Journal of Discrete Mathematical Sciences & Cryptography
ISSN 0972-0529 (Print), ISSN 2169-0065 (Online)
Vol. 29 (2026), No. 4, pp. 1541–1564
DOI : 10.47974/JDMSC-2226

Predicting reactivity and properties of glass using Revan indices and M-polynomial

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Abstract

The primary use of topological indices (TIs) in chemical sciences lies in estimating the physical and chemical attributes of molecular graphs. The refractive, reflective, and transmission properties of glass has made it suitable for the production of optical lenses, prisms, and optical electronic materials and have wide applications in the optical industry.

In this article, first, Revan indices are obtained using M-polynomial, and their relationships are proved. Afterwards, utilizing these relationships, Revan indices for the molecular structure, Para-line structure, and Subdivision structure of Glass are computed. In conclusion, utilizing MATLAB programming, we can demonstrate that the presence of an M-polynomial (M-P) enables the computation of various Revan indices for any molecular graph.

Subject Classification: *05C09, 05Cxx, 05C99, 92E10, 82D30.*

Keywords: *Topological indices, Revan index, M-polynomial, Para-line graph, Subdivision graph, Glass.*

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1. Introduction

Finding the properties of compounds in laboratories is expensive and requires time and expertise.

Therefore, to evaluate the physical and chemical characteristics of the molecular framework, we use TIs and M-polynomials [8,11,12,24].

TIs are employed to predict the structural and compositional characteristics of a molecular graph. Various frameworks have been analyzed using topological indices [14, 20, 21].

As a result, chemical graph theory plays a significant role in advancing chemical sciences. Glass is one of the oldest man-made substances, and in modern times, it serves as an element for embellishing architecture and buildings (Figure 1).

The most famous and oldest known glass is "silicate glass". Glass fibers are used in the optical fiber of communication networks, thermal insulation materials, and fiberglass [9].

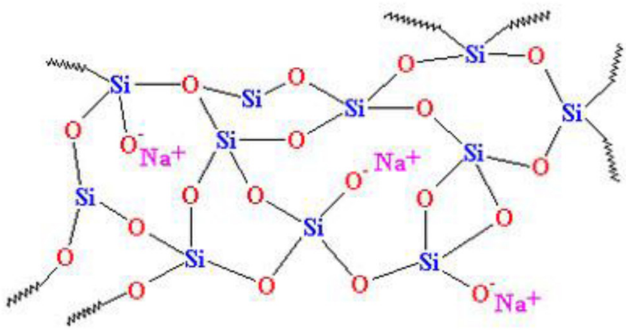

Figure 1

Molecular composition of silica.

Glass can be formed through various techniques, including vapor-phase deposition [4], solid-phase reactions (thermochemical approaches [17, 26] and mechanochemical processes [3]), liquid-phase reactions (such as the sol-gel method [15, 18]), irradiation of crystalline structures (radiative amorphization [1,6]), along with the application of high pressures (pressure-induced amorphization [16,25]). Numerous studies have been conducted on the Revan indices [13, 22, 23]. Revan indices have been extensively analyzed in earlier research. In this article, hyper Revan indices, modified Revan indices, and Y-Revan index are computed using M-P for Glass.

2. Fundamentals

In this article, H = (V, E) is treated as a simple and connected network, with V(H) representing the vertex set and E(H) representing the edge set. The degree associated with an arbitrary vertex r is represented as d_r. In graph H, the minimum vertex degree is represented as δ(H), and the maximum vertex degree is represented as Δ(H).

Definition 2.1: The description of the M-polynomial within network H is provided in the form of [5]:

$$M(H;\mathrm{x},\mathrm{y}) = \sum_{\delta \le i \le j \le \Delta} m_{ij}(H) x^i y^j.$$

Where d(r) represents the count of edges e such that e connects to r, a few operators, which will be utilized in subsequent sections, are defined as follows:

$$D_x M(H;\mathrm{x},\mathrm{y}) = x \times \frac{\partial M(H;\mathrm{x},\mathrm{y})}{\partial x},\ D_y M(H;\mathrm{x},\mathrm{y}) = y \times \frac{\partial M(H;\mathrm{x},\mathrm{y})}{\partial y},$$

$$Q_{x(\alpha)} M(H;\mathrm{x},\mathrm{y}) = x^{\alpha} M(H;\mathrm{x},\mathrm{y}),\ Q_{y(\alpha)} M(H;\mathrm{x},\mathrm{y}) = y^{\alpha} M(H;\mathrm{x},\mathrm{y}),$$

$$S_x M(H;\mathrm{x},\mathrm{y}) = \int_0^x \frac{\mathrm{M}(H;t,\mathrm{y})}{t} dt,\ S_y M(H;\mathrm{x},\mathrm{y}) = \int_0^y \frac{M(H;\mathrm{x},t)}{t} dt,$$

$$J\,M(H;\mathrm{x},\mathrm{y}) = M(H;\mathrm{x},\mathrm{x}).$$

The M-P can be utilized to determine TIs. The evaluation of TIs through the M-P within molecular graphs has likewise been investigated [2,19].

The Revan degree for a vertex u in network H is defined as follows: $r_u = \Delta + \delta - \mathrm{d}_u$.

Definition 2.2: The primary and secondary hyper Revan indices for graph H are specified as [22]:

$$R_1(H) = \sum_{uv \in E(H)} [r_u + r_v]^2,\ R_2(H) = \sum_{uv \in E(H)} [r_u . r_v]^2.$$

Definition 2.3: The primary and secondary modified Revan indices for graph H are specified as [23]:

$${}^{m}R_1(H) = \sum_{uv \in E(H)} \frac{1}{r_u + r_v},\ {}^{m}R_2(H) = \sum_{uv \in E(H)} \frac{1}{r_u r_v}.$$

Definition 2.4: The Y-Revan index for the structure H is expressed as:

$$YR(H) = \sum_{uv \in E(H)} (r_u^{\,3} + r_v^{\,3}).$$

Definition 2.5: The line structure H is denoted as L(H), representing a structure that depicts the adjacencies between the edges of H. The vertices of graph L(H) correspond to the edges of graph H [10,24].

This graph is found in multiple foundational quantum chemical models. The vertices of the Para-line structures in the molecular framework correspond to its atomic hybrid orbitals, while the links indicate the more robust interactions among pairs of these orbitals [24]. There are multiple approaches to associating a molecular structure with a chemical framework, such as considering a fundamental set of localized orbitals (vertices), with links representing more significant interactions among pairs of orbitals [7].

