Journal of Discrete Mathematical Sciences & Cryptography
ISSN 0972-0529 (Print), ISSN 2169-0065 (Online)
Vol. 29 (2026), No. 5, pp. 2131–2138
DOI : 10.47974/JDMSC-2715

Computing topological descriptors of the graph A4(Co3, 3B)

Ellaf Hassan Mahmood †
Ali Abd Aubad *
Department of Mathematics
College of Science
University of Baghdad
Baghdad
Iraq

Abstract

This paper presents a quantification of the complex geometry of symmetric A4-graph constructed from Conway group Co_3 and a particular category of Co_3 by calculating its complete topological descriptors. The primary objective is to characterize this graph using molecular topology by calculating a suite of significant topological indices. Furthermore, we derive the Hosoya and Schultz polynomials, which serve as generating functions for the full distance graph and degree spectra. A principal result is the establishment of closed-form relationships between these topological descriptors and fundamental group characteristics, notably conjugacy-class cardinality and vertex-degree distribution, revealing that the graph's metric architecture is predetermined by its algebraic origin.

Subject Classification: *20D08, 05C25, 05C09, 05C31.*

Keywords: *Conway group, A4-graph, Topological indices, Hosoya polynomial.*

1. Introduction

Topological indicators yield an objective statistic that reflects the degree of interconnectedness within the network. The numerical results serve as tools for analyzing and evaluating the characteristics associated with methodological frameworks. Graph descriptors are organized according to core structural attributes, such as node valence, geodesic distance, and vertex eccentricity. The preliminary pair of species captures close association, while the latter type deals with broad-spectrum communication. Consequently, three principal indices exist: distance-dependent (DISI), evaluated from shortest-route arrays; degree-dependent indices (DEGI), built from degree sequence arrangements; and

† *E-mail:* **elaf.hasan1203a@sc.uobaghdad.edu.iq**
* *E-mail:* **ali.abd@sc.uobaghdad.edu.iq (Corresponding Author)**

eccentricity-dependent (ECCI), which fuse central distance with valency or path-length factors, see [1-9]. Many recent studies address the calculation of topological descriptors for graphs constructed from the components of a given group. Some are presented in [10-15]. Pick a subset $\mathcal{S}$ of group G, the $A4$ -graph, officially called $A4(G, \mathcal{S})$ see [16-18]. In this graph, the nodes set is $\mathcal{S}$, and an arc is established across vertices c and k whenever (ck^{-1}) comprises an involution. Computational techniques implemented in Gap [19] supply a complete depiction of the disc designs in the graph $A4(Co_3, \mathcal{S})$, in case $\mathcal{S} = 3B$. This research intends to elucidate this graph through molecular topology. To accomplish this, we will compute a set of essential topological indices and subsequently determine their Hosoya and Schultz polynomials. A key outcome is the formulation of proper formulae that connect these descriptors to essential group parameters, particularly the enumeration of conjugacy classes and the ordering of vertex degrees. This shows that the structure of the graph distance arises from its algebraic basis. Moreover, the technique can be applied to study other types of graphs, such as the one mentioned in [20]. The important works contain further information about these topological indices [14].

2. Topological Indices for $A_4(Co_3, 3\mathrm{B})$

Our procedure proceeds as follows: we initially identify the core features of the A_4-graph layout, which then establishes the calculation of the connectivity index of $\nabla\wedge = A_4(Co_3, 3\mathrm{B})$. Observe that $\rho_r(c)$ denotes the number of nodes sitting at a distance of no more than r from the node c. The initial part of the preceding proposition employs techniques identical to [16], whereas the latter part derives from the vertex-transitivity of $\nabla\wedge$.

Proposition 1: The graph $\nabla\wedge$, holds the following properties for any $c, k \in V(\nabla\wedge)$:

1- Diam $(\nabla\wedge) = 4$, for any $c \in V(\nabla\wedge)$, $\rho_r(c)$ takes the values $1701, 1679967, 15319901, 30$ when $r = 1,2,3,4$ respectively.

