

Fundamental properties and results of interval type-2 fuzzy rhotrix : A research perspective

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Abstract

This research paper discusses the concept of interval type - 2 fuzzy rhotrix, by focusing its definitions, properties, and significant results. The study begins by defining interval type - 2 fuzzy rhotrix as an extension of traditional rhotrix theory that incorporates interval type - 2 fuzzy logic, with uncertainty and imprecision. The algebraic properties such as addition, multiplication, identity elements, determinant, transpose, codeterminant function and Index of IT2FR are systematically discussed, with rigorous proofs and examples provided to substantiate these fundamental characteristics, this work aims to enhance the understanding of interval type - 2 fuzzy rhotrix in both mathematical theory and real-world situations.

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1. Introduction

“Interval Type-2 Fuzzy Rhotrix” (IT2FR) is an advanced mathematical framework that extends the traditional fuzzy logic by incorporating uncertainty. This research paper gives the foundational concepts of interval type-2 fuzzy rhotrix with its properties. The paper begins by defining interval type-2 fuzzy rhotrix by expressing their ability to project uncertainty effectively than type-1 fuzzy rhotrix. It discusses the algebraic operations such as addition,

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Multiplication, inverse, determinant, coupled IT2FM (Interval Type 2 Fuzzy Matrix), Codeterminant, and Index of IT2FR. Overall, this study gives the comprehensive introduction to IT2FR, for future research and applications in many fields.

2. Literature review

Sola et al. [1] explore the concepts of “interval type 2 fuzzy sets and interval-valued fuzzy sets”, by discussing their generalization properties. Ajibade [2] describes mathematical constructs that extend classical matrix theory by arranging elements in a rhomboidal fashion and examines the fundamental operations. Sani [3] presents a novel multiplication technique in rhotrices. Isere [4] introduces the concept of rhotrix with natural numbers. Isere [5] discusses the properties of natural rhotrix, using diagonal function. Sani [6] describes a coupled matrix that involves rotating the columns of rhotrices by 45 degrees. Shyamal and Pal [7] introduced the concept of interval valued fuzzy matrices and studied their fundamental algebraic properties, providing a framework to handle uncertainty in matrix structures. Pal [8] presented several theoretical results on fuzzy matrices, including their operations and applications in decision making and optimization problems. Kumar and Gupta [9] explored the statistical applications of fuzzy matrices, demonstrating their usefulness in analyzing imprecise and uncertain data. Khan and Ahmed [10] examined the applications of rhotrix theory in discrete mathematical structures, particularly in combinatorial analysis and cryptography.

3. Interval Type 2 Fuzzy Rhotrix (IT2FR)

IT2FM is a mathematical structure that extends the concept of traditional fuzzy matrices by incorporating IT2F sets into its framework. In this context, each element of the matrix is represented as an IT2FE, which is characterized by upper and lower membership functions.

Definition 3.1: An IT2FM is $P_{IT2FM} = \begin{bmatrix} \widetilde{t}_{11} & \widetilde{t}_{12} & \cdots & \widetilde{t}_{1n} \\ \widetilde{t}_{21} & \widetilde{t}_{22} & \cdots & \widetilde{t}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{t}_{m1} & \widetilde{t}_{m2} & \cdots & \widetilde{t}_{mn} \end{bmatrix}$ with each

element $\widetilde{t}_{ij}$ is an IT2FE (Interval Type 2 Fuzzy Element) defined by its upper and lower membership functions.

Example 3.1: Consider a weather forecast of a particular location for 3 days, and the parameters are temperature, humidity, and precipitation.

	<i>Temperature</i>	<i>Humidity</i>	<i>Preci</i>
Day 1	[0.8, 1.0]	[0.6, 0.9]	[0.0, 0.2]
Day 2	[0.7, 1.0]	[0.5, 0.8]	[0.1, 0.3]
Day 3	[0.6, 0.9]	[0.7, 1.0]	[0.0, 0.1]

Definition 3.2: The IT2FR can be defined as follows:

$$R_{IT_2FR} = \begin{pmatrix} & & \tilde{r}_{11} & & \\ & \tilde{r}_{21} & \tilde{r}_{22} & \tilde{r}_{23} & \\ \tilde{r}_{31} & \tilde{r}_{32} & \tilde{r}_{33} & \tilde{r}_{34} & \tilde{r}_{35} \\ & & \vdots & & \\ & \tilde{r}_{n-1,1} & \tilde{r}_{n-1,2} & \tilde{r}_{n-1,3} & \\ & & \tilde{r}_{n1} & & \end{pmatrix}$$

Each element $\tilde{r}_{ij}$ is an IT2FE defined between $\mu_A^-(x)$ and $\mu_A^+(x)$ with n is odd.

Example 3.2: $P_{IT_2FR^3} = \begin{pmatrix} & [0.2, 0.5] & \\ [0.1, 0.6] & [0.3, 0.9] & [0.2, 0.5] \\ & [0.4, 0.5] & \end{pmatrix}$ is an example of IT2FR of order 3 and $[0.3, 0.9]$ is the heart of the IT2FR.

