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Wiener index of C_6C_4 mesh topology

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Abstract

In graph theory, there are many quantitative values that are usually graph invariant and one such invariant that characterizes the structure or topology of a graph is called as topological index. Wiener index is one of the significant indices that quantifies the total of all shortest-path distances in the topology of a graph. After gaining recognition in the mid 20th century, its applications continue range across various fields including network theory, chemical graph theory, social network analysis and combinatorics. Here, we analyse and determine the Wiener index for the $C_6\,C_4$ mesh topology-a combination of graphs constructed by interconnecting cycles of order 6 (C_6) and order 4 (C_4) in a mesh-like arrangement. We present unique formulas for the Wiener index of $C_6\,C_4$ meshes, obtained from the wirelength of their respective embeddings.

Subject Classification (2020): *05C12, 05C38, 05C76, 05C85, 05C90.*

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1. Introduction

Graph theory is an essential interdisciplinary tool for many scientific fields such as network theory, computer science, relational data structures, biological, social networks and algorithm optimization. In this paper, graphs are used to model molecular structures in chemistry, where each vertex stands for an atom and each edge denotes a chemical bond. The physical and chemical characteristics of the molecules can be systematically predicted using topological indices, which are derived from these graph representations. This topological index, known as the Wiener index, sums the distances between all pairs of vertices in the graph [1, 2].

In 1947, chemist Harold Wiener proposed the Wiener index, sometimes referred to as the path number, which is considered among the earliest topological indices for analyzing molecular branching. As a significant graph invariant, it calculates the overall sum of the minimum path lengths connecting every pair of vertices in the graph. Initially, it was used to determine the thermodynamic properties including boiling points and molecular weights of alkanes, but its use and other such graph descriptors has since broadened successfully to include network theory and computational science due to its effectiveness in assessing connectivity efficiency within networks. The Wiener index and other related topological indices have been studied for several classes of graphs, including windmill graphs, mk-graphs, and polyomino chains [3, 4, 5]. These indices, based on the distance matrix of a graph, have found extensive use in various chemical applications, including drug design and materials science [6].

The Wiener index is frequently employed to estimate different characteristics of alkanes, including molar volumes, heats of vaporization, boiling points, and molar refractions [7]. John and Diudea [8] has extended the application of topological indices to nanostructure research, sparking further investigations into methods for calculating the Wiener index of chemical compounds [9], including the use of matrix distances for direct computation [10].

In theoretical computer science, the Wiener index has come out as a critical metric for analysing interconnection networks, offering insights on the average distance between nodes. In unweighted graphs, the wiener index is calculated directly by using algorithms including the breadth-first search algorithm whereas in weighted graphs, Floyd-Warshall's algorithm or Johnson's algorithm may be used. With the help of these algorithms, the Wiener index is found by computing all pairwise distances in a graph [11].

The C_6C_4 mesh topology, which combines hexagonal (C_6) and square (C_4) tiling patterns, offers a unique balance between connectivity, path diversity, and structural regularity. Unlike pure square or hexagonal meshes, the C_6C_4 structure enhances routing flexibility while maintaining efficient node placement and reduced communication distance. This hybrid pattern can lead to lower congestion and improved fault tolerance, making it especially valuable for parallel architectures and network-on-chip (NoC) designs. In comparison to traditional tiling patterns, such as square-only grids (which may suffer from higher average distances) or hexagonal meshes (which may be harder to embed in rectangular hardware layouts), the C_6C_4 mesh provides a practical compromise that supports efficient embedding, better load distribution, and scalable interconnection frameworks [11].

Various tiling patterns have been studied in terms of their Wiener index to evaluate their efficiency as network topologies. Square grids (C_4) generally yield higher Wiener index values due to longer average distances between nodes. Hexagonal meshes (C_6) offer slightly better communication efficiency by reducing average path lengths but can be more complex to implement in rectangular layouts. Triangular tilings tend to provide dense connectivity but may result in irregular node placement. In contrast, the C_6C_4 mesh topology, by combining the benefits of both square and hexagonal tilings, achieves a more balanced structure with improved distance metrics. In this paper, we determines the Wiener index of the C_6C_4 mesh topology. This index highlights its effectiveness in minimizing total communication cost, making it a promising candidate for scalable and low-latency network designs [12].

In Section 2, we review the mathematical preliminaries and available literature on the Wiener index and graph invariants for composite topologies. Section 3 explains in detail the construction of the C_6C_4 mesh topology and the computation of the Wiener index of the C_6C_4 mesh topology. Finally, in Section 4, we discuss the inferences of our results and propose directions for future research.

