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Maximum subgraph problem of hiked hypercube and its linear arrangement

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Abstract

Among interconnection topologies, the hypercube is both popular and highly attractive, extensively used in parallel computing and network design due to its versatility and scalability. Its desirable properties have inspired the development of several variants, including the folded hypercube, crossed cube, augmented cube, locally twisted cube, enhanced hypercube, twisted cube, extended hypercube, Mobius cube, and Fibonacci cube, each improving specific aspects such as fault tolerance, communication efficiency, and scalability. A generalized hypercube Q_n (S), where $S \subseteq \{1, 2, \dots, n\}$, is an extension of the classic hypercube graph. It has a vertex set $\{0, 1\}^n$, and edges are formed between vertices whose Hamming distance belongs to the set S. This structure allows for flexible modeling of relationships and connectivity, making it a valuable tool in parallel computing, network design, and combinatorial optimization. This paper focuses on the generalized structure of the hiked hypercube when $S = \{1, 2, \dots, j\} \subseteq \{1, 2, \dots, n\}$, and addresses the maximum subgraph problem. Additionally, we determine the minimum linear arrangement of this topology using graph embedding techniques, highlighting its potential in optimizing interconnection networks.

Subject Classification (2020): *05C60, 05C62, 05C75.*

Keywords: *Maximum subgraph problem, Wirelength, Linear arrangement, Hiked hypercube.*

1. Introduction

Graph embedding serves as an effective analytical approach in which a guest graph is mapped onto a host graph for analyzing and simulating complex networks. It finds extensive applications in areas such as VLSI design, data structure optimization, biological system modeling, parallel computing, and structural engineering. Through this technique, designers can assess performance measures, detect potential bottlenecks, and refine network structures to enhance efficiency and scalability [1].

The hypercube is a widely used interconnection topology in parallel computing because of its scalability, symmetry, and low diameter [3]. Various hypercube variants have been proposed to enhance communication efficiency, fault tolerance, and reduce diameter. The folded hypercube connects opposite nodes to reduce diameter and improve fault tolerance [5]. The augmented cube adds extra edges for redundancy and reliability, the crossed cube rearranges connections to reduce diameter and enhance communication efficiency, and the twisted cube modifies connections to improve routing and lower diameter [4]. The enhanced hypercube boosts performance and robustness through additional links [6]. Several recent studies have explored structural and domination-related properties of graph networks, as well as techniques related to knowledge graph embeddings and power domination, which contribute to understanding structural characteristics and optimization in complex networked systems [11, 12, 13, 14].

The generalized hypercube $Q_n(S)$ is a flexible extension of the traditional n-dimensional hypercube graph. In this form, vertices are represented as binary n-tuples, and edges are defined based on a subset $S \subseteq \{1,2,\dots,n\}$ [2], influencing the Hamming distance between connected vertices. This flexibility allows for different graph structures, such as the classical hypercube when $S = \{1\}$, Laborde-Mulder graphs when $S = \{1,2k\}$, halfcubes when $S = \{1,2k-1\}$), hyper-octahedron graphs when $S = \{1,2,\dots,n-1\}$, and a complete graph K_{2^n} when $S = \{1,2,\dots,n\}$ [1, 2]. $Q_n(S)$ is connected for all choices of S, except when $S = n$ or S consists entirely of even numbers [1].

A significant variation of the generalized hypercube is called the hiked hypercube, when $S = \{1,2,\dots,j\}$, with $1 \le j \le n$. This modification further enhances the flexibility of the graphs structure and connectivity. The generalized hypercube is characterized by its regularity and scalability, making it applicable in fields like parallel computing, combinatorial optimization, and communication systems. It offers advantages such as fault tolerance, scalability, and adaptability, making it well-suited for complex network designs.

The hiked hypercube allows connections between nodes whose binary addresses differ by up to j bits, increasing flexibility and creating more direct communication paths. This improves fault tolerance and reduces the network diameter, leading to lower communication latency. As j increases, the network becomes denser, enhancing performance but also increasing the number of connections per node.

For example, consider a 4-dimensional hypercube Q^4 with nodes labeled as binary strings of length 4. In the normal hypercube, each node connects to 4 others, differing by exactly one bit. In a hiked hypercube with $j = 2$, each node could connect to others differing by one or two bits, significantly increasing the number of connections and providing additional communication paths. The hiked hypercube improves on the classical hypercube by enabling fast communication in parallel computing and scalable connectivity in distributed systems. It is also robust and adaptable, making it useful for high-performance networks. By changing the parameter j, the topology can be adjusted to achieve better scalability, fault tolerance, and communication efficiency.

