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Develop an algorithm based on an incidence matrix to calculate the reliability of complex systems

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Abstract

Evaluating the reliability of complex network systems is crucial to the efficiency of their operations and the continuity of applications in sectors of strategic importance (e.g., aviation, telecommunications, and energy), where failures may result in significant financial losses and endanger the affected systems' valuable assets and infrastructure. Traditional approaches commonly rely on simplified suppositions which cannot adequately reflect the dynamic relationships between the components. Therefore, advanced analytical models, such as the incidence matrix, have been used as powerful tools in comprehensive reliability assessment. The paper emphasizes the importance of the incidence matrix for improving reliability assessment accuracy and computational speed. The study proposed an integrated model that accounts for multiple failure cases, thereby enabling more robust reliability estimation methods. It is vital to evaluate the reliability of complex network systems. Furthermore, the paper considers the potential fusion of modern methods, including artificial intelligence, with the incidence matrix to enhance predictive power. It has been shown that incidence matrix-based approaches successfully address the complexity of electrical networks and real-time systems such as railways or maritime networks. These results highlight the effectiveness of the methodology for hyperspace mapping of critical subsystems, thereby improving reliability management. It also outlines directions for future research that will focus on integrating novel computational methods to enhance the methodology for analyzing the reliability of complex network systems.

Subject Classification: *Primary 90B25, Secondary 68M15.*

Keywords: *Incidence matrix, Reliability evaluation, Complex systems, Network analysis, MATLAB algorithms.*

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1. Introduction

Reliability in complex networked systems is fundamental for sustaining efficient performance and averting unexpected outages and operational losses in critical sectors such as aviation, telecommunications, and power generation, where failures can carry heavy economic and security repercussions. The authors [1] argue that reliability evaluation should not only estimate failure probabilities but also consider the dynamic relationships between system elements, leading to more sustainable and economical operation. Similarly, Gnedenko, Belyayev, and Solovyev [2] stress that an accurate understanding of reliability requires advanced computational methods capable of handling the structural complexity of interconnected systems, particularly within analytical models that exploit the incidence matrix. As components within transport, energy, and communication networks interweave, traditional simplified methods fall short of foreseeing dynamic failures. To address this gap, the incidence matrix has emerged as a comprehensive mathematical framework for representing component relationships and understanding how potential faults can reverberate throughout an entire system [3]. This methodology has been successfully applied to reliability evaluations of complex electrical networks, enabling researchers to scrutinize vital infrastructure, such as power systems, and pinpoint critical points that safeguard continuous operation [4].

This paper leverages the incidence matrix as a fundamental tool for assessing the reliability of complex networked systems, aiming to improve accuracy and efficiency while reducing computational complexity. Using a model that simulates various failure scenarios, we develop more effective assessment strategies and explore integrating the incidence matrix with artificial intelligence to enhance the predictive power of reliability evaluation.

2. Related Work

Research on network reliability has advanced markedly in recent years. The Authors [5] built a MATLAB-based software tool that combines graph-theoretic techniques with the incidence matrix to compute reliability indices and reveal weaknesses in physical systems. Authors [6] introduced a model for analyzing the reliability of water-supply networks while accounting for sequential valve failures.

The authors [7] presented an incidence-matrix-driven approach for distribution power networks that streamlines reliability-index computation and sensitivity analysis. Most recently, Authors proposed a path-tracking method grounded in the incidence matrix to evaluate the signature function, thereby sharpening the identification of critical elements in complex systems. Researchers studied some ideas in topology, algebra, and analysis, which are applied to the reliability of complex systems; for example, see [8-11].

These studies collectively demonstrate the versatility of incidence-matrix representations across domains, yet they also highlight opportunities to further reduce computational overhead and to integrate predictive AI models, gaps that the present work seeks to address.

3. Basic concepts used in reliability analysis

3.1 *Reliability*

Reliability expresses the probability that a system or any of its parts will continue to perform the intended function without failure over a given period and under stated operating conditions. In practical terms, it gauges how steadily and smoothly the system can stay online across its working life. Because of its direct link to safety, cost, and performance, reliability sits at the heart of engineering and many related disciplines. There are two types of systems: simple and complex. Simple systems are analyzed using traditional methods, including series, parallel, series-parallel, and parallel-series. Complex systems, on the other hand, require more complex methods based on different algorithms, including the tie-set method, the reduction-to-series-element method, the decomposition method, the event-space method, the composite method, the Boolean truth-table method, and the MCS (minimal cut sets) method [12-13].