2. Basic concepts

Consider a simple graph $A = (V, E)$, where $V(A)$ and $E(A)$ represent its vertices and edges. The distance $d_A(x, y)$ between two vertices $x, y \in V(A)$ is the length of the shortest path joining them. The *Wiener index* $W(A)$ of graph A is given by [14, 15]:

$$W(A) = \frac{1}{2}\sum_{\{x,y\}\subseteq V(A)} d_A(x, y),$$

where the summation extends over all distinct pairs of vertices in $V(A)$. Alternatively, if $V(A)$ is linearly ordered, the Wiener index can be expressed as:

$$W(A) = \sum_{x<y,\{x,y\}\subseteq V(A)} d_A(x, y).$$

The Wiener index measures the aggregate distance between all pairs of vertices, making it an essential metric for evaluating communication costs within a network. This property makes it particularly useful for analyzing the efficiency of molecular structures and representing computer network systems. In the context of lattice-based hardware architectures, a lower Wiener index implies shorter overall communication paths, which can enhance performance and reduce latency. This is especially valuable in cryptographic hardware, where efficient and uniform communication helps mitigate timing and power-based side-channel attacks, thereby contributing to more secure hardware design.

Graph embedding is a beneficial technique for deploying parallel algorithms and simulations in network design, where multiple computational tasks are executed simultaneously across processors, and efficient mapping reduces communication cost and enhances overall performance.

An embedding [16] incorporates the guest graph A into the host graph B using an injective function $f: V(A) \to V(B)$. This is accompanied by a mapping P_f that assigns a path $P_f((xy))$ in B to each edge (xy) in $E(A)$, ensuring the path connects $f(x)$ and $f(y)$ in B.

The congestion of a mapping f that embeds guest graph A into host graph B is defined as the maximum number of edges in A whose corresponding paths in B share a common edge. Let $C_f(A, B, e)$ denote the number of edges $(xy) \in E(A)$ for which the path $P_f((xy))$ in B passes through the edge e. Mathematically

$$C_f(A, B, e) = |\{(xy) \in E(A): e \in P_f((xy))\}|.$$

For a subset $S \subseteq E(B)$, the total congestion is the cumulative congestion over all edges in S:

$$C_f(A, B(S)) = \sum_{e \in S} C_f(A, B, e).$$

The total congestion over all edges of A is measured by the *wirelength* of the embedding f.

$$WL_f(A, B) = \sum_{e \in E(A)} C_f(A, B, e).$$

The wirelength of graph A embedded into graph B is defined as the minimum wirelength over all possible embeddings [17]:

$$WL(A, B) = \min WL_f(A, B).$$

Lemma 2.1: [15] *Let A be a finite graph, and let $\{S_1, S_2, \dots, S_t\}$ denote a partition of its edge set $E(A)$. For each S_i, the set constitutes an edge cut whose removal separates A into two disjoint subgraphs, A_i^1 and A_i^2. The sets S_i must satisfy the following conditions:*

- *For $x, y \in A_i^1$, the shortest path between x and y does not pass through any edges in S_i.*
- *Similarly, for $x, y \in A_i^2$, the shortest path between x and y avoids all edges in S_i.*
- *For $x \in A_i^1$ and $y \in A_i^2$, the shortest path between x and y includes exactly one edge from S_i.*

Under these conditions, the Wiener index of A can be represented as:

$$W(A) = \sum_{i=1}^{t} |V(A_i^1)| \times (n - |V(A_i^1)|).$$

3. Computing the Wiener Index

Recent research has probed the connection between the graph invariant Wiener index, denoted as $W(B)$, and the wirelength of graph embeddings, represented as $WL(A, B)$. Kumar et al. [19] investigated this connection, specifically when A is a complete bipartite graph and B is formed by the

Cartesian product of graphs. Expanding on this work, Nandini et al. [20] precisely determined the wirelength for embedding Cartesian products of complete graphs into structures created by the Cartesian product of paths, cycles. Their study also included an analysis of the Wiener index of generalized books and its correlation with embedding wirelengths. This current study further extends this line of research by considering A as a complete graph and B as a mesh topology denoted as $C_6C_4(n,n)$. Here, we calculate the Wiener index using the wirelength, providing new insights on the relationship between these two graph properties.