2- deg $(c) = 1701$, $e(c) = 4$, and $\mathrm{T}D(c) = \sum_{r=1}^{\mathrm{Diam}(\nabla\wedge)} r * \rho_r(c)$ where $\mathrm{T}D(c)$ represents the overall distance c.

Now, by using **Proposition 1**, it follows that $|E(\nabla\wedge)| = 14460370800$ and $|\mathrm{E}(\nabla\wedge)^c| = 14459860800$. And $|V(\nabla\wedge)| = 17001600$.

2.1 *Computing DEGI of* $\nabla\wedge$

Here, we outline the calculation of the DEGI for $\nabla\wedge$:

(1) The first Zagreb index:

$$M1(\nabla\wedge) = \sum_{c \in V(\nabla\wedge)} \deg(c)^2 = (1701)^2 * 17001600$$
$$= 49192446441600 \quad (2)$$

(2) The second Zagreb index:

$$M2(\nabla\wedge) = \sum_{ck\,\in E(\nabla\wedge)} \deg(c)\deg(k) = (1701)^2 * 14460370800$$
$$= 41839651333090800 \tag{2}$$

(3) The forgotten topological index or F-index:

$$F(\nabla\wedge) = \sum_{c\,\in V(\nabla\wedge)} \deg(c)^3 = (1701)^3 * 17001600$$
$$= 83676351397161600 \tag{3}$$

(4) The first Zagreb co-index:

$$\bar{M}1(\nabla\wedge) = \sum_{ck\,\notin E(\nabla\wedge)}(\deg(c) + \deg(k))$$
$$= 2(1701) * 14459860800 = 49192446441600 \tag{4}$$

(5) The second Zagreb co-index:

$$\bar{M}2(\nabla\wedge) = \sum_{ck\,\notin E(\nabla\wedge)} \deg(c)\deg(k)$$
$$= (1701)^2 * 14459860800 = 41838175698580800 \tag{5}$$

(6) The reduced second Zagreb index:

$$RM2(\nabla\wedge) = \sum_{ck\,\in E(\nabla\wedge)}[\deg(c) - 1][\deg(k) - 1]$$
$$= (1700)^2 * 14459860800 = 4.178899712000000 \tag{6}$$

(7) The Narumi-Katayama index:

$$Nk(\nabla\wedge) = \prod_{c\,\in v(\nabla\wedge)} \deg(c) = 1701 * 17001600$$
$$= \log_{10} 34.54530900 \tag{7}$$

(8) The first multiplicative Zagreb index:

$$\prod\nolimits_1(\nabla\wedge) = \prod_{c\,\in v(\nabla\wedge)} \deg(c)^2 = (1701)^2 * 17001600$$
$$= \log_{10} 7.787656500 \tag{8}$$

(9) The second multiplicative Zagreb index:

$$\prod\nolimits_2(\nabla\wedge) = \prod_{ck\,\in E(\nabla\wedge)} \deg(c)\,\deg(k) = (1701)^2 * 14460370800$$
$$= \log_0 51.1056564090 \tag{9}$$

(10) The Randic index:

$$R(\nabla\wedge) = \sum_{ck\,\in E(\nabla\wedge)} \frac{1}{\deg(c)\deg(\mathrm{k})} = \frac{1}{(1701)^2} * 14460370800$$
$$= 4997.70713 \tag{10}$$

(11) The reciprocal Randic index:

$$RR(\nabla\wedge) = \sum_{ck\,\in E(\nabla\wedge)} \sqrt{\deg(c)\,\deg(k)} = 1701 * 14460370800$$
$$= 245970907308000 \tag{11}$$

(12) The reduced reciprocal Randic Index:

$$RRR(\nabla\wedge) = \sum_{ck\,\in E(\nabla\wedge)} \sqrt{[\deg(c) - 1][\deg(k) - 1]}$$
$$= 1700 * 14460370800 = 245826303608000 \tag{12}$$