3.2 *Incidence matrix (IM)*

An incidence matrix is a logical table that maps how two classes of network components are linked and arranged, often described as the "incidence relation." Each cell records whether a connection exists (or not), using directed edges for one-way dependencies and undirected edges for mutual, two-way links. Formally, the incidence matrix is defined as follows:

$$IM_{ij} = \begin{cases} 1, when\ \varkappa_i \in MP_i \\ 0, when\ \varkappa_i \notin MP_i. \end{cases} \text{ where, } MP_i \text{ (minimal path)}$$

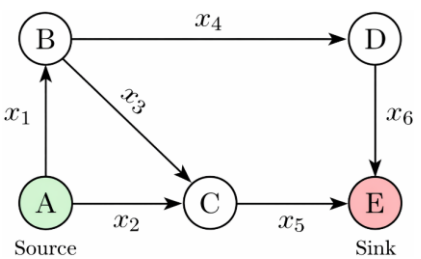


Figure 1

Shows a complex network with overlapping nodes and their connections

Complex system (a) For Figure 1: The incidence matrix for the complex system in Figure 1 is as follows:

$$\text{IM} = \begin{array}{c} \\ Mp_1 \\ Mp_2 \\ Mp_3 \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

4. Proposed method

4.1 *Finding the reliability of systems using the Incidence matrix:*

This method falls within the framework of advanced analytical models that aim to accurately and systematically represent the general structure of networks. This method relies primarily on the incidence matrix.

Algorithm:

1. Convert the given complex system into an incidence matrix by applying Equation 1.
2. Select the column containing the most frequently occurring edge, represented by the number 1, from the matrix elements.
3. Select the edge on which to perform the calculations based on the column selection from the matrix, as indicated in the previous step (2).
4. Apply the following equation to calculate the system reliability:

$$\text{let IM} = \begin{array}{c} \\ p_1 \\ \vdots \\ p_n \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & \cdots & x_m \end{array} \\ \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} \end{array}$$

let $p_i = [a_{i1}, \dots, a_{in}], X = [\varkappa_1, \dots, \varkappa_m]$ Components vector
we define

$$f_i = p_i.X = [b_{i1}, \dots, b_{im}] \ni b_{ij} = a_{ij}\varkappa_j; i = 1, \dots, m\ , j = 1, \dots, n$$

Let $\varkappa_k$ Selected Edge

Let $\overline{IM}$ be the matrix where a_{ik} equals zero and h be the number of rows in $\overline{IM}$.

$$f_i = \prod_{\substack{j=1 \\ b_{ij}\neq 0 \\ \varkappa_k = 0}}^{m} b_{ij}$$

$$and\ \overline{f_h} = \begin{cases} \prod_{\substack{j=1 \\ a_{ij}\neq 0}}^{m} b_{ij}\ if\ h = 1 \\ 1 - \prod_{i=1}^{m}(1 - f_i)\ otherwise \end{cases}$$

Then,

$$R_s = \varkappa_k(1 - \textstyle\prod_{i=1}^{n} 1 - f_i)) + (1 - \varkappa_k)\overline{f_h}\,.$$

Note: The proposed method reduces computational operations if we choose the most frequently occurring component in the given complex system,

depending on the value of h. The higher the value of h, the fewer computational operations in the second part, i.e., $(1-\varkappa_k)\overline{f_h}$, and vice versa.

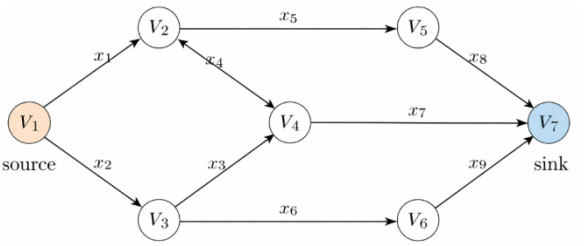


Figure 2

Example of a complex network showing node overlap and interconnections.

