

Some properties of k -even mean graph

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Abstract

In this Paper we establish proved several properties related to k – EMG. Properties are very much useful for some cycle related graphs.

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1. Introduction

For a detailed survey, refer to [1]. The concept of Mean labeling of graphs has been explored and analyzed in [2,3,4].

2. Observation

If $s(u)$ and $s(v)$ be the labels of two adjacent vertices u and v of a k – even mean graph then $s^*(uv) = \left\lceil \frac{s(u)+s(v)}{2} \right\rceil$

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Illustration: We present below some examples for the clear understanding of the concept of different kinds of mean labeling.

$K_{1,3}$ is a 2-EMG but not a 1-EMG of shown in Figure 1.

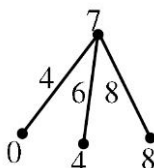


Figure 1
2-even mean labeling of $K_{1,3}$

$K_{1,4}$ is a 3-EMG but not a 2-EMG of shown in Figure 2.

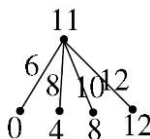


Figure 2
2-even mean labeling of $K_{1,4}$

3. Main Results

Lemma 3.1: In every k -EMG, $G = (a, b)$, the numbers $2k + 2b - 3$ and $2k + 2b - 2$ must occur as labels of the adjacent vertices.

Proof: The edge label $2k + 2b - 2$ is possible only when the vertices with labels $2k + 2b - 3$ and $2k + 2b - 2$ occur and are adjacent.

Lemma 3.2: In every k -EMG, $G = (a, b)$, $2k + 2b - 3$ and $2k + 2b - 2$ cannot be the labels of vertices of the same Δ contained in G .

Proof: Let a_0, a_1 and a_2 be the vertices. If a_0 and a_1 are labeled with $2k + 2b - 3$ and $2k + 2b - 2$ respectively and a_2 is labeled with some x where $0 \leq x \leq 2k + 2b - 2$, then clearly $x \neq 2k + 2b - 3$ and $x \neq 2k + 2b - 2$ since G is a k -EMG. Now these vertex labels will induce the edge labels in the following cases.

Case 1: x even

In this case, the edge labels are $\frac{x+2k+2b-2}{2}, 2k + 2b - 2, \frac{x+2k+2b-2}{2}$ which are not distinct.

Case 1: x odd

In this case, the edge labels are

$$\frac{x+2k+2b-3}{2}, 2k+2b-2, \frac{x+2k+2b-1}{2} \quad \text{---(1)}$$

Subcase 1: k odd, b even or k even, b odd

Then $k+b$ is odd and $2(k+b) \equiv 2(\text{mod } 4)$

$$\begin{aligned} &\Rightarrow 2k+2b \equiv 2(\text{mod } 4) \\ &\Rightarrow 2k+2b-1 \equiv 1(\text{mod } 4) \end{aligned} \quad \text{---(2)}$$

As x is odd, $x \equiv 1$ or $3(\text{mod } 4)$

Subcase a: $x \equiv 1(\text{mod } 4)$ ---(3)

From (2) and (3)

$$\begin{aligned} x+2k+2b-1 &\equiv 2(\text{mod } 4) \\ \frac{x+2k+2b-1}{2} &\equiv 1 \text{ or } 3(\text{mod } 4) \end{aligned} \quad \text{---(4)}$$

Therefore, $\frac{x+2k+2b-1}{2}$ is odd

a $\Rightarrow \Leftarrow$ to G is a $k-EMG$.

Subcase b: $x \equiv 3(\text{mod } 4)$ ---(5)

From (2)

$$\begin{aligned} x+2k+2b-1 &\equiv 1(\text{mod } 4) \\ &\Rightarrow 2k+2b-3 \equiv 3(\text{mod } 4) \end{aligned} \quad \text{---(6)}$$

From (5) and (6),

$$\begin{aligned} x+2k+2b-3 &\equiv 6(\text{mod } 4) \equiv 2(\text{mod } 4) \\ &\Rightarrow \frac{x+2k+2b-3}{2} \equiv 1 \text{ or } 3(\text{mod } 4) \end{aligned}$$

Therefore, $\frac{x+2k+2b-3}{2}$ is odd

a $\Rightarrow \Leftarrow$ to G is a $k-EMG$.