Theorem 2.1: *Assume denotes the M-polynomial of graph H. The primary hyper-Revan index is subsequently determined in the following manner:*

$$HR_1(H) = [(D_x^2 + D_y^2 + 2D_xD_y)(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y))]_{|(x,y)=(1,1)}.$$

Proof:

$$\begin{aligned} HR_1(H) &= \sum_{uv \in E(H)} (r_u + r_v)^2 = \sum_{uv \in E(H)} (r_u^{\,2} + r_u^{\,2} + 2r_ur_v) \\ &= \sum_{uv \in E(H)} [(\Delta+\delta-d_u)^2 + (\Delta+\delta-d_v)^2 + 2(\Delta+\delta-d_u)(\Delta+\delta-d_v)] \\ &= \sum_{uv \in E(H)} [(d_u-\Delta-\delta)^2 + (d_v-\Delta-\delta)^2 + 2(d_u-\Delta-\delta)(d_v-\Delta-\delta)], \end{aligned} \quad (1)$$

$$Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y) = \sum_{\delta \le i \le j \le \Delta} m_{ij} x^{i-\Delta-\delta} y^{j-\Delta-\delta},$$

$$D_x^{\,2}\Big(\sum_{\delta \le i \le j \le \Delta} m_{ij} x^{i-\Delta-\delta} y^{j-\Delta-\delta}\Big) = \sum_{\delta \le i \le j \le \Delta} (i-\Delta-\delta)^2 m_{ij} x^{i-\Delta-\delta} y^{j-\Delta-\delta},$$

$$D_y^{\,2}\Big(\sum_{\delta \le i \le j \le \Delta} m_{ij} x^{i-\Delta-\delta} y^{j-\Delta-\delta}\Big) = \sum_{\delta \le i \le j \le \Delta} (j-\Delta-\delta)^2 m_{ij} x^{i-\Delta-\delta} y^{j-\Delta-\delta},$$

$$D_xD_y\Big(\sum_{\delta \le i \le j \le \Delta} m_{ij} x^{i-\Delta-\delta} y^{j-\Delta-\delta}\Big) = \sum_{\delta \le i \le j \le \Delta} (i-\Delta-\delta)(j-\Delta-\delta) m_{ij} x^{i-\Delta-\delta} y^{j-\Delta-\delta},$$

$$(D_x^2 + D_y^2 + 2D_xD_y)\Big(\sum_{\delta \le i \le j \le \Delta} m_{ij} x^{i-\Delta-\delta} y^{j-\Delta-\delta}\Big)_{|(x,y)=(1,1)} =$$

$$\sum_{\delta \le i \le j \le \Delta} \begin{bmatrix} (i-\Delta-\delta)^2 + (j-\Delta-\delta)^2 \\ +(2(i-\Delta-\delta)(j-\Delta-\delta)) \end{bmatrix} m_{ij}, \quad (2)$$

Hence, according to relations (1) and (2), we have,

$$HR_1(H) = [(D_x^2 + D_y^2 + 2D_xD_y)(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y))]_{|(x,y)=(1,1)}.$$

Theorem 2.2: *Assume represents the M-polynomial associated with graph H. The second hyper-Revan index is then determined as:*

$$HR_2(H) = [(D^2{}_xD^2{}_y)(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y))]_{|(x,y)=(1,1)}.$$

Proof:

$$\begin{aligned} HR_2(H) &= \sum_{uv\in E(H)} (r_ur_v)^2 = \sum_{uv\in E(H)} r_u{}^2 r_v{}^2 \\ &= \sum_{uv\in E(H)} [(\Delta+\delta-du)^2(\Delta+\delta-dv)^2] \\ &= \sum_{uv\in E(H)} [(-(du-\Delta-\delta))^2(-(dv-\Delta-\delta))^2], \end{aligned} \tag{3}$$

$$Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y) = \sum_{\delta\le i\le j\le\Delta} m_{ij}x^{i-\Delta-\delta}y^{j-\Delta-\delta},$$

$$D_x{}^2\Big(\sum_{\delta\le i\le j\le\Delta} m_{ij}x^{i-\Delta-\delta}y^{j-\Delta-\delta}\Big) = \sum_{\delta\le i\le j\le\Delta} (i-\Delta-\delta)^2 m_{ij}x^{i-\Delta-\delta}y^{j-\Delta-\delta},$$

$$D_y{}^2\Big(\sum_{\delta\le i\le j\le\Delta} m_{ij}x^{i-\Delta-\delta}y^{j-\Delta-\delta}\Big) = \sum_{\delta\le i\le j\le\Delta} (j-\Delta-\delta)^2 m_{ij}x^{i-\Delta-\delta}y^{j-\Delta-\delta},$$

$$D^2{}_xD^2{}_y\Big(\sum_{\delta\le i\le j\le\Delta} m_{ij}x^{i-\Delta-\delta}y^{j-\Delta-\delta}\Big) = \sum_{\delta\le i\le j\le\Delta} (i-\Delta-\delta)^2(j-\Delta-\delta)^2 m_{ij}x^{i-\Delta-\delta}y^{j-\Delta-\delta},$$

On the other hand, we have,

$$[D_x{}^2D_y{}^2(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y))]_{|(x,y)=(1,1)} = \sum_{\delta\le i\le j\le\Delta} m_{ij}(i-\Delta-\delta)^2(j-\Delta-\delta)^2, \tag{4}$$

Based on relations (3) and (4), it follows that:

$$HR_2(H) = [D_x{}^2D_y{}^2(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y))]_{|(x,y)=(1,1)}.$$

Theorem 2.3: *Assume is the M-polynomial corresponding to graph H. The initial modified Revan index is then calculated as:*

$${}^mR_1(H) = [-S_xJ(Q_{(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y)]_{|(x,y)=(1,1)}.$$

Proof:

$$
\begin{aligned}
{}^{m}R_1(H) &= \sum_{uv\in E(H)} \frac{1}{r_u + r_v} = \sum_{uv\in E(H)} \frac{1}{(\Delta+\delta-d_u)+(\Delta+\delta-d_v)} \\
&= \sum_{uv\in E(H)} \frac{-1}{(d_u-\Delta-\delta)+(d_v-\Delta-\delta)} = -\sum_{uv\in E(H)} \frac{1}{(d_u-\Delta-\delta)+(d_v-\Delta-\delta)},
\end{aligned} \tag{5}
$$

$$
J(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y)) = \sum_{\delta\le i\le j\le\Delta} m_{ij} x^{i-\Delta-\delta} x^{j-\Delta-\delta} = \sum_{\delta\le i\le j\le\Delta} m_{ij} x^{i+j-2\Delta-2\delta},
$$

$$
\begin{aligned}
S_x J(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y)) &= \int_0^x \frac{\sum_{\delta\le i\le j\le\Delta} m_{ij} t^{(i+j-2\Delta-2\delta)}}{t} dt = \sum_{\delta\le i\le j\le\Delta} m_{ij} \int_0^x \frac{t^{(i+j-2\Delta-2\delta)}}{t} dt \\
&= \sum_{\delta\le i\le j\le\Delta} m_{ij} \left(\frac{x^{(i+j-2\Delta-2\delta)}}{i+j-2\Delta-2\delta} \right),
\end{aligned}
$$

$$
-[S_x J(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y))]\big|_{x=1} = -\sum_{\delta\le i\le j\le\Delta} m_{ij} \frac{1}{i+j-2\Delta-2\delta}, \tag{6}
$$

$$
J(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y)) = \sum_{\delta\le i\le j\le\Delta} m_{ij} x^{i-\Delta-\delta} x^{j-\Delta-\delta} = \sum_{\delta\le i\le j\le\Delta} m_{ij} x^{i+j-2\Delta-2\delta},
$$

$$
\begin{aligned}
S_x J(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y)) &= \int_0^x \frac{\sum_{\delta\le i\le j\le\Delta} m_{ij} t^{(i+j-2\Delta-2\delta)}}{t} dt = \sum_{\delta\le i\le j\le\Delta} m_{ij} \int_0^x \frac{t^{(i+j-2\Delta-2\delta)}}{t} dt \\
&= \sum_{\delta\le i\le j\le\Delta} m_{ij} \left(\frac{x^{(i+j-2\Delta-2\delta)}}{i+j-2\Delta-2\delta} \right),
\end{aligned}
$$

$$
-[S_x J(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y))]\big|_{x=1} = -\sum_{\delta\le i\le j\le\Delta} m_{ij} \frac{1}{i+j-2\Delta-2\delta}, \tag{6}
$$