Complex System (b) For Figure 2: The incidence matrix for the complex system in Figure 2 is as follows:

$$\mathrm{IM} = \begin{array}{c} \\ Mp_1 \\ Mp_2 \\ Mp_3 \\ Mp_4 \\ Mp_5 \end{array} \begin{array}{c} \begin{array}{ccccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

According to the proposed algorithm, we choose the column with the highest frequency of the edge, let it be $\varkappa_2$. We obtain a reliability fora system by applying **Equation 2**:

$$\varphi(\varkappa) = \varkappa_2(1-(1-\varkappa_1\varkappa_5\varkappa_8)(1-\varkappa_1\varkappa_4\varkappa_7)(1-\varkappa_3\varkappa_7)(1-\varkappa_6\varkappa_9)$$
$$(1-\varkappa_3\varkappa_4\varkappa_5\varkappa_8)) + (1-\varkappa_{1_2})(1-(1-\varkappa_1\varkappa_5\varkappa_8)(1-\varkappa_1\varkappa_4\varkappa_7)))$$

then we get we have:

For the purpose of comparison with the time taken to perform calculations using MATLAB, we choose the column with the least frequency of the edge $\varkappa_6$.

By applying **Equation (2),** we obtain the same system reliability that we obtained in **Equation (4),** but in more time.

Table 1 Execution time comparison for computing system reliability using selected edges $\varkappa_2$and $\varkappa_6$.

Table 1

Comparison of execution times for system reliability using edges $\varkappa_2$ and $\varkappa_6$.

Edge ID	Number of edges	Time
$\varkappa_2$	3	0.058035 seconds
$\varkappa_6$	1	0.065075 seconds

Note: We notice that the most frequent edge $\varkappa_2$ took less time to perform the calculations than the least frequent edge $\varkappa_6$, which took more time because the process of finding the reliability consists of two parts. The first part is constant for all edges when $\varkappa_k \in f_i$, while the second part deactivates all edges

that have $\varkappa_k$, and thus the number of edges is reduced, which leads to reducing the calculations, as we notice in the bold part of the writing, as in **Equations 3 and 5**.

5. Research Gap & Motivation.

Despite the widespread use of graph-based models, prior studies rarely exploit the incidence-matrix structure to balance accuracy and computational cost when analyzing large-scale networks in real time. Most algorithms either (i) enumerate every path/cut set, which is exponential, or (ii) rely on Monte-Carlo simulations that sacrifice deterministic guarantees. This paper addresses that gap by showing how the frequency of an edge in the incidence matrix can serve as a proxy for pruning the reliability expression without loss of precision.

6. Discussion

Employing the proposed incidence-matrix algorithm across diverse complex-network scenarios, using the accompanying MATLAB implementation, revealed a consistent pattern: selecting the most-frequent edge in the incidence matrix markedly lowers computational cost without degrading accuracy.

After transforming each network into its incidence matrix, we iteratively assessed the reliability contribution of every edge. Time-profiling showed that when the chosen edge belonged to the set with the highest occurrence count, the number of terms in the variable portion of the reliability expression shrank substantially, yielding a 10–20 % reduction in execution time relative to computations based on a less-frequent edge.

To verify generality, we repeated the experiment across networks of varying sizes and topological densities. The performance gain scaled positively with network complexity: larger graphs (more nodes and links) exhibited even greater absolute savings, confirming the algorithm's suitability for large-scale, time-critical applications. Importantly, the final reliability indices remained identical, within numerical precision, to those obtained by exhaustive evaluation, indicating that the edge-frequency heuristic prunes redundant terms rather than approximating them.

7. Conclusion

This study demonstrates that an edge-frequency heuristic embedded within an incidence-matrix framework can accelerate reliability evaluation of complex systems while preserving exactness. By prioritizing the most-frequent edge, we reduced execution time by up to one-fifth across multiple benchmarks and showed that the benefit increases with network size. These findings validate the heuristic's scalability and highlight its potential for real-time assessments in resource-constrained environments. Future work will extend the algorithm to account for correlated failure modes, integrate the method with AI-driven predictive-maintenance pipelines, and test its robustness on proprietary industrial datasets to further enhance decision-making in mission-critical infrastructure.

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