Two separate formulations of the edge isoperimetric problem on a graph G has been examined in the literature both of which are proven to be NP-complete [7]. These problems have significant applications, such as determining precise wirelength in graph-based embeddings for parallel networks, biological models, and data structures. Additionally, they are used to solve graph partitioning problems and calculate the bisection width of networks, both of which are essential for maximizing resource allocation and communication effectiveness across a variety of systems [8]. This paper investigates the *Maximum Subgraph Problem (MSP)*, which involves finding a subgraph of a graph G with n vertices, subject to a size constraint $m < n$. find a subset $K \subseteq V$ with $|K| = m$ that induces the maximum possible number of edges. i.e., $I_G(m) = \max_{\substack{K\subseteq V \\ |K|=m}} |I_G(K)|$, where $I_G(K) = \{(uv) \in E | u, v \in K\}$. A subset K is called an *optimal set* if $|K| = m$ and $I_G(K) = I_G(m)$ [8].

2. Preliminaries

This section covers the basic terminology and underlying framework for graph embedding problems. Graph embedding refers to the process of mapping the vertices and edges of a guest graph A into a host graph B, which is often utilized for modeling interconnection networks. The embedding of one finite graph A into another finite graph B involves mapping the nodes and edges of A into the nodes and edges of B while preserving their connections are defined as follows.

Definition 2.1: [9] An embedding of A into B is given by (ϕ, P_{ϕ}), where:

- ϕ is an injective mapping from $V(A)$ to $V(B)$.
- P_{ϕ} assigns each edge $e = (xy) \in E(A)$ to a path in B joining the images of x and y under ϕ, i.e., $P_{\phi}(e)$ is the corresponding path in the host graph.

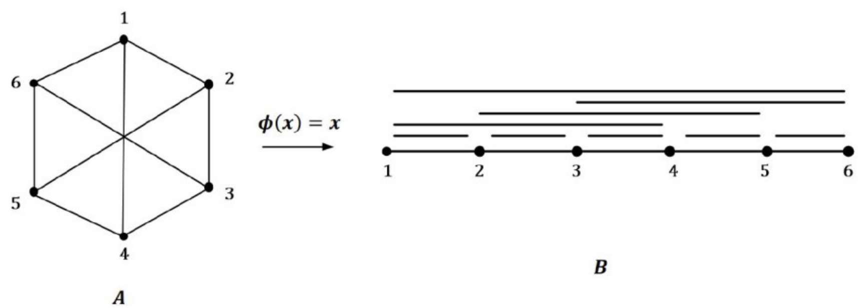

Figure 1
The wiring diagram for embedding circulant graph A into the path B has $WL_{\phi}(A, B) = 19$

Definition 2.2: [8] Let $\phi: A \to B$ be an embedding. For an edge $e \in E(B)$, its edge congestion is the number of edges $(xy) \in E(A)$ that ϕ maps onto e whose corresponding path $P_{\phi}(xy)$, connecting $\phi(x)$ and $\phi(y)$, passes through e, and it is represented by $EC_{\phi}(e)$. That is, $EC_{\phi}(e) = |(xy) \in E(A): e \in P_{\phi}(xy)|$. Hence,

$$EC(A, B) = \min_{\phi: A \to B} \max_{e \in E(B)} EC_{\phi}(e).$$

Also, if D is any subset of $E(B)$ then

$$EC_{\phi}(D) = \sum_{e \in D} EC_{\phi}(e).$$

Definition 2.3: [8] Given an embedding ϕ of A into B, the wirelength is measured as the sum of edge congestions across all edges of B, $WL_{\phi}(A, B) = \sum_{e \in E(B)} EC_{\phi}(e)$. The minimal wirelength achieved over all possible embeddings is denoted by

$$WL(A,B) = \min_{\phi:A\to B} WL_{\phi}(A,B).$$

Figure 1 shows the embedding of A into B and the corresponding wirelength under the mapping ϕ.

Definition 2.4: [8] Let $S \subseteq V(A)$ be a nonempty subset of vertices. The induced subgraph on S, denoted by $A[S]$, includes all vertices in S and every edge of A whose endpoints are both in S. The expression $A\backslash S$ represents the subgraph induced by the vertices in $V\backslash S$, that is, $A[V\backslash S]$.