Based on relations (5) along with (6), we have,

$$
{}^{m}R_1(H) = [-S_x J(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y)]\big|_{(x,y)=(1,1)}.
$$

Theorem 2.4: *Assume represents the M-polynomial associated with graph H. The second modified Revan index is then calculated as:*

$$
{}^{m}R_2(H) = [S_x S_y (Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y)]\big|_{(x,y)=(1,1)}.
$$

Proof:

$$^{m}R_2(H) = \sum_{uv\in E(H)} \frac{1}{r_u r_v} = \sum_{uv\in E(H)} \frac{-1}{(d_u - \Delta - \delta)(d_v - \Delta - \delta)}, \tag{7}$$

$$S_y(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y)) = \sum_{\delta\le i\le j\le\Delta} m_{ij}\int_0^y \frac{x^{i-\Delta-\delta}t^{j-\Delta-\delta}}{t}dt = \sum_{i<J} m_{ij}x^{i-\Delta-\delta}\frac{y^{j-\Delta-\delta}}{j-\Delta-\delta},$$

$$(S_xS_y(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y))) = \sum_{\delta\le i\le j\le\Delta} m_{ij}\frac{y^{j-\Delta-\delta}}{j-\Delta-\delta}\int_0^x \frac{t^{i-\Delta-\delta}}{t}dt$$

$$= \sum_{\delta\le i\le j\le\Delta} m_{ij}\frac{y^{j-\Delta-\delta}x^{i-\Delta-\delta}}{(i-\Delta-\delta)(j-\Delta-\delta)},$$

$$(S_xS_y(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y)))\big|_{(x,y)=(1,1)} = \sum_{\delta\le i\le j\le\Delta} m_{ij}\frac{1}{(i-\Delta-\delta)(j-\Delta-\delta)}, \tag{8}$$

According to relations (7) and (8), we have,

$$^{m}R_2(H) = [S_xS_y(Q_{x(-\Delta-\partial)}Q_{y(-\Delta-\delta)}M(H;x,y)]\big|_{(x,y)=(1,1)}.$$

Theorem 2.5: *Let represent the M-polynomial associated with graph H. The Y-Revan index is then calculated as:*

$$YR(H) = \sum_{uv\in E(H)} (r_u^{\,3} + r_v^{\,3}) = -[(D_x^{\,3} + D_y^{\,3})(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y)]\big|_{(x,y)=(1,1)}.$$

Proof:

$$\begin{aligned} YR = \sum_{uv\in E(H)} (r_u^{\,3} + r_v^{\,3}) &= \sum_{uv\in E(H)} [(\Delta+\delta-d_u)^3 + (\Delta+\delta-d_v)^3] \\ &= \sum_{uv\in E(H)} [(-(d_u-\Delta-\delta)) + (-(d_v-\Delta-\delta))] \\ &= -\sum_{uv\in E(H)} [(d_u-\Delta-\delta) + (d_v-\Delta-\delta)], \end{aligned} \tag{9}$$

$$D_x^{\,3}(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y)) = \sum_{\delta\le i\le j\le\Delta} (i-\Delta-\delta)^3 m_{ij}x^{i-\Delta-\delta}y^{j-\Delta-\delta},$$

$$D_x^{\,3}(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y)) = \sum_{\delta\le i\le j\le\Delta} (j-\Delta-\delta)^3 m_{ij}x^{i-\Delta-\delta}y^{j-\Delta-\delta},$$

$$-[(D_x{}^3+D_y{}^3)(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y)]|_{(x,y)=(1,1)} = -\sum_{\delta\le i\le j\le\Delta} m_{ij}((i-\Delta-\delta)^3+(j-\Delta-\delta)^3), \tag{10}$$

According to relations (9) and (10), we have,

$$YR(H) = -[(D_x{}^3+D_y{}^3)(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y)]|_{(x,y)=(1,1)}.$$

Building upon the findings of the preceding theorems, Table 1 is derived for calculating Revan indices using the M-polynomial.

Table 1

Calculation of Revan indices derived from the M-polynomial.

Topological Index name	Topological Index formula	Derivation from M(H; x, y)
First hyper Revan	$\sum_{uv\in E(H)}(r_u+r_v)^2$	$[(D_x^2+D_y^2+2D_xD_y)(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(H;x,y))]|_{(x,y)=(1,1)}.$
Second hyper Revan	$\sum_{uv\in E(H)}(r_ur_v)^2$	$[(D^2{}_xD^2{}_y)(Q_{x(-\Delta-\delta)}Q_{y(-\Delta-\delta)}M(\mathrm{H};x,y))]|_{(x,y)=(1,1)}.$
First modified Revan	$\sum_{uv\in E(H)}\frac{1}{r_u+r_v}$	$[-S_xJ(Q_{x(-\Delta-\partial)}Q_{y(-\Delta-\partial)}M(H;x,y)]|_{(x,y)=(1,1)}.$
Second modified Revan	$\sum_{uv\in E(H)}\frac{1}{r_ur_v}$	$[S_xS_y(Q_{x(-\Delta-\partial)}Q_{y(-\Delta-\partial)}M(H;x,y)]|_{(x,y)=(1,1)}.$
Y-Revan	$\sum_{uv\in E(H)}(r_u{}^3+r_v{}^3)$	$-[(D_x{}^3+D_y{}^3)(Q_{x(-\Delta-\partial)}Q_{y(-\Delta-\partial)}M(H;x,y)]|_{(x,y)=(1,1)}.$

3. Topological indices and their calculations for Glass

In this part, we compute various types of TIs for the molecular graph, subdivision graph, and para-line graph of Glass using the M-polynomial and the relationships established in Section 2.

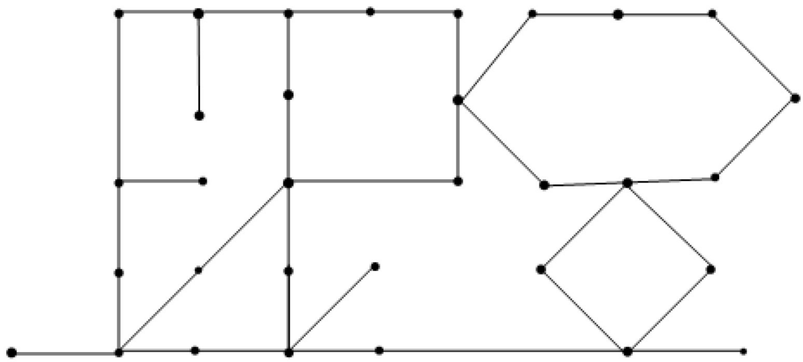

Figure 2

The chemical framework of Glass.