3.1 C_6C_4 *mesh topology*

The C_6C_4 mesh topology is represented by a graph $A=(V,E)$, where V and E correspond to the vertex set and edge set, respectively. It is constructed as follows:

- Arrange cycles C_6 (hexagonal graphs) and C_4 (square graphs) in a two-dimensional grid, alternating in a checkerboard pattern.
- Each C_6 consists of 6 vertices $x_1,x_2,\dots,x_6$, and each C_4 consists of 4 vertices y_1,y_2,y_3,y_4, with edges forming closed cycles.
- Adjacent C_6 and C_4 cycles are interconnected through edges between specific vertices such that the entire graph forms a single connected component.
- The topology can be designed with either open boundaries (finite grid) or wrapped boundaries (toroidal grid), depending on the desired application.

The mesh topology is a structured interconnection network characterized by several notable graph-theoretic properties. It consists of $6n^2$ vertices and $10n^2-4n$ edges, forming a complex yet organized framework. The diameter of the topology is $5n-2$, indicating a relatively short maximum distance between any two nodes, which is beneficial for efficient communication. While the topology is non-regular meaning not all vertices have the same degreeâ€"it maintains a connectivity of 2, ensuring fault tolerance as the network remains connected even after the failure of one node or edge. Furthermore, the topology is Hamiltonian, allowing the construction of Hamiltonian cycles for efficient traversal. However, it is neither bipartite nor pancyclic, implying limitations in certain cycle-based algorithms. Despite this, the topology is Eulerian, as all its vertices have even degrees, enabling the traversal of each edge exactly once without repetition.

Due to these properties, the mesh is particularly well-suited for parallel computing systems, VLSI design, and network-on-chip (NoC) architectures. Its moderate diameter and Eulerian nature support efficient routing and data transfer, while Hamiltonicity ensures complete cycle-based communication protocols. The network's structural robustness and redundancy make it an attractive choice in scenarios requiring high performance and fault-tolerant communication frameworks [13].

These emerging applications highlight the versatility of the Wiener index as a graph-based metric across interdisciplinary fields. In robotics, for example, environments are often modeled as discrete grids or graphs where minimizing total travel distance is crucial for efficient navigation and energy conservation. The Wiener index helps evaluate how well an environment supports optimal routing for multiple agents [18, 21]. In computational linguistics, syntactic structures and semantic networks are analyzed using graph theory, where the Wiener index can be used to measure structural complexity or coherence in language models [22]. Similarly, in blockchain and peer-to-peer systems, where decentralized nodes must communicate effectively, the Wiener index serves as a metric for analyzing propagation delays and optimizing transaction pathways [23]. These examples demonstrate that the Wiener index is not only a theoretical tool but also a practical one, applicable to real-world systems that rely on structured, efficient communication. For illustration the mesh topology $C_6C_4(3,3)$ and $C_6C_4(4,4)$ is given in Fig. 1.

This topology is a multidimensional structure that combines cycles and a mesh, making it suitable for certain network architectures and parallel computing applications.

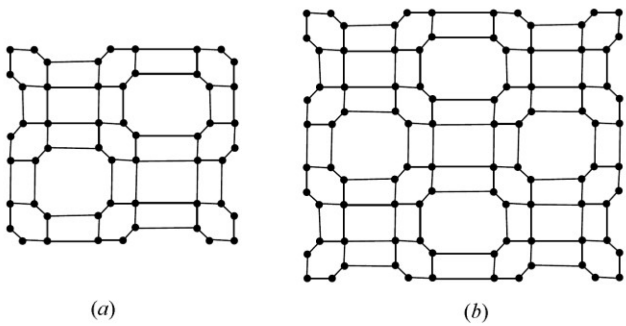

Figure 1
(*a*) $C_6C_4(3,3)$ (*b*) $C_6C_4(4,4)$

3.2 *Computing Wiener index via wirelength*

Using the existing techniques in graph theory and embedding, the Wiener index of the C_6C_4 topology can be shown the Wiener index $W(A) = WL(K_n, A)$, where $|V(A)| = n$ and K_n is a complete graph of n vertices.

Theorem 3.1: *Let A be a $C_6C_4(n,n)$ graph, where $n \geq 2$ and n is an even number. Then the Wiener index of A is shown as*

$$W(A) = WL(K_n, A) = \frac{6}{5}n(27n^4 - 5n^2 - 7).$$

Proof: Let K_n be a complete graph, A be a mesh topology $C_6C_4(n,n)$ and we have the following edge cuts T_l, S_l, U_l, W_l, see Figure 2. Consider $T_l, 1 \leq l \leq 2n-1$ be a horizontal cut in $C_6C_4(n,n), n \geq 2$ which divides A into two connected components, A_l^1 and A_l^2, where the vertices of $A_l^1 = l \times 3n$ and vertices of $A_l^2 = 6n^2 - l \times 3n$. Let $B_l^1 = K_n[f^{-1}(V(A_l^1))]$ and $B_l^2 = K_n[f^{-1}(V(A_l^2))]$. Clearly, B_l^1 is optimal set in K_n.

