}^c) = ext(\mathrm{F})$. So, we have $\mathfrak{t}(\mathrm{F}) = ext(\mathrm{F})$.

## 4. Conclusion

We considered the idea for the interior operator and the exterior operator in the $\mathcal{L}$ −spaces that are extensions of the classical topological space. We obtained an important result, which is, if $\mathrm{V}$ is any nonempty set and

$\mathfrak{h} : (\mathrm{V}, \mathcal{L}o_{\mathrm{V}}) \to (\mathrm{V}, \mathcal{L}o_{\mathrm{V}})$, is the any $\mathcal{L}$ −interior operator. Define a class $\mathfrak{F}$ as follows:$\mathfrak{F} = \{\mathrm{F} \epsilon \mathcal{L}o_{\mathrm{V}} : \mathrm{F} = \mathfrak{h}(\mathrm{F})\}$. Then $\mathfrak{F}$ is a topology on $\mathrm{V}$ and

$$\mathfrak{h}(\mathrm{F}) = lnt\,(\mathrm{F}) \text{ for all } \mathrm{F} \epsilon \mathrm{V}.$$

In the future, we plan to characterize novel types of sets related to the $\mathcal{L}$ −open by employing soft sets.

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