The molecular graph of glass is denoted as H. In the chemical graph of glass, there are seven distinct types of edges, as listed below:

$E_1 = \{rs \in E(H), d_H(r) = 1, d_H(s) = 3\}, E_2 = \{rs \in E(H), d_H(r) = 1, d_H(s) = 4\},$
$E_3 = \{rs \in E(H), d_H(r) = 2, d_H(s) = 2\}$, $E_4 = \{rs \in E(H), d_H(r) = 2, d_H(s) = 3\},$
$E_5 = \{rs \in E(H), d_H(r) = 2, d_H(s) = 4\}$, $E_6 = \{rs \in E(H), d_H(r) = 3, d_H(s) = 3\},$
$E_7 = \{rs \in E(H), d_H(r) = 3, d_H(s) = 4\}.$

Based on Figure 2, Table 2 is provided for the chemical graph of glass:

Table 2

The number of edges in the molecular graph of Glass.

Edge type	**Number of edges**
E_1	2
E_2	3
E_3	5
E_4	4
E_5	20
E_6	1
E_7	1

Therefore, the edge count within the chemical framework of glass corresponds to:

$$|E(H)| = 2 + 3 + 5 + 4 + 20 + 1 + 1 = 36.$$

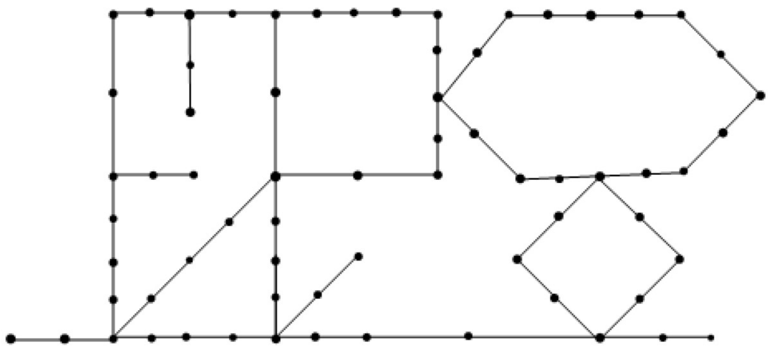

Figure 3

The subdivided structure of Glass.

The refined structure of glass is represented by S (H). In S (H), four different categories of edges exist, as outlined:

$$E_1' = \{rs \in E(S(H)), d_{S(H)}(r) = 1, d_{S(H)}(s) = 2\},$$
$$E_2' = \{rs \in E(S(H)), d_{S(H)}(r) = 2, d_{S(H)}(s) = 2\},$$
$$E_3' = \{rs \in E(S(H)), d_{S(H)}(r) = 2, d_{S(H)}(s) = 3\},$$
$$E_4' = \{rs \in E(S(H)), d_{S(H)}(r) = 2, d_{S(H)}(s) = 4\}.$$

Figure 3 illustrates the refined structure of glass.

According to Figure 3, we have Table 3 for S(H).

Table 3

The number of edges in the S(H).

Edge type	Number of edges
E_1'	5
E_2'	34
E_3'	9
E_4'	24

Therefore, the number of edges in the refined structure is equivalent to:

$$|E(S(H))| = 5 + 34 + 9 + 24 = 72.$$

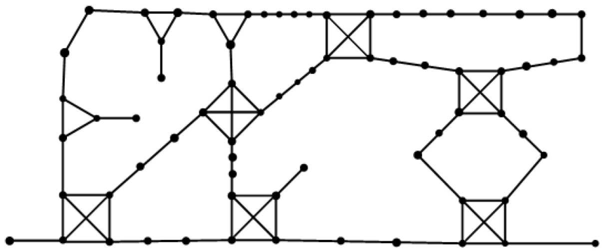

Figure 4

Shows the para-line graph.

The para-line structure of glass is denoted by L (S (H)). In L (S (H)), there are eight distinct types of edges, as follows:

$$E_1'' = \{rs \in E(L(S(H))), d_{L(S(H))}(r) = 1, d_{L(S(H))}(s) = 3\},$$
$$E_2'' = \{rs \in E(L(S(H))), d_{L(S(H))}(r) = 1, d_{L(S(H))}(s) = 4\},$$
$$E_3'' = \{rs \in E(L(S(H))), d_{L(S(H))}(r) = 2, d_{L(S(H))}(s) = 2\},$$
$$E_4'' = \{rs \in E(L(S(H))), d_{L(S(H))}(r) = 2, d_{L(S(H))}(s) = 3\},$$
$$E_5'' = \{rs \in E(L(S(H))), d_{L(S(H))}(r) = 2, d_{L(S(H))}(s) = 4\},$$
$$E_6'' = \{rs \in E(L(S(H))), d_{L(S(H))}(r) = 3, d_{L(S(H))}(s) = 3\},$$
$$E_7'' = \{rs \in E(L(S(H))), d_{L(S(H))}(r) = 3, d_{L(S(H))}(s) = 4\},$$
$$E_8'' = \{rs \in E(L(S(\mathrm{H}))), d_{L(S(H))}(r) = 4, d_{L(S(H))}(s) = 4\}.$$

Based on Figure 4, the following table 4 is provided for the L (S (H)) structure of glass:

Table 4

The number of edges in the L(S(H)).

Type of connection	Edge count
E_1''	2
E_2''	3
E_3''	22
E_4''	3
E_5''	19
E_6''	10
E_7''	2
E_8''	36

Therefore, the count of connections in the para-line structure is equivalent to:

$$\left|E(L(S(H))\right| = 2+3+22+3+19+10+2+36 = 97.$$

Theorem 3.1: *Let H, S(H), and L(S(H)) represent the molecular structure, subdivision structure, and para-line structure of Glass, respectively. Then their M-Ps are obtained as follows:*

i) $M(H;x,y) = 2x^1y^3 + 3x^1y^4 + 5x^2y^2 + 4x^2y^3 + 20x^2y^4 + 1x^3y^3 + 1x^3y^4$,

ii) $M(\mathrm{S}(H);x,y) = 5x^1y^2 + 34x^2y^2 + 9x^2y^3 + 24x^2y^4$,

iii) $M(L(S(H));x,y) = 2x^1y^3 + 3x^1y^4 + 22x^2y^2 + 3x^2y^3 + 19x^2y^4 + 10x^3y^3$ $+2x^3y^4 + 36x^4y^4$.

Proof: By placing the values of tables 2, 3, and 4 in definition 2.1, the relationships above are easily calculated.