Consider $S_l, 1 \le l \le 2n-1$ be a acute cut in $C_6C_4(n,n), n \ge 2$, which divides A into two connected components, A_l^1 and A_l^2, where the vertices of $A_l^1 = l \times 3n$ and vertices of $A_l^2 = 6n^2 - l \times 3n$. Let $B_l^1 = K_n[f^{-1}(V(A_l^1))]$ and $B_l^2 = K_n[f^{-1}(V(A_l^2))]$. Clearly, B_l^1 is optimal set in K_n.

Consider $U_l, 1 \le l \le \frac{n}{2}$ be a vertical cut in $C_6C_4(n,n), n \ge 2$, which divides A into two connected components, A_l^1 and A_l^2, where the vertices of $A_l^1 = 3(2l-1)^2$ and vertices of $A_l^2 = 6n^2 - \times 3(2l-1)^2$. Let $B_l^1 = K_n[f^{-1}(V(A_l^1))]$ and $B_l^2 = K_n[f^{-1}(V(A_l^2))]$. Clearly, B_l^1 is optimal set in K_n.

Consider $W_l, 1 \le l \le \frac{n}{2}$ be a vertical cut in $C_6C_4(n,n), n \ge 2$, which divides A into two connected components, A_l^1 and A_l^2, where the vertices of $A_l^1 = 3(2l-1)^2$ and vertices of $A_l^2 = 6n^2 - \times 3(2l-1)^2$. Let $B_l^1 = K_n[f^{-1}(V(A_l^1))]$ and $B_l^2 = K_n[f^{-1}(V(A_l^2))]$. Clearly, B_l^1 is optimal set in K_n.

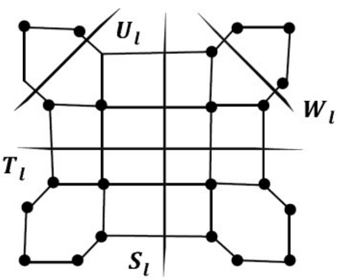

Figure 2
(*a*) $C_6C_4(3,3)$ (*b*) $C_6C_4(4,4)$

Hence, the edge cuts $\{T_l, S_l, U_l, W_l\}$ fulfills the conditions stated in Lemma 2.1. Therefore, $C_f(T_l), C_f(S_l), C_f(U_l), C_f(W_l)$ is minimum and the edge congestions are as follows:

1. $C_f(T_l) = (l \times 3n) \times (6n^2 - l \times 3n)$ *for* $1 \le l \le 2n-1$;
2. $C_f(S_l) = (l \times 3n) \times (6n^2 - l \times 3n)$ *for* $1 \le l \le 2n-1$;
3. $C_f(U_l) = 3(2l-1)^2 \times (6n^2 - 3(2l-1)^2)$ *for* $1 \le l \le \frac{n}{2}$;
4. $C_f(W_l) = 3(2l-1)^2 \times (6n^2 - 3(2l-1)^2)$ *for* $1 \le l \le \frac{n}{2}$.

Then by Lemma 2.1, we have $W(A) = WL(K_n, A)$ and is given by

$$W(A) = \sum_{l=1}^{2n-1} C_f(T_l) + \sum_{l=1}^{2n-1} C_f(S_l) + 2\sum_{l=1}^{\frac{n}{2}} C_f(U_l) + 2\sum_{l=1}^{\frac{n}{2}} C_f(W_l)$$
$$= 2\sum_{l=1}^{2n-1} (l \times 3n) \times (6n^2 - l \times 3n) + 4\sum_{l=1}^{\frac{n}{2}} 3(2l-1)^2$$
$$\times (6n^2 - 3(2l-1)^2)$$

$$= 6n^3(4n^2-1)+\frac{42}{5}n(n^4-1)$$
$$=\frac{6}{5}n(27n^4-5n^2-7).$$

Theorem 3.2: *Let A be a $C_6C_4(n,n)$ graph, where $n \geq 2$ and n is an odd number. Then the Wiener index of A is expressed as*

$$W(A)=WL(K_n,A)=\frac{3}{16}(169n^5-66n^4+34n^3-12n^2+13n+6).$$

Proof: Let K_n be a complete graph, A be a mesh topology $C_6C_4(n,n)$ and we have the following edge cuts T_l, S_l, U_l, W_l, see Figure 2. Consider $T_l, 1 \leq l \leq 2n-1$ be a horizontal cut in $C_6C_4(n,n), n \geq 2$ which divides A into two connected components, A_l^1 and A_l^2, where the vertices of $A_l^1 = l \times 3n$ and vertices of $A_l^2 = 6n^2 - l \times 3n$. Let $B_l^1 = K_n[f^{-1}(V(A_l^1))]$ and $B_l^2 = K_n[f^{-1}(V(A_l^2))]$. Clearly, B_l^1 is optimal set in K_n.