Subcase 2: k even, b even or k odd, b odd

Then $k+b$ is even and $2(k+b) \equiv 0(\text{mod } 4)$

$$\begin{aligned} &\Rightarrow 2k+2b \equiv 0(\text{mod } 4) \\ &\Rightarrow 2k+2b-1 \equiv 3(\text{mod } 4) \end{aligned} \quad \text{---(7)}$$

Subcase a: $x \equiv 1(\text{mod } 4)$ ---(8)

From (7), $2k+2b \equiv 0(\text{mod } 4)$ ---(9)

From (8) and (9) $x+2k+2b-3 \equiv 2(\text{mod } 4)$

$$\frac{x+2k+2b-3}{2} \equiv 1 \text{ or } 3(\text{mod } 4)$$

Therefore, $\frac{x+2k+2b-3}{2}$ is odd

a $\Rightarrow \Leftarrow$ to G is a $k-EMG$.

Subcase b: $x \equiv 3 \pmod{4}$ —(10)

From (2)

$$x + 2k + 2b - 1 \equiv 6 \equiv 2 \pmod{4}$$

$$\Rightarrow x + 2k + 2b - 1 \equiv 2 \pmod{4}$$

$$\Rightarrow \frac{x+2k+2b-1}{2} \equiv 1 \text{ or } 3 \pmod{4}$$

Therefore, $\frac{x+2k+2b-1}{2}$ is odd

$a \Rightarrow \Leftarrow$ to G is a $k - EMG$.

Lemma 3.3: In every $k - EMG$, either $2k - 2$ or $2k - 1$ has to be one of the vertex labels.

Proof: The edge labels $2k$ is possible only when the adjacent vertex labels are of the form: $(2k - 2, 2k + 2), (2k - 2, 2k + 1), (2k - 1, 2k + 1), (2k - 1, 2k)$. Thus the vertex label $2k - 2$ or $2k - 1$ is mandatory.

Theorem 3.1: Let G be a $k - EMG$ with $k - EML$ of s . Let ' c ' represent the number of edges connecting a vertex with an even label to a vertex and the other is odd. Then $\sum_{v \in V(G)} r(v)s(v) + c = 2b[2k + b - 1]$ where $r(v)$ denotes the degree of a vertex v .

Proof: w.k.t. $s^*(uv) = \left\lceil \frac{s(u)+s(v)}{2} \right\rceil$

$$\begin{aligned} \text{Now, } \sum r(v)s(v) &= 2 \left[\sum_{xy \in E} s^*(xy) - \frac{c}{2} \right] \\ &= 2[2k + 2k + 2 + \dots + 2k + 2(b - 1)] - c \\ &= 2 \left[\frac{b}{2} \{2k + 2k + 2(b - 1)\} \right] - c \\ &= 2b[2k + b - 1] - c \\ \sum r(v)s(v) + c &= 2b[2k + b - 1] \end{aligned}$$

Theorem 3.2: If G is a $k - EMG$ containing a ΔC_3 and one of the vertex label of C_3 is even then the other two vertices cannot hold adjacent numbers as labels

Proof: Let G be a $k - EMG$ containing the $\Delta C_3 = u_1u_2u_3u_1$.

Without loss of generality assume $s(u_1) = x$ is even.

If possible, let u_2 and u_3 hold adjacent numbers as labels say y and $y + 1$.

Case 1: y is even

Then $s^*(c_1) = \frac{x+y}{2}, s^*(c_2) = \frac{x+y+2}{2}$ and $s^*(c_3) = y + 1$ which is odd, a $\Rightarrow \Leftarrow$.

Case 1: y is odd

Then $s^*(c_1) = \frac{x+y+1}{2}$, $s^*(c_2) = \frac{x+y+1}{2}$, $s^*(c_1) = s^*(c_2)$, $a \Rightarrow \Leftarrow$.

Therefore u_2 and u_3 cannot hold adjacent numbers as vertex labels.

References

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