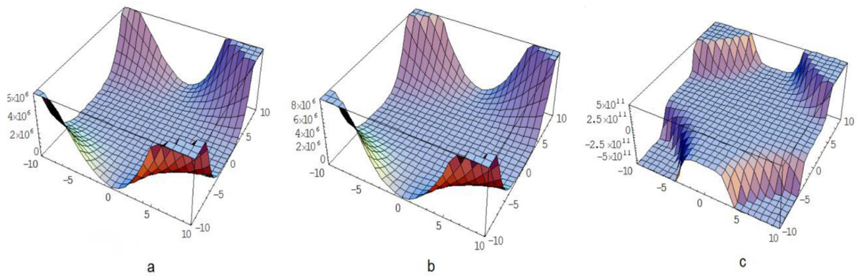

Figure 5

M-polynomial of (a) molecular graph, (b) subdivision graph, and (c) para-line graph of Glass, respectively.

Figure 5 illustrates the M-P behavior for glass graphs. By examining Figure 5, it is clear that in the molecular graph and subdivision graph, the M-polynomial is always positive, but in the line graph, the value of the polynomial for $y \le -5$ is not always positive.

3.1 *Molecular structure of glass*

A molecular graph emerges by considering atoms as nodes and bonds as connections from the corresponding chemical representation. Utilizing the M-polynomial and the relationships proved in Section 2, the following TI for this graph can be determined:

- The first hyper Revan index,
- The second hyper Revan index,
- The first modified Revan index,
- The second modified Revan index,
- The Y-Revan index.

Theorem 3.2: *Consider H to represent the molecular structure of glass, with the M-polynomial of H denoted as. Then, the Revan indices are computed as:*

1. $HR_1(H) = 772$,
2. $HR_2(H) = 925$,
3. ${}^{m}R_1(H) = 8.14$,
4. ${}^{m}R_2(H) = 9.56$,
5. $YR(H) = 1334$,

Proof: Let

$$M(H;x,y) = 2x^1y^3 + 3x^1y^4 + 5x^2y^2 + 4x^2y^3 + 20x^2y^4 + 1x^3y^3 + 1x^3y^4.$$

Then

$$Q_{x(-5)}Q_{y(-5)}M(H;x,y) = 2x^{-4}y^{-2} + 3x^{-4}y^{-1} + 5x^{-3}y^{-3} + 4x^{-3}y^{-2} + 20x^{-3}y^{-1} + x^{-2}y^{-2} + x^{-2}y^{-1},$$

$$D_x^{\,2}(Q_{x(-5)}Q_{y(-5)}M(H;x,y)) = 32x^{-4}y^{-2} + 48x^{-4}y^{-1} + 45x^{-3}y^{-3} + 36x^{-3}y^{-2} + 180x^{-3}y^{-1} + 4x^{-2}y^{-2} + 4x^{-2}y^{-1},$$

$$D_y^{\,2}(Q_{x(-5)}Q_{y(-5)}M(H;x,y)) = 8x^{-4}y^{-2} + 3x^{-4}y^{-1} + 45x^{-3}y^{-3} + 16x^{-3}y^{-2} + 20x^{-3}y^{-1} + 4x^{-2}y^{-2} + x^{-2}y^{-1},$$

$$D_xD_y(Q_{x(-5)}Q_{y(-5)}M(H;x,y)) = 16x^{-4}y^{-2} + 12x^{-4}y^{-1} + 45x^{-3}y^{-3} + 24x^{-3}y^{-2} + 60x^{-3}y^{-1} + 4x^{-2}y^{-2} + 2x^{-2}y^{-1},$$

$$D^2{}_xD^2{}_y(Q_{x(-5)}Q_{y(-5)}M(H;x,y)) = 128x^{-4}y^{-2} + 48x^{-4}y^{-1} + 405x^{-3}y^{-3} + 144x^{-3}y^{-2} + 180x^{-3}y^{-1} + 16x^{-2}y^{-2} + 4x^{-2}y^{-1},$$

$$J(Q_{x(-5)}Q_{y(-5)}M(H;x,y)) = 2x^{-6} + 3x^{-5} + 5x^{-6} + 4x^{-5} + 21x^{-4} + x^{-3},$$

$$S_x J(Q_{x(-5)}Q_{y(-5)}M(H;x,y)) = \int_0^x \frac{2t^{-6}}{t}dt + \int_0^x \frac{3t^{-5}}{t}dt + \int_0^x \frac{5t^{-6}}{t}dt + \int_0^x \frac{4t^{-5}}{t}dt + \int_0^x \frac{20t^{-4}}{t}dt$$
$$+\int_0^x \frac{t^{-4}}{t}dt + \int_0^x \frac{t^{-3}}{t}dt$$
$$= \frac{-x^{-6}}{3} + \frac{-3x^{-5}}{5} + \frac{-5x^{-6}}{6} + \frac{-4x^{-5}}{5} + \frac{-21x^{-4}}{4} + \frac{-x^{-3}}{3},$$

$$S_y(Q_{x(-5)}Q_{y(-5)}M(H;x,y))$$
$$= \int_0^y \frac{2x^{-4}t^{-2}}{t}dt + \int_0^y \frac{3x^{-4}t^{-1}}{t}dt + \int_0^y \frac{5x^{-3}t^{-3}}{t}dt + \int_0^y \frac{4x^{-3}t^{-2}}{t}dt + \int_0^y \frac{20x^{-3}t^{-1}}{t}dt$$
$$+\int_0^y \frac{x^{-2}t^{-2}}{t}dt + \int_0^y \frac{x^{-2}t^{-1}}{t}dt$$
$$= \frac{2x^{-4}y^{-2}}{-2} + \frac{3x^{-4}y^{-1}}{-1} + \frac{5x^{-3}y^{-3}}{-3} + \frac{4x^{-3}y^{-2}}{-2} + \frac{20x^{-3}y^{-1}}{-1} + \frac{x^{-2}y^{-2}}{-2} + \frac{x^{-2}y^{-1}}{-1},$$

$$(S_xS_y(Q_{x(-5)}Q_{y(-5)}M(H;x,y)))$$
$$= \frac{2y^{-2}}{-2}\int_0^x \frac{t^{-4}}{t}dt + \frac{3y^{-1}}{-1}\int_0^x \frac{t^{-4}}{t}dt + \frac{5y^{-3}}{-3}\int_0^x \frac{t^{-3}}{t}dt + \frac{4y^{-2}}{-2}\int_0^x \frac{t^{-3}}{t}dt + \frac{20y^{-1}}{-1}\int_0^x \frac{t^{-3}}{t}dt$$
$$+ \frac{y^{-2}}{-2}\int_0^x \frac{t^{-2}}{t}dt + \frac{y^{-1}}{-1}\int_0^x \frac{t^{-2}}{t}$$
$$= \frac{y^{-2}x^{-4}}{4} + \frac{3y^{-1}x^{-4}}{4} + \frac{5y^{-3}x^{-3}}{9} + \frac{2y^{-2}x^{-3}}{3} + \frac{20y^{-1}x^{-3}}{3} + \frac{y^{-2}x^{-2}}{4} + \frac{y^{-1}x^{-2}}{2},$$

$$D_x^{\ 3}(Q_{x(-5)}Q_{y(-5)}M(H;x,y))$$
$$= -128x^{-4}y^{-2} - 192x^{-4}y^{-1} - 135x^{-3}y^{-3} - 108x^{-3}y^{-2} - 540x^{-3}y^{-1} - 8x^{-2}y^{-2}$$
$$-8x^{-2}y^{-1},$$

$$D_y^{\ 3}(Q_{x(-5)}Q_{y(-5)}M(H;x,y))$$
$$= -16x^{-4}y^{-2} - 3x^{-4}y^{-1} - 135x^{-3}y^{-3} - 32x^{-3}y^{-2} - 20x^{-3}y^{-1} - 8x^{-2}y^{-2} - x^{-2}y^{-1},$$