Lemma 2.5: (Congestion Lemma) [10] *Let $\phi: A \to B$ be an embedding, and let $S \subseteq E(B)$ be an edge cut that splits B into two connected subgraphs, B_1 and B_2. Let $A_1 = A[\phi^{-1}(V(B_1))]$ and $A_2 = A[\phi^{-1}(V(B_2))]$ denote the subgraphs of A induced by the preimages of B_1 and B_2, respectively. The edge cut S adheres to the following criteria,*

- *Each edge xy within A_i for $i = 1,2$ is mapped by $P_\phi(xy)$ along a path that avoids all edges in S.*
- *Any edge xy with $x \in A_1$ and $y \in A_2$ is assigned by $P_\phi(xy)$ to a path that crosses exactly one edge in S.*
- *The vertex set of A_1 forms an optimal set.*

Then, $EC_\phi(S) = r|V(A_1)| - 2|E(A_1)|$ and $EC_\phi(S)$ is minimum.

Lemma 2.6: (Partition Lemma) [9] *Consider an embedding $\phi: A \to B$, where the edge set $E(B)$ is partitioned into $S_1, S_2, \dots, S_l$, with each S_k corresponding to an edge cut of the host graph that satisfies the conditions of Lemma 2.5. Then,*

$$WL(A,B) = WL_{\phi}(A,B) = \sum_{k=1}^{l} EC_{\phi}(S_k).$$

3. The Hiked Hypercube

The hypercube is a highly symmetric structure fundamental to graph theory, computer science, and high-dimensional studies. Various modifications, such as the folded, augmented, enhanced, crossed, and twisted hypercubes, have been proposed to improve fault tolerance, communication efficiency, and scalability. These variants enhance the classical hypercube for network applications. Building on them, the hiked hypercube allows connections between nodes differing by up to j bits. This increases flexibility and robustness and introduces generalized topological indices.

Definition 3.1: [1] Let n be a positive integer and $S \subseteq \{1,2,\cdots,n\}$. The generalized hypercube $Q_n(S)$ has vertex set $V(Q_n)$, with edges between vertices at Hamming distances in S, $E(Q_n(S)) = \{(u,v) | d_{Q_n}(u,v) \in S\}$. It is a regular graph of degree $\sum_{i\in S} \binom{n}{i}$.

In a similar manner, we define another variation of the generalized hypercube, known as the hiked hypercube, which is mathematically expressed as follows.

Definition 3.2: For integers $n \geq j \geq 1$, the n-dimensional hiked hypercube, denoted by HQ_j^n, is defined on the vertices of Q^n with the edge set $E(HQ_j^n) = \{(uv) | d_{Q^n}(u,v) \leq j\}$.

Note that, when $j = 1$ then the hiked hypercube HQ_j^n is a normal hypercube Q^n and when $j = n$ the hiked hypercube HQ_j^n is a complete graph on 2^n vertices, K_{2^n}. For illustration, the hiked hypercubes HQ_1^4 (is isomorphic to Q^4), HQ_2^4, and HQ_3^4 is in Figures 2 and 3.