(13) The ABC-index or Atom-Bond-Connectivity index:

$$ABC(\nabla\wedge) = \sum_{ck\in E(\nabla\wedge)} \sqrt{\frac{\deg(c)+\deg(k)-2}{\deg(c)\deg(k)}}$$

$$= \sqrt{\frac{3400}{2893401}} * 14460370800 = 495695041.25 \tag{13}$$

(14) The harmonic index:

$$H(\nabla\wedge) = \sum_{ck\in E(\nabla\wedge)} \frac{2}{\deg(c)+\deg(\mathrm{k})} = \frac{2}{2(1701)} * 14460370800$$

$$= 8501099.82 \tag{14}$$

(15) The augmented Zagreb index:

$$AZI(\nabla\wedge) = \sum_{ck\in E(\nabla\wedge)} \left(\frac{\deg(c)\deg(k)}{\deg(c)+\deg(k)-2}\right)^3$$

$$= \left(\frac{2893401}{3400}\right)^3 * 14460370800 = 8911549889 \tag{15}$$

(16) The geometric-arithmetic index:

$$GA(\nabla\wedge) = \sum_{ck\in E(\nabla\wedge)} \frac{2\sqrt{\deg(c)\deg(k)}}{\deg(c)+\deg(k)}$$

$$= \frac{2\sqrt{(1701)^2}}{2(1701)} * 1446037080014460370800 \tag{16}$$

(17) The sum-connectivity index:

$$SCI(\nabla\wedge) = \sum_{ck\in E(\nabla\wedge)} \frac{1}{\sqrt{\deg(c)+\deg(k)}}$$

$$= \frac{1}{58.326} * 14460370800 = 247920406.237 \tag{17}$$

2.2 *Computing DISI of $\nabla\wedge$*

Here we outline the calculation of the DISI for $\nabla\wedge$:

(1) Wiener index:

$$W(\nabla\wedge) = \sum_{\{c,k\}\subseteq V(\nabla\wedge)} d(c,k) = \frac{v}{2}(\mathrm{T}D(c)) = 8500800$$

$$(1701 + 3359934 + 45959703 + 120) = 67848778166400 \tag{18}$$

(2) Wiener polarity index:

$$Wp(\nabla\wedge) = d\big((\nabla\wedge), 3\big) = 1445986000 * 15319901$$

$$= 22155156777786000 \tag{19}$$

(3) The terminal Wiener index:

$$TW(\nabla\wedge) = \sum_{\{c,k\}\subseteq V1(\nabla\wedge)} d(c,k) = 0 \tag{20}$$

(4) The hyper Wiener index:

$$WW(\nabla\wedge) = \frac{1}{2}\sum_{ck\subseteq V\nabla\wedge}[d(c,k) + d(c,k)^2]$$

$$= \frac{v}{4}([1 * \rho_1(c) + 2 * \rho_2(c) + 3 * \rho_3(c) + 4 * \rho_4(c)]$$

$$+[1^2 * \rho_1(c) + 2^2 * \rho_2(c) + 3^2 * \rho_3(c) + 4^2 * \rho_4(c)])$$

$$= 121402543046400 \quad (21)$$

(5) The reciprocal complementary Wiener index:

$$RCW(\nabla\wedge) = \sum_{\{c,k\}\subseteq V(\nabla\wedge)} \frac{1}{diam((\nabla\wedge))-1+d(c,k)}$$

$$= \frac{v}{2}\left(\frac{\rho_1(c)}{4} + \frac{\rho_2(c)}{5} + \frac{\rho_3(c)}{6} + \frac{\rho_4(c)}{7}\right) = 5041500000000 \quad (22)$$