Now we have,

1. $[(D_x^2 + D_y^2 + 2D_xD_y)(Q_{x(-5)}Q_{y(-5)}M(H;x,y))]|_{(x,y)=(1,1)} = 772,$
2. $[D_x^{\ 2}D_y^{\ 2}(Q_{x(-5)}Q_{y(-5)}M(H;x,y))]|_{(x,y)=(1,1)} = 925,$
3. $-[S_xJ(Q_{x(-5)}Q_{y(-5)}M(H;x,y))]|_{x=1} = 8.14,$
4. $(S_xS_y(Q_{x(-5)}Q_{y(-5)}M(H;x,y)))|_{(x,y)=(1,1)} = 9.56,$
5. $-[(D_x^{\ 3} + D_y^{\ 3})(Q_{x(-\Delta-\partial)}Q_{y(-\Delta-\partial)}M(H;x,y)]|_{(x,y)=(1,1)} = 1334,$

3.2 *Subdivision structure of Glass*

The subdivision structure of Glass is derived by placing a new vertex on each edge of the molecular structure of Glass and connecting it to the endpoints of that edge [4]. The subdivision graph of Glass is shown in Figure 2.

Using the M-polynomial and the relationships proved in Section 2, the following TIs for this graph can be determined.

- The first hyper Revan index,
- The second hyper Revan index,
- The first modified Revan index,
- The second modified Revan index,
- The Y-Revan index.

Theorem 3.3: *Let S (H) be the subdivision graph of Glass and* $M(\mathrm{S}(H);\mathrm{x},\mathrm{y})$ *be the M-P of S (H). Then the Revan indices are computed as,*

1) $HR_1(\mathrm{S}(H)) = 2078$,
2) $HR_2(\mathrm{S}(H)) = 4014$,
3) ${}^{m}R_1(\mathrm{S}(H)) = 14.18$,
4) ${}^{m}R_2(\mathrm{S}(H)) = 13.69$,
5) $YR(\mathrm{S}(H)) = 3278$,

Proof: Let $M(S(H);x,y) = 5x^1y^2 + 34x^2y^2 + 9x^2y^3 + 24x^2y^4$,. Then

$$Q_{x(-5)}Q_{y(-5)}M(S(H);x,y)$$
$$= 5x^{-4}y^{-3} + 34x^{-3}y^{-3} + 9x^{-3}y^{-2} + 24x^{-3}y^{-1},$$

$$D_x{}^2(Q_{x(-5)}Q_{y(-5)}M(S(H);x,y))$$
$$= 80x^{-4}y^{-3} + 306x^{-3}y^{-3} + 81x^{-3}y^{-2} + 216x^{-3}y^{-1},$$

$$D_y{}^2(Q_{x(-5)}Q_{y(-5)}M(S(H);x,y))$$
$$= 45x^{-4}y^{-3} + 306x^{-3}y^{-3} + 369(-2)(-2)x^{-3}y^{-2} + 24x^{-3}y^{-1},$$

$$D_xD_y(Q_{x(-5)}Q_{y(-5)}M(S(H);x,y))$$
$$= 60x^{-4}y^{-3} + 306x^{-3}y^{-3} + 54x^{-3}y^{-2} + 72x^{-3}y^{-1},$$

$$\begin{aligned}
&D^2{}_x D^2{}_y (Q_{x(-5)}Q_{y(-5)}M(S(H);x,y)) \\
&= 720x^{-4}y^{-3} + 2754x^{-3}y^{-3} + 324x^{-3}y^{-2} + 216x^{-3}y^{-1},
\end{aligned}$$

$$\begin{aligned}
&J(Q_{x(-5)}Q_{y(-5)}M(S(H);x,y)) \\
&= 5x^{-4}x^{-3} + 34x^{-3}x^{-3} + 9x^{-3}x^{-2} + 24x^{-3}x^{-1},
\end{aligned}$$

$$\begin{aligned}
&S_x J(Q_{x(-5)}Q_{y(-5)}M(S(H);x,y)) \\
&= \int_0^x \frac{5t^{-7}}{t}dt + \int_0^x \frac{34t^{-6}}{t}dt + \int_0^x \frac{9t^{-5}}{t}dt + \int_0^x \frac{24t^{-4}}{t}dt \\
&= \frac{-5x^{-7}}{7} + \frac{-34x^{-6}}{6} + \frac{-9x^{-5}}{5} + \frac{-24x^{-4}}{4},
\end{aligned}$$

$$\begin{aligned}
&S_y (Q_{x(-5)}Q_{y(-5)}M(S(H);x,y)) \\
&= \int_0^y \frac{5x^{-4}t^{-3}}{t}dt + \int_0^y \frac{34x^{-3}t^{-3}}{t}dt + \int_0^y \frac{9x^{-3}t^{-2}}{t}dt + \int_0^y \frac{24x^{-3}t^{-1}}{t}dt \\
&= \frac{5x^{-4}y^{-3}}{-3} + \frac{34x^{-4}y^{-3}}{-3} + \frac{9x^{-3}y^{-2}}{-2} + \frac{24x^{-3}y^{-1}}{-1},
\end{aligned}$$

$$\begin{aligned}
&(S_x S_y (Q_{x(-5)}Q_{y(-5)}M(S(H);x,y))) \\
&= \frac{5y^{-3}}{-3}\int_0^x \frac{t^{-4}}{t}dt + \frac{34y^{-3}}{-3}\int_0^x \frac{t^{-4}}{t}dt + \frac{9y^{-2}}{-2}\int_0^x \frac{t^{-3}}{t}dt + \frac{24y^{-1}}{-1}\int_0^x \frac{t^{-3}}{t}dt \\
&= \frac{5y^{-3}x^{-4}}{12} + \frac{34y^{-3}x^{-4}}{12} + \frac{9y^{-2}x^{-3}}{6} + \frac{24y^{-1}x^{-3}}{3},
\end{aligned}$$

$$\begin{aligned}
&D_x{}^3 (Q_{x(-5)}Q_{y(-5)}M(S(H);x,y)) \\
&= -320x^{-4}y^{-3} - 918x^{-3}y^{-3} - 243x^{-3}y^{-2} - 648x^{-3}y^{-1},
\end{aligned}$$

$$\begin{aligned}
&D_y{}^3 (Q_{x(-5)}Q_{y(-5)}M(S(H);x,y)) \\
&= -135x^{-4}y^{-3} - 918x^{-3}y^{-3} - 72x^{-3}y^{-2} - 24x^{-3}y^{-1},
\end{aligned}$$