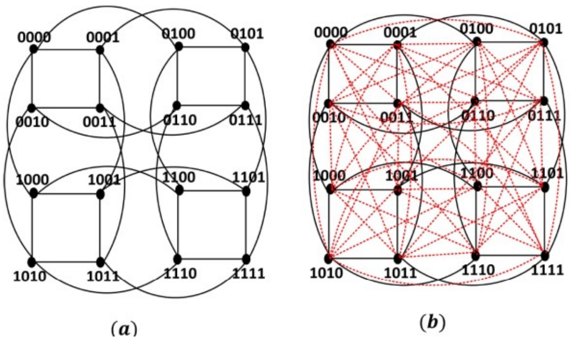

Figure 2
Hiked hypercube (a) HQ_1^4 (b) HQ_2^4

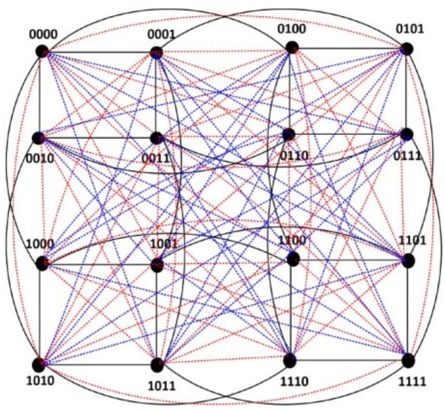

Figure 3
Hiked hypercube HQ_3^4

The n-dimensional hiked hypercube has a total number of vertices given by $|V| = 2^n$. The overall edge count is determined by the number of differing bits, denoted by j, and is given by $|E| = 2^{n-1} \sum_{i=1}^{j} \binom{n}{i}$, where $\binom{n}{i}$ denotes the binomial coefficient, representing the number of ways to select i differing bits from n dimensions. Each node in the hiked hypercube is connected to $d = \sum_{i=1}^{j} \binom{n}{i}$ vertices, representing the total possible connections up to j differing bits.

The diameter of the hiked hypercube represents the maximum distance, which is the largest number of hops needed to travel between any two nodes, and is given by $D = \left\lceil \frac{n}{j} \right\rceil$. when $j = n$, the hiked hypercube becomes a complete graph K_{2^n} and the diameter is 1. For $j < n$, the diameter becomes j, as connections are restricted to paths involving at most j differing bits.

The hiked hypercube is Eulerian if all vertex degrees $d = \sum_{i=1}^{j} \binom{n}{i}$ are even. It is always Hamiltonian due to its symmetric and recursive structure, allowing a cycle that visits each vertex exactly once. Like the standard hypercube, it maintains strong traversal properties. As the parameter j increases, the graph's connectivity and fault tolerance improve. This results in greater robustness and more alternative paths between nodes. The topological properties of hiked hypercube and comparison with hypercube variants are summarized in Table 1.

Two graphs are isomorphic if a bijective mapping exists between their vertex sets that preserve the adjacency relationship. For hiked hypercubes, the isomorphism properties arise from their symmetric and recursive structure.

The recursive construction of hiked hypercubes implies that subgraphs within a hiked hypercube, such as lower-dimensional hypercubes, are also isomorphic to standard hypercubes of corresponding dimensions.

Table 1
Topological properties for hiked hypercube HQ_j^n and comparison with hypercube variants

Parameters	Q^n [3]	CQ^n [4]	AQ^n [4]	FQ^n [5]	TQ^n[4]	$Q_{n,k}$ [6]	HQ_j^n
Nodes	2^n	2^n	2^n	2^n	2^n	2^n	2^n
Edges	$n2^{n-1}$	$n2^{n-1}$	$2^{n-1} \times (2n-1)$	$(n+1) \times 2^{n-1}$	$n2^{n-1}$	$(n+1) \times 2^{n-1}$	$2^{n-1} \times \sum_{i=1}^{j} \binom{n}{i}$
Regularity	n	n	$(2n-1)$	$(n+1)$	n	$(n+1)$	$\sum_{i=1}^{j} \binom{n}{i}$
Diameter	n	$\left\lceil \frac{n+1}{2} \right\rceil$	$\left\lceil \frac{n}{2} \right\rceil$	$\left\lceil \frac{n}{2} \right\rceil$	$\left\lceil \frac{n+1}{2} \right\rceil$	$\left\lceil \frac{n-k}{2} \right\rceil + k$	$\left\lceil \frac{n}{j} \right\rceil$
Connectivity	n	n	$2n-1$	$n+1$	n	$n+1$	$\sum_{i=1}^{j} \binom{n}{i}$
Eulerian	Yes	Yes	No	No	Yes	No	Yes
Hamiltonian	Yes	Yes	Yes	Yes	Yes	Yes	Yes

4. Main Results

We first focus on solving the maximum subgraph problem for the hiked hypercube. Then, we find its minimum linear arrangement.

4.1 *Maximum subgraph problem*

The maximum subgraph problem for a given graph A focuses on identifying the maximum induced subgraph K with exactly l vertices, which contains the maximum number of edges present in any subgraphs that can be induced by l vertices. In the context of embedding, this problem refers to finding the maximum subgraph within an embedded graph that maintains a specific desired property or structure. The process involves extracting the most relevant or critical segment of the embedded data while retaining its connections within the overall graph.

In this work, we study the MSP in hiked hypercubes, highlighting its strong link to the wirelength problem. These connections are crucial for analyzing edge congestion in path embeddings and optimizing vertex arrangement within the embedded graph [10].