(6) The old Harary index:

$$Hold(\nabla\wedge) = \sum_{\{c,k\}\subseteq V(\nabla\wedge)} \frac{1}{d(c,k)^2} = \frac{v}{2}\left(\frac{\rho_1(c)}{1^2} + \frac{\rho_2(c)}{2^2} + \frac{\rho_3(c)}{3^2} + \frac{\rho_4(c)}{4^2}\right)$$

$$= 5039229492733300 \quad (23)$$

(7) The Harary index:

$$H(\nabla\wedge) = \sum_{\{c,k\}\subseteq V(\nabla\wedge)} \frac{1}{d(c,k)} = \frac{v}{2}\left(\frac{\rho_1(c)}{1} + \frac{\rho_2(c)}{2} + \frac{\rho_3(c)}{3} + \frac{\rho_4(c)}{4}\right)$$

$$= 50565500000000 \quad (24)$$

(8) The Schultz index or degree distance index:

$$DD(\nabla\wedge) = \sum_{c\neq k}(\deg(c) + \deg(k))d(c,k)$$

$$= \frac{17001600}{2}(3402)(\mathrm{T}D(c)) = 230821543322092800 \quad (25)$$

(9) Gutman index:

$$Gut(\nabla\wedge) = \sum_{c\neq k} \deg(c)\deg(k)d(c,k) = \frac{17001600}{2}(2893401)(\mathrm{T}D(c))$$

$$= 196313722595439926400 \quad (26)$$

(10) The additively weighted Harary index or reciprocal degree distance index:

$$H_A(\nabla\wedge) = \sum_{c\neq k} \frac{(\deg(c)+\deg(k))}{d(c,k)} = \frac{17001600}{2}(3402)$$

$$\left(\frac{\rho_1(c)}{1} + \frac{\rho_2(c)}{2} + \frac{\rho_3(c)}{3} + \frac{\rho_4(c)}{4}\right) = 172023912616587296 \quad (27)$$

(11) Multiplicatively weighted Harary index:

$$H_M(\nabla\wedge) = \sum_{c\neq k} \frac{(\deg(c)\deg(k))}{d(c,k)} = \frac{17001600}{2}(1701)^2$$

$$\left(\frac{\rho_1(c)}{1} + \frac{\rho_2(c)}{2} + \frac{\rho_3(c)}{3} + \frac{\rho_4(c)}{4}\right) = 145640308643669582346 6.4 \quad (28)$$

2.3 *Computing ECCI of $\nabla\wedge$*

Here we outline the calculation of the ECCI for $\nabla\wedge$:

(1) Total eccentricity:

$$\xi(\nabla\wedge) = \sum_{c\in V(\nabla\wedge)} e(c) = 4 * 17001600 = 68006400 \quad (29)$$

(2) The first Zagreb eccentricity index:

$$E_1(\nabla\wedge) = \sum_{c\in V(\nabla\wedge)} (e(c))^2 = 4^2 * 17001600 = 272025600 \quad (30)$$

(3) The second Zagreb eccentricity index:

$$E_2(\nabla\wedge) = \sum_{ck\in E(\nabla\wedge)} e(c)e(k) = 4^2 * 14460370800 = 231365932800$$

(31)

(4) The eccentric connectivity:

$$\xi^c(\nabla\wedge) = \sum_{c\,\in V(\nabla\wedge)} e(c)\deg(c) = 4 * 1701 * 17001600$$
$$= 115678886400 \tag{32}$$

(5) The connective eccentricity index:

$$C^{\xi}(\nabla\wedge) = \sum_{c\,\in V(\nabla\wedge)} \frac{\deg(c)}{e(c)} = \frac{1701}{4} * 17001600 = 7229930400 \tag{33}$$

(6) The eccentric distance sum index:

$$\xi^d(\nabla\wedge) = \sum_{c\,\in V(\nabla\wedge)} e(c)D(c) = 4v(\mathrm{T}D(c)) = 3354174801331200 \tag{34}$$

(7) The adjacent eccentric distance sum:

$$\xi^{sv}(\nabla\wedge) = \sum_{c\,\in V(\nabla\wedge)} \frac{e(c)D(c)}{\deg(c)} = 17001600 * \frac{4}{1701}\left(\mathrm{T}D(c)\right)$$
$$= 1341600000000000000 \tag{35}$$

Corollary 1.