Now we have,

1. $[(D_x^2 + D_y^2 + 2D_x D_y)(Q_{x(-5)}Q_{y(-5)}M(S(H);x,y))]_{|(x,y)=(1,1)} = 2078,$
2. $[D_x{}^2 D_y{}^2 (Q_{x(-5)}Q_{y(-5)}M(S(H);x,y))]_{|(x,y)=(1,1)} = 4014,$
3. $-[S_x J(Q_{x(-5)}Q_{y(-5)}M(S(H);x,y))]_{|x=1} = 14.18,$

4. $(S_x S_y (Q_{x(-5)} Q_{y(-5)} M(S(H);x,y)))|_{(x,y)=(1,1)} = 13.69,$
5. $-[(D_x{}^3 + D_y{}^3)(Q_{x(-\Delta-\partial)} Q_{y(-\Delta-\partial)} M(S(H);x,y)]|_{(x,y)=(1,1)} = 3278,$

3.3 *Para-line structure*

The para-line structure of Glass is obtained by replacing each edge of the molecular structure of Glass with a two-step path [7]. The para-line graph of Glass is shown in Figure 3.

Using the M-p and the relationships proved in Section 2, we can calculate the following TI for this graph:

- The first hyper Revan index,
- The second hyper Revan index,
- The first modified Revan index,
- The second modified Revan index,
- The Y-Revan index.

Theorem 3.4: *Let L(S(H)) be the para-line structure of Glass and* $M(L(S(H));\mathrm{x},\mathrm{y})$ *be the M-P of L(S(H)). Then the Revan indices are computed as,*

1) $HR_1(\mathrm{L(S(}H)) = 1640,$
2) $HR_2(\mathrm{L(S(}H)) = 2441,$
3) ${}^m R_1(\mathrm{L(S(}H)) = 31.11,$
4) ${}^m R_2(\mathrm{L(S(}H)) = 50.075,$
5) $YR(\mathrm{L(S(}H)) = 2414.$

Proof: Let

$$M(L(S(H));\mathrm{x},\mathrm{y}) = 2x^1y^3 + 3x^1y^4 + 22x^2y^2 + 3x^2y^3 + 19x^2y^4 + 10x^3y^3 + 2x^3y^4 + 36x^4y^4.$$

Then

$$Q_{x(-5)} Q_{y(-5)} M(L(S(H));\mathrm{x},\mathrm{y}) = 2x^{-4}y^{-2} + 3x^{-4}y^{-1} + 22x^{-3}y^{-3} + 3x^{-3}y^{-2} + 19x^{-3}y^{-1} + 10x^{-2}y^{-2} + 2x^{-2}y^{-1} + 36x^{-1}y^{-1},$$

$$D_x^{\,2}(Q_{x(-5)}Q_{y(-5)}M(L(S(H));\mathrm{x},\mathrm{y}))$$
$$= 32x^{-4}y^{-2}+48x^{-4}y^{-1}+198x^{-3}y^{-3}+27x^{-3}y^{-2}+171x^{-3}y^{-1}+40x^{-2}y^{-2}$$
$$+8x^{-2}y^{-1}+36x^{-1}y^{-1},$$

$$D_y^{\,2}(Q_{x(-5)}Q_{y(-5)}M(L(S(H));\mathrm{x},\mathrm{y}))$$
$$= 8x^{-4}y^{-2}+3x^{-4}y^{-1}+198x^{-3}y^{-3}+12x^{-3}y^{-2}+19x^{-3}y^{-1}+40x^{-2}y^{-2}$$
$$+2x^{-2}y^{-1}+36x^{-1}y^{-1},$$

$$D_xD_y(Q_{x(-5)}Q_{y(-5)}M(L(S(H));\mathrm{x},\mathrm{y}))$$
$$= 16x^{-4}y^{-2}+12x^{-4}y^{-1}+198x^{-3}y^{-3}+18x^{-3}y^{-2}+57x^{-3}y^{-1}+40x^{-2}y^{-2}$$
$$+4x^{-2}y^{-1}+36x^{-1}y^{-1},$$

$$D_x^2D_y^2(Q_{x(-5)}Q_{y(-5)}M(L(S(H));\mathrm{x},\mathrm{y}))$$
$$= 128x^{-4}y^{-2}+48x^{-4}y^{-1}+1782x^{-3}y^{-3}+108x^{-3}y^{-2}+171x^{-3}y^{-1}$$
$$+160x^{-2}y^{-2}+8x^{-2}y^{-1}+36x^{-1}y^{-1},$$

$$J(Q_{x(-5)}Q_{y(-5)}M(L(S(H));\mathrm{x},\mathrm{y}))$$
$$= 2x^{-4}x^{-2}+3x^{-4}x^{-1}+22x^{-3}x^{-3}+3x^{-3}x^{-2}+19x^{-3}x^{-1}+10x^{-2}x^{-2}+2x^{-2}x^{-1}$$
$$+36x^{-1}x^{-1}$$
$$= 2x^{-6}+3x^{-5}+22x^{-6}+3x^{-5}+19x^{-4}+10x^{-4}+2x^{-3}+36x^{-2},$$

$$S_xJ(Q_{x(-5)}Q_{y(-5)}M(L(S(H));\mathrm{x},\mathrm{y}))$$
$$= \int_0^x\frac{2t^{-6}}{t}dt+\int_0^x\frac{3t^{-5}}{t}dt+\int_0^x\frac{22t^{-6}}{t}dt+\int_0^x\frac{3t^{-5}}{t}dt+\int_0^x\frac{19t^{-4}}{t}dt+\int_0^x\frac{10t^{-4}}{t}dt$$
$$+\int_0^x\frac{2t^{-3}}{t}dt+\int_0^x\frac{36t^{-2}}{t}dt$$
$$= \frac{-2x^{-6}}{6}+\frac{-3x^{-5}}{5}+\frac{-22x^{-6}}{6}+\frac{-3x^{-5}}{5}+\frac{-19x^{-4}}{4}+\frac{-10x^{-4}}{4}$$
$$+\frac{-2x^{-3}}{3}+\frac{-36x^{-2}}{2},$$

$$S_y(Q_{x(-5)}Q_{y(-5)}M(L(S(H));\mathrm{x},\mathrm{y}))$$
$$= \int_0^y\frac{2x^{-4}t^{-2}}{t}dt+\int_0^y\frac{3x^{-4}t^{-1}}{t}dt+\int_0^y\frac{22x^{-3}t^{-3}}{t}dt+\int_0^y\frac{3x^{-3}t^{-2}}{t}dt$$
$$+\int_0^y\frac{19x^{-3}t^{-1}}{t}dt+\int_0^y\frac{10x^{-2}t^{-2}}{t}dt+\int_0^y\frac{2x^{-2}t^{-1}}{t}dt+\int_0^y\frac{36x^{-1}t^{-1}}{t}dt$$

$$= \frac{2x^{-4}y^{-2}}{-2}+\frac{3x^{-4}y^{-1}}{-1}+\frac{22x^{-3}y^{-3}}{-3}+\frac{3x^{-3}y^{-2}}{-2}+\frac{19x^{-3}y^{-1}}{-1}+\frac{10x^{-2}y^{-2}}{-2}$$
$$+\frac{2x^{-2}y^{-1}}{-1}+\frac{36x^{-1}y^{-1}}{-1},$$