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Consider $U_l, 1 \leq l \leq \frac{n-1}{2} 1 \leq l \leq \frac{n+1}{2}$ be a vertical cut in $C_6C_4(n,n), n \geq 2$ which divides A into two connected components, A_l^1 and A_l^2, where the vertices of $A_l^1 = 3(2l-1)^2$ and vertices of $A_l^2 = 6n^2 - \times 3(2l-1)^2$. Let $B_l^1 = K_n[f^{-1}(V(A_l^1))]$ and $B_l^2 = K_n[f^{-1}(V(A_l^2))]$. Clearly, B_l^1 is optimal set in K_n.

Consider $W_l, 1 \leq l \leq \frac{n-1}{2}$ be a vertical cut in $C_6C_4(n,n), n \geq 2$, which divides A into two connected components, A_l^1 and A_l^2, where the vertices of $A_l^1 = 3(l-1)^2$ and vertices of $A_l^2 = 6n^2 - \times 3(l-1)^2$. Let $B_l^1 = K_n[f^{-1}(V(A_l^1))]$ and $B_l^2 = K_n[f^{-1}(V(A_l^2))]$. Clearly, B_l^1 is optimal set in K_n.

Hence, the edge cuts $\{T_l, S_l, U_l, W_l\}$ fulfills the conditions stated in Lemma 2.1. Therefore, $C_f(T_l), C_f(S_l), C_f(U_l), C_f(W_l)$ is minimum and the edge congestions are as follows:

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3. $C_f(U_l) = 3(2l-1)^2 \times (6n^2 - 3(2l-1)^2)$ *for* $1 \leq l \leq \frac{n-1}{2} 1 \leq l \leq \frac{n+1}{2}$;
4. $C_f(W_l) = 3(2l-1)^2 \times (6n^2 - 3(2l-1)^2)$ *for* $1 \leq l \leq \frac{n-1}{2}$.

Then, by Lemma 2.1, we have $W(A) = WL(K_n, A)$ and is given by

$$W(A) = \sum_{l=1}^{2n-1} C_f(T_l) + \sum_{l=1}^{2n-1} C_f(S_l) + 2\sum_{l=1}^{\frac{n-1}{2}} C_f(U_l) + \sum_{l=1}^{\frac{n+1}{2}} C_f(U_l)$$
$$+2\sum_{l=1}^{\frac{n-1}{2}} C_f(W_l)$$

$$= 2\sum_{l=1}^{2n-1} (l \times 3n) \times (6n^2 - l \times 3n) + 2\sum_{l=1}^{\frac{n-1}{2}} 3(2l-1)^2$$
$$\times (6n^2 - 3(2l-1)^2) + \sum_{l=1}^{\frac{n+1}{2}} 3(2l-1)^2 \times (6n^2 - 3(2l-1)^2)$$
$$+2\sum_{l=1}^{\frac{n-1}{2}} 3(l-1)^2 \times (6n^2 - 3(l-1)^2)$$
$$= 6n^3(4n^2-1) + \frac{3}{5}n(7n^4 - 15n^3 + 8) + \frac{3}{10}n(7n^4 + 15n^3 + 8)$$
$$+\frac{3}{80}(37n^5 - 210n^4 + 330n^3 - 60n^2 - 127n + 30)$$
$$= \frac{3}{16}(169n^5 - 66n^4 + 34n^3 - 12n^2 + 13n + 6).$$

4. Concluding Remarks

This paper studies in detail the Wiener index for the C_6C_4 mesh topology. The formulas of the same highlights the fact that this topology is more efficient and scalable, making it a great option for applications in communication networks and computational modeling. In the future, it is planned to extend these results to other hybrid graph structures and exploring dynamic behaviors in C_6C_4 meshes under different conditions.

Open Problem:

Finding the Wiener index for the $C_6C_4(m,n)$ mesh topology for any arbitrary m,n remains a unsolved challenge.

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