1- The Hosoya polynomial of $\nabla\wedge$ is $\{17001600+1701b + 1679967\,b^2 + 15319901b^3 + 30\,b^4\}$.

2- The Schultz polynomial of $\nabla\wedge$ is $\{1701\,|3B|\,|(H(\nabla\wedge, b) - |3B|))\}$.

Proof: Let $c \in V(\nabla\wedge)$, then

$$\mathcal{H}(\nabla\wedge, b) = \sum_{y=0}^{diam(\nabla_\Lambda)} d(\nabla_\Lambda, y)b^y = \sum_{y=0}^{4} d(\nabla_\Lambda, y)b^y$$
$$= 17001600+\rho_1(c)\,b + \rho_2(c)\,b^2 + \rho_3(c)\,b^3$$
$$= 17001600+1701b + 1679967\,b^2 + 15319901b^3 + 30\,b^4.$$

Moreover,

$$\mathfrak{S}c(\nabla\wedge, b) = \sum_{\mu\neq\gamma\in 3A}(deg(c) + de\,g(k))b^{d(c,k)}$$
$$=2 * 1701 \sum_{c\neq k\in V(\nabla\wedge)} b^{d(c,k))}$$
$$= 2 * 1701\,\frac{|3B|}{2}\,(\rho_1(c)\,b + \rho_2(c)\,b^2 + \rho_3(c)\,b^3 + \rho_4(c)\,b^4)$$
$$=1701\,|3B|\,(1701b + 1679967\,b^2 + 15319901b^3 + 30\,b^4)$$
$$= 1701\,|3B|\,|(H(\nabla\wedge, b) - |3B|)$$

3. Conclusion

We compute an extensive topological descriptor of symmetric A_4-graphs resulting from Co_3. By illustrating this graph through molecular topology, we obtain a vital connectivity index along with calculating the Hosoya and Schultz polynomials as expanding functions from the distance and the degree parameters of the graph. Our major study includes explicit formulations that link these descriptors to group parameters, such as the size of the conjugacy class and the distribution of vertex degrees. Consequently, the underlying algebraic structure of the group directly determines the metric architecture of the graphs.

References

[1] R. Durgut and T. Turaci, "Computing Some Eccentricity-Based Topological Indices of Para-Line Graphs of Hexagonal Cactus Chains," *Polycycl. Aromat. Compd.*, vol. 43, no. 1, pp. 115–130 (2021), doi: 10.1080/10406638.2021.2011330.

[2] H. Shooshtari, S. M. Sheikholeslami, and M. Cancan, "On the topological indices and graph operation," *Discrete Math. Algorithms Appl.*, vol. 17, no. 03, p. 11 (2024), doi: 10.1142/s1793830924500472.

[3] A. Aytaç and B. Coşkun, "Eccentric connectivity index in complementary prisms," *Discrete Mathematics, Algorithms and Applications*, vol. 17, no. 5, Art. no. 2450083 (2024), doi: 10.1142/S1793830924500836.

[4] S. Rajendran, R. R. Iyer, A. Asiri, and K. Somasundaram, "Detour Eccentric Sum Index for QSPR Modeling in Molecular Structures," *Symmetry (Basel)*, vol. 17, no. 11, p. 1897 (2025), doi: 10.3390/sym17111897.

[5] F. Deng and T. Wu, "Algorithms for Two Types of Topological Indices," *Algorithms*, vol. 18, no. 11, p. 673 (2025), doi: 10.3390/a18110673.

[6] P. Nathiya, M. Suresh, and J. H. H. Bayati, "Topological Indices and Quantitative Structure-Property Analysis of Novel Drugs for Acute Lymphoblastic Leukemia Treatment," *Baghdad Science Journal*, vol. 22, no. 8, pp. 2723–2737 (2025), doi: 10.21123/2411-7986.5033.