Definition 3.3: Addition and Subtraction of two IT2FR

Let P_{IT_2FR} , Q_{IT_2FR} be the two IT2FR and it is given by

$$P_{IT_2FR} = \begin{pmatrix} & [\alpha_{2L}, \alpha_{2U}] & [\alpha_{3L}, \alpha_{3U}] & [\alpha_{4L}, \alpha_{4U}] & \\ [\alpha_{5L}, \alpha_{5U}] & [\alpha_{6L}, \alpha_{6U}] & [\alpha_{7L}, \alpha_{7U}] & [\alpha_{8L}, \alpha_{8U}] & [\alpha_{9L}, \alpha_{9U}] \\ & [\alpha_{10L}, \alpha_{10U}] & [\alpha_{11L}, \alpha_{11U}] & [\alpha_{12L}, \alpha_{12U}] & \\ & & [\alpha_{13L}, \alpha_{13U}] & & \end{pmatrix}$$

$$Q_{IT_2FR} = \begin{pmatrix} & [\beta_{2L}, \beta_{2U}] & [\beta_{3L}, \beta_{3U}] & [\beta_{4L}, \beta_{4U}] & \\ [\beta_{5L}, \beta_{5U}] & [\beta_{6L}, \beta_{6U}] & [\beta_{7L}, \beta_{7U}] & [\beta_{8L}, \beta_{8U}] & [\beta_{9L}, \beta_{9U}] \\ & [\beta_{10L}, \beta_{10U}] & [\beta_{11L}, \beta_{11U}] & [\beta_{12L}, \beta_{12U}] & \\ & & [\beta_{13L}, \beta_{13U}] & & \end{pmatrix}$$

$$\text{Then } P_{IT_2FR} + Q_{IT_2FR} = \begin{pmatrix} & \widetilde{\kappa}_2 & \widetilde{\kappa}_3 & \widetilde{\kappa}_4 & \\ \widetilde{\kappa}_5 & \widetilde{\kappa}_6 & \widetilde{\kappa}_7 & \widetilde{\kappa}_8 & \widetilde{\kappa}_9 \\ & \widetilde{\kappa}_{10} & \widetilde{\kappa}_{11} & \widetilde{\kappa}_{12} & \\ & & \widetilde{\kappa}_{13} & & \end{pmatrix}$$

Where $\widetilde{\kappa}_l = [(\alpha_{iL} + \beta_{iL}), (\alpha_{iU} + \beta_{iU})]$ is IT2FE. Similarly, $P_{IT_2FR} - Q_{IT_2FR}$ is the IT2FR and $\widetilde{\kappa}_l = [(\alpha_{iL} - \beta_{iL}), (\alpha_{iU} - \beta_{iU})]$ is IT2FE. After operating the addition and subtraction, validate that each resulting interval is within the range of $[0, 1]$. This can be accomplished by clamping the values. Adjust the resulting interval $[x, y]$ as follows replace x by 0 if x is less than 0, and y by 1 if y is greater than 1. Suppose $x > y$, then adjust it to a default value $[0, 0]$.

Example 3.3:

$$\text{If } P_{IT2FR} = \left\langle \begin{array}{ccccc} & & [0.5, 0.9] & & \\ & [0.1, 0.9] & [0.3, 0.5] & [0.1, 0.5] & \\ [0.3, 0.7] & [0.4, 0.7] & [0, 0.5] & [0.4, 0.7] & [0.3, 0.6] \\ & [0.5, 0.6] & [0.2, 0.9] & [0.2, 0.9] & \\ & & [0.2, 0.3] & & \end{array} \right\rangle \text{ and}$$

$$Q_{IT2FR} = \left\langle \begin{array}{ccccc} & & [0.4, 0.8] & & \\ & [0.1, 0.8] & [0.4, 0.5] & [0.3, 0.4] & \\ [0.4, 0.9] & [0.3, 0.5] & [0.4, 0.6] & [0.2, 0.4] & [0.1, 0.4] \\ & [0.1, 0.2] & [0.3, 0.5] & [0.4, 0.5] & \\ & & [0.2, 0.6] & & \end{array} \right\rangle \text{ then}$$

$$P_{IT2FR} + Q_{IT2FR} = \left\langle \begin{array}{ccccc} & & [0.9, 1] & & \\ & [0.2, 1] & [0.7, 1] & [0.4, 0.9] & \\ [0.7, 1] & [0.7, 1] & [0.4, 1] & [0.8, 0.9] & [0.4, 1] \\ & [0.6, 0.8] & [0.5, 1] & [0.9, 1] & \\ & & [0.4, 0.9] & & \end{array} \right\rangle$$

$$P_{IT2FR} - Q_{IT2FR} = \left\langle \begin{array}{ccccc} & & [0.1, 0.1] & & \\ & [0, 0.1] & [0, 0] & [0, 0.1] & \\ [0, 0] & [0.1, 0.2] & [0, 0.1] & [0.2, 0.3] & [0, 0] \\ & [0.4, 0.4] & [0, 0.4] & [0, 0.4] & \\ & & [0, 0] & & \end{array} \right\rangle$$