Theorem 4.1: *Let HQ_j^n be the n-dimensional hiked hypercube, $1 \leq j \leq n$ and $n \geq 1$. Let $m = 2^{t_1} + 2^{t_2} + \cdots + 2^{t_l}$ such that $n \geq t_1 > t_2 > \cdots > t_l \geq 0$. Then*

$$I_m(HQ_j^n) = \left[2^{t_1-1} \times \sum_{i=1}^{j} \binom{t_1}{i} + 2^{t_2-1} \times \sum_{i=1}^{j} \binom{t_2}{i} + \cdots + 2^{t_l-1} \times \sum_{i=1}^{j} \binom{t_l}{i}\right] + \left[\sum_l \left[2^{t_l} \sum_{p=1}^{i-1} \sum_{i=1}^{j} \binom{t_p}{i-1}\right]\right].$$

Proof: Let's examine the structure of $I_m(HQ_j^n)$, here $m = 2^{t_1} + 2^{t_2} + \cdots + 2^{t_l}$. Since $I_m(HQ_j^n)$ contains subcubes $HQ_j^{t_1}, HQ_j^{t_2}, \dots, HQ_j^{t_l}$ such that $HQ_j^{t_i}$ is adjacent to $HQ_j^{t_{i-1}}$ for $i = 2,3,\dots,l$. This implies that there are $2^{t_i} \times \sum_{p=1}^{i-1} \sum_{i=1}^{j} \binom{t_p}{i-1}$ edges between $HQ_j^{t_i}$ and $HQ_j^{t_k}$ for all k, $1 \leq k \leq i-1$. Thus $HQ_j^{t_i}$ contributes $\sum_{i=2}^{l} \left[2^{t_i} \times \sum_{p=1}^{i-1} \sum_{i=1}^{j} \binom{t_p}{i-1}\right]$ edges to $I_m(HQ_j^n)$. Also, $HQ_j^{t_r}$ has $2^{t_r-1} \times \sum_{i=1}^{j} \binom{t_r}{i}$ edges within itself, $1 \leq r \leq l$. Thus $HQ_j^{t_r}$ contributes $\sum_{r=1}^{l} \left[2^{t_r-1} \times \sum_{i=1}^{j} \binom{t_r}{i}\right]$ edges to $I_m(HQ_j^n)$. As a result, the total number of edges in HQ_j^n induced by m vertices is

$$\sum_{r=1}^{l} \left[2^{t_r-1} \times \sum_{i=1}^{j} \binom{t_r}{i}\right] + \sum_{i=2}^{l} \left[2^{t_i} \times \sum_{p=1}^{i-1} \sum_{i=1}^{j} \binom{t_p}{i-1}\right].$$

Hence, the theorem is proved.

Remark: When $j = 1$, HQ_1^n is isomorphic to Q^n, and the maximum subgraph problem is also explored in [9, 10].

4.2 *Minimum linear arrangement*

The *minimum linear arrangement* (MinLA) optimizes the placement of tasks or processors along a line to minimize communication costs by reducing the total distance between communicating nodes [9]. This approach decreases delays and data transfer overhead, improving system efficiency and execution speed in distributed systems. MinLA is particularly useful when tasks are interdependent. First introduced by Harper in 1964, it was initially studied in the context of designing error-correcting codes with minimal errors. The process involves embedding a graph G onto a path [7]. The resulting wirelength is the same as the minimum linear arrangement of G, represented as $\mathrm{MinLA}(G)$ [9].

Theorem 4.2: *Let A be the n-dimensional hiked hypercube HQ_j^n, $n \geq j \geq 1$, and B be the path on 2^n vertices P_{2^n}. Then*

$$WL(A,B) = \sum_{m=1}^{2^n-1} \left[\sum_{i=1}^{j} \binom{n}{i}(m) - 2 \times I_m(A_k)\right].$$

Proof: Let $a_0 a_1 \cdots a_{n-1}$ be the n^{th}-bit binary representation of $HQ_j^n, n \geq 1$. Since HQ_1^n is a spanning subgraph of HQ_j^n, $j \geq 1$, and label the vertices of HQ_j^n by $x = \sum_{i=0}^{n-1} a_i \times 2^i$, where a_i is the i^{th} bit string of HQ_j^n. Since B is a path on 2^n vertices, from left to right, label the vertices $1, 2, \ldots, 2^n$ with each vertex connected sequentially to its neighbors, forming a linear arrangement. Let $\phi(x) = x + 1$ for every $x \in V(A)$, and let $P_\phi(uv)$ be the shortest path from $\phi(u)$ to $\phi(v)$ in P_{2^n} for each edge $(uv) \in E(A)$.