$$(S_xS_y(Q_{x(-5)}Q_{y(-5)}M(L(S(H));\mathrm{x},\mathrm{y})))$$
$$= \frac{2y^{-2}}{-2}\int_0^x \frac{t^{-4}}{t}dt+\frac{3y^{-1}}{-1}\int_0^x \frac{t^{-4}}{t}dt+\frac{22y^{-3}}{-3}\int_0^x \frac{t^{-3}}{t}dt$$
$$+\frac{3y^{-2}}{-2}\int_0^x \frac{t^{-3}}{t}dt+\frac{19y^{-1}}{-1}\int_0^x \frac{t^{-3}}{t}dt+\frac{10y^{-2}}{-2}\int_0^x \frac{t^{-2}}{t}+\frac{2y^{-1}}{-1}\int_0^x \frac{t^{-2}}{t}+\frac{36y^{-1}}{-1}\int_0^x \frac{t^{-1}}{t}$$
$$= \frac{2y^{-2}x^{-4}}{8}+\frac{3y^{-1}x^{-4}}{4}+\frac{22y^{-3}x^{-3}}{9}+\frac{3y^{-2}x^{-3}}{6}+\frac{19y^{-1}x^{-3}}{3}$$
$$+\frac{10y^{-2}x^{-2}}{4}+\frac{2y^{-1}x^{-2}}{2}+\frac{36y^{-1}x^{-1}}{1},$$

$$D_x^{\,3}(Q_{x(-5)}Q_{y(-5)}M(L(S(H));\mathrm{x},\mathrm{y}))$$
$$= -128x^{-4}y^{-2}-192x^{-4}y^{-1}-594x^{-3}y^{-3}-81x^{-3}y^{-2}-513x^{-3}y^{-1}$$
$$-80x^{-2}y^{-2}-16x^{-2}y^{-1}-288x^{-1}y^{-1},$$

$$D_y^{\,3}(Q_{x(-5)}Q_{y(-5)}M(L(S(H));\mathrm{x},\mathrm{y}))$$
$$= -16x^{-4}y^{-2}-3x^{-4}y^{-1}-594x^{-3}y^{-3}-24x^{-3}y^{-2}-19x^{-3}y^{-1}$$
$$-80x^{-2}y^{-2}-2x^{-2}y^{-1}-36x^{-1}y^{-1},$$

Now we have,

1. $[(D_x^2+D_y^2+2D_xD_y)(Q_{x(-5)}Q_{y(-5)}M(L(S(H));\mathrm{x},\mathrm{y}))]\big|_{(x,y)=(1,1)}=1640,$
2. $[D_x^{\,2}D_y^{\,2}(Q_{x(-5)}Q_{y(-5)}M(L(S(H));\mathrm{x},\mathrm{y}))]\big|_{(x,y)=(1,1)}=2441,$
3. $-[S_xJ(Q_{x(-5)}Q_{y(-5)}M(L(S(H));\mathrm{x},\mathrm{y}))]\big|_{x=1}=31.11,$
4. $(S_xS_y(Q_{x(-5)}Q_{y(-5)}M(L(S(H));\mathrm{x},\mathrm{y})))\big|_{(x,y)=(1,1)}=50.075,$
5. $-[(D_x^{\,3}+D_y^{\,3})(Q_{x(-\Delta-\partial)}Q_{y(-\Delta-\partial)}M(L(S(H));\mathrm{x},\mathrm{y})]\big|_{(x,y)=(1,1)}=2414,$

4. MATLAB coding for topological indices

In this section, we provide MATLAB coding for the calculation of TIs using M-polynomial and the relationships proved in Section 2.

The first hyper Revan index from M-polynomial coding is computed as,

```
clear
clc
n=input('Enter number of M-polynomial sentence:')
A=zeros(n,3)
for i=1:n
   for j=1:3
      A(i,j)=input('For  each  term,  enter  the  polynomial
coefficient, the x power, and the y power, respectively ');
   end
end
syms x y
mHxy=0
for i=1:n
      mHxy=mHxy+A(i,1)*x^A(i,2)*y^A(i,3);
   end
mHxy
Delta=input('enter maximum degree of graph:')
dlta=input('enter minimum deg
ree of graph:')
f=-Delta-dlta
B=zeros(n,3)
for i=1:n
   for j=1:3
      if j==1
      B(i,j)=A(i,j)
      else
         B(i,j)=A(i,j)+f
      end
   end
end
B
FA=0
for i=1:n

      FA=FA+B(i,1)*x^B(i,2)*y^B(i,3);
   end
FA
syms x y
DX=diff(FA,x)*x
syms x y
DX2=diff(DX,x)*x
subs(DX2,[x,y],[1,1])
syms x y
DY=diff(FA,y)*y
DY2=diff(DY,y)*y
```

```
subs(DY2,[x,y],[1,1])
DXDY=0
for i=1:n
DXDY=DXDY+B(i,1)*B(i,2)*B(i,3)*x^B(i,2)*y^B(i,3)
end
syms x y
subs(DXDY,[x,y],[1,1])
HR1=[(DX2+DY2+2*DXDY)]
syms x y
subs(HR1,[x,y],[1,1]).
```

The second hyper Revan index from M-polynomial coding is computed as,

```
clear
clc
n=input(' Enter number of M-polynomial sentence ');

A=zeros(n,3);
for i=1:n
   for j=1:3
      A(i,j)=input(' For each term, enter the polynomial
coefficient, the x power, and the y power, respectively ');
   end
end
syms x y
mHxy=0;
for i=1:n

      mHxy=mHxy+A(i,1)*x^A(i,2)*y^A(i,3);
   end
mHxy
Delta=input('enter maximum degree of graph:');
dlta=input('enter minimum degree of graph:');
f=-Delta-dlta;
B=zeros(n,3);
for i=1:n
   for j=1:3
      if j==1
      B(i,j)=A(i,j);
      else
         B(i,j)=A(i,j)+f;
      end
   end
end
B;
FA=0;
```

```
for i=1:n

     FA=FA+B(i,1)*x^B(i,2)*y^B(i,3);
  end
FA
DX=diff(FA,x)*x
DX2=diff(DX,x)*x
DY=diff(FA,y)*y
DY2=diff(DY,y)*y
DX2DY2=0;
for i=1:n
DX2DY2=DX2DY2+B(i,1)*B(i,2)^2*B(i,3)^2*x^B(i,2)*y^B(i,3);
end
DX2DY2
HR2=DX2DY2
syms x y
subs(HR2,[x,y],[1,1]).
```

6. Conclusion

In this article, different types of TIs were calculated for the molecular structure, subdivision structure, and para-line structure of Glass using M-polynomial, and their relationships were proved. MATLAB coding was also provided for the calculation of these indices. These topological indices can be used to predict various physical and chemical properties of Glass and other compounds, providing an efficient and cost-effective alternative to laboratory testing.

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Received July, 2023