[7] J. H. H. Bayati, A. Mahboob, L. Amin, M. W. Rasheed, and A. Alameri, "Predictive modeling of asthma drug properties using machine learning and topological indices in a MATLAB based QSPR study," *Scientific Reports*, vol. 15, Art. no. 30373 (2025), doi: 10.1038/s41598-025-07022-5.

[8] R. Li and Y. Zhang, "On the sum of the second Zagreb index and its reciprocal index for bicyclic graphs," *Discrete Appl. Math.*, vol. 380, no. 01, pp. 612–618 (2026), doi: 10.1016/j.dam.2025.11.009.

[9] N. A. Abd Aziz,"A note on graphs with integer Sombor index," *Communications in Combinatorics and Optimization*, vol. 11, no. 1, pp. 205–209 (2026), doi: 10.22049/cco.2024.29474.2013.

[10] N. A. Supu, I. Muchtadi-Alamsyah, and E. Suwastika, "Topological Indices of Relative g-noncommuting Graph of Dihedral Groups," *Journal of the Indonesian Mathematical Society*, vol. 29, no. 03, pp. 271–288 (2023), doi: 10.22342/jims.29.3.1594.271-288.

[11] R. Ismail, F. Ali, R. Qasim, M. Naeem, W. K. Mashwani, and S. Khan, "Several Zagreb indices of power graphs of finite non-abelian

groups," *Heliyon*, vol. 9, no. 9, Art. no. e19560 (2023), doi: 10.1016/j.heliyon.2023.e19560.

[12] M. S. Makki, "Topological Indices of Inverse Graph for Generalized Quaternion Group," *International Journal of Analysis and Applications,* vol. 22, p. 226 (2024), doi: 10.28924/2291-8639-22-2024-226.

[13] P. Rana, A. Sehgal, P. Bhatia, and P. Kumar, "Topological Indices and Structural Properties of Cubic Power Graph of Dihedral Group," *Contemporary Mathematics*, vol. 5, no. 1, pp. 761–779 (2024), doi: 10.37256/cm.5120243029.

[14] M. M. Iftekhan and A. A. Aubad, "Some parameters for the resize graph of G2(4)," *Journal of Discrete Mathematical Sciences and Cryptography,* vol. 28, no. 4-B, pp. 1375–1383 (2025), doi: 10.47974/jdmsc-2277.

[15] R. Binthiya, "Some new topological indices on unitary cayley quotient graphs," *Int. J. Appl. Math.*, vol. 38, no. 4s, pp. 519–524 (2025), doi: 10.12732/ijam.v38i4s.255.

[16] A. A. Aubad, "A4-Graph of Finite Simple Groups," *Iraqi Journal of Science,* vol. 62, no. 01, pp. 289–294 (2021), doi: 10.24996/ijs.2021.62.1.27.

[17] Z. H. Msheree, M. M. Abed, and W. F. Abid," Studying the A4-Graphs for elements of order 3 in tits group T and Mathieu group M20," *International Journal of Nonlinear Analysis and Applications,* vol. 12, no. 02, pp. 1855–1860 (2021), doi: 10.22075/IJNAA.2021.5321.

[18] S. M. Kasim, S. C. Soh, and S. N. A. M. Aslam, "Construction of the A4-Graph of Mathieu Group M11," *Mathematics and Statistics*, vol. 11, no. 1, pp. 179–182 (2023), doi: 10.13189/ms.2023.110120.

[19] The GAP Group, *GAP – Groups, Algorithms, and Programming*, ver. 4.15.1 (2025). [Online]. Available: https://www.gap-system.org. [Accessed: May 12, 2026].

[20] A. T. Jawad and A. A. Aubad, "Studying the result involution graphs for HS and McL leech lattice groups," *Journal of Interdisciplinary Mathematics*, vol. 26, no. 5, pp. 829–834 (2023), doi: 10.47974/JIM-1503.

Received January, 2026