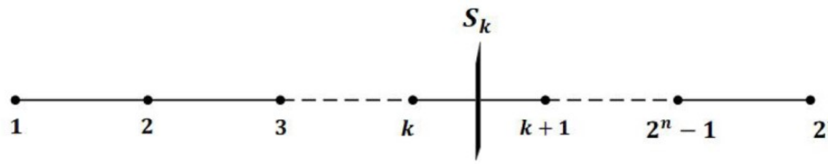

Figure 4
Edge cut of P_{2^n}

For P_{2^n}, let S_k be an edge cut, given as $S_k = (k, k+1)$, where $1 \leq k \leq 2^n - 1$, see Figure 4. For $1 \leq k \leq 2^n - 1$, the edge set $E(P_{2^n}) \backslash S_k$ has two components B_k and $\overline{B_k}$. Let $V(B_k) = \{1, 2, \ldots, k\}$. Let $A_k = A[\phi^{-1}(V(B_k))]$ and $\overline{A_k} = A[\phi^{-1}(V(\overline{B_k}))]$. By Lemma 2.5, A_k is optimal with k vertices. Then, for $1 \leq k \leq 2^n - 1$, $EC_\phi(S_k)$ is minimum and is given by

$$EC_\phi(S_k) = \sum_{i=1}^{j} \binom{n}{i}(m) - 2 \times I_m(A_k).$$

Since Lemma 2.6 states that the wirelength is minimum, we obtain

$$WL(A,B) = \sum_{m=1}^{2^n-1} \left[\sum_{i=1}^{j} \binom{n}{i}(m) - 2 \times I_m(A_k)\right].$$

5. Implementation of Hiked Hypercube

The hiked hypercube offers a flexible and scalable interconnection network that is applicable in a wide range of real-world systems. Its unique properties, including reduced diameter, increased fault tolerance, and efficient routing, make it suitable for numerous applications, particularly in parallel and distributed computing, communication systems, and large-scale networks.

In *Parallel computing systems*, hiked hypercubes are used as interconnection networks in multiprocessor systems. Their j^{th}-bit links reduce communication latency by creating shortcuts between nodes. This improves data transfer efficiency and load balancing and reduces congestion. They are ideal for parallel tasks such as matrix multiplication and graph traversal.

Modern data centers hiked hypercubes reduce the average path length between servers, improving throughput and minimizing communication delays. The j^{th}-bit connectivity enhances fault tolerance and maintains reliable data transfer during failures. These properties support efficient data replication, redundancy, and optimized routing.

In *fault-tolerant communication networks*, hiked hypercubes provide alternative communication paths during link or node failures, increasing the fault tolerance of critical systems. Their rich connectivity allows efficient rerouting, ensuring continuous communication. This is vital in military systems, distributed sensor networks, and disaster recovery frameworks.

In *Neural Networks and IoT Applications*, hiked hypercubes enable fast, hierarchical communication in AI accelerators, supporting synchronized deep learning operations. In IoT networks, they offer scalable, low-latency data transfer with high robustness. These features make them suitable for smart cities, industrial automation, and smart grid applications.

6. Concluding Remarks

The hiked hypercube HQ_j^n can be viewed as a specific case of the generalized hypercube $Q_n(S)$, where $S = \{1,2, \dots, j\}$. While the generalized hypercube encompasses a broader range of graph structures and applications, the hiked hypercube is focused on specific structural properties and practical optimization problems. This paper addresses the maximum subgraph problem for the hiked hypercube that differs up to the j^{th}-bit. Furthermore, the study explores the minimum linear arrangement of the hiked hypercube and some of the open problems are listed here.

Problem 1: Finding the wirelength of embedding between variants of hypercubes such as Q^n, CQ^n, AQ^n, FQ^n, TQ^n, $Q_{n,k}$, and HQ_j^n for all n and j, $1 \leq j \leq n$.

Problem 2: The problem of determining the wirelength of an embedding for hiked hypercube into other host graphs such as grid, cylinder, torus, interval graph, chordal graph, and Knödel graph.

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