Definition 3.4: Multiplication of two IT2FR
Heart Oriented Multiplication (HOM)

$$\text{Consider two IT2FR, } P_{IT2FR} = \left\langle \begin{array}{ccc} [\alpha_{1L}, \alpha_{1U}] & & \\ [\alpha_{2L}, \alpha_{2U}] & [\alpha_{3L}, \alpha_{3U}] & [\alpha_{4L}, \alpha_{4U}] \\ & [\alpha_{5L}, \alpha_{5U}] & \end{array} \right\rangle$$

$$Q_{IT2FR} = \left\langle \begin{array}{ccc} [\beta_{1L}, \beta_{1U}] & & \\ [\beta_{2L}, \beta_{2U}] & [\beta_{3L}, \beta_{3U}] & [\beta_{4L}, \beta_{4U}] \\ & [\beta_{5L}, \beta_{5U}] & \end{array} \right\rangle \text{ then the HOM defined}$$

by

$$P_{IT2FR} \circ Q_{IT2FR} = \left\langle \begin{array}{ccc} [\alpha_{1L}, \alpha_{1U}][\beta_{3L}, \beta_{3U}] + [\beta_{1L}, \beta_{1U}][\alpha_{3L}, \alpha_{3U}] & & \\ [\alpha_{2L}, \alpha_{2U}][\beta_{3L}, \beta_{3U}] + [\alpha_{3L}, \alpha_{3U}][\beta_{3L}, \beta_{3U}] & & [\alpha_{4L}, \alpha_{4U}][\beta_{3L}, \beta_{3U}] + \\ [\beta_{2L}, \beta_{2U}][\alpha_{3L}, \alpha_{3U}] & [\alpha_{5L}, \alpha_{5U}][\beta_{3L}, \beta_{3U}] + [\beta_{5L}, \beta_{5U}][\alpha_{3L}, \alpha_{3U}] & [\beta_{4L}, \beta_{4U}][\alpha_{3L}, \alpha_{3U}] \end{array} \right\rangle$$

Row Column Multiplication (RCM)

$$P_{IT2FR} \circ Q_{IT2FR} = \left\langle \begin{array}{ccc} [\alpha_{1L}, \alpha_{1U}][\beta_{1L}, \beta_{1U}] + [\alpha_{4L}, \alpha_{4U}][\beta_{2L}, \beta_{2U}] & & \\ [\alpha_{2L}, \alpha_{2U}][\alpha_{4L}, \alpha_{4U}] + [\alpha_{3L}, \alpha_{3U}][\beta_{3L}, \beta_{3U}] & [\alpha_{1L}, \alpha_{1U}][\beta_{4L}, \beta_{4U}] + & \\ [\alpha_{5L}, \alpha_{5U}][\beta_{2L}, \beta_{2U}] & [\alpha_{2L}, \alpha_{2U}][\beta_{4L}, \beta_{4U}] + [\alpha_{5L}, \alpha_{5U}][\beta_{5L}, \beta_{5U}] & [\alpha_{4L}, \alpha_{4U}][\beta_{5L}, \beta_{5U}] \end{array} \right\rangle$$

where

$$[\alpha_{iL}, \alpha_{iU}][\beta_{jL}, \beta_{jU}] = [\min((\alpha_{iL} * \beta_{jL}), (\alpha_{iL} * \beta_{jU}), (\beta_{jL} * \alpha_{jU}), (\beta_{jU} * \alpha_{jU})), \max((\alpha_{iL} * \alpha_{jL}), (\alpha_{iL} * \beta_{jU}), (\beta_{jL} * \alpha_{jU}), (\beta_{jU} * \alpha_{jU}))].$$

Example 3.4 : Find $P_{IT2FR} \circ Q_{IT2FR}$

$$\text{for } P_{IT2FR} = \left\langle \begin{array}{ccc} & [0.2, 0.5] & \\ [0.1, 0.6] & [0.3, 0.9] & [0.2, 0.5] \\ & [0.4, 0.5] & \end{array} \right\rangle$$

$$Q_{IT2FR} = \left\langle \begin{array}{ccc} & [0.4, 0.5] & \\ [0.3, 0.4] & [0.4, 0.6] & [0.2, 0.4] \\ & [0.3, 0.5] & \end{array} \right\rangle \text{ under HOM and RCM.}$$

$$\textbf{Solution: HOM: } P_{IT2FR} \circ Q_{IT2FR} = \left\langle \begin{array}{ccc} & C_1 & \\ C_2 & C_3 & C_4 \\ & C_5 & \end{array} \right\rangle$$

$$C_1 = [0.2, 0.5][0.4, 0.6] + [0.4, 0.5][0.3, 0.9]$$

$$[0.2, 0.5][0.4, 0.6] = [\min\{0.2 * 0.4, 0.2 * 0.6, 0.5 * 0.4, 0.5 * 0.6\},$$

$$\max\{0.2 * 0.4, 0.2 * 0.6, 0.5 * 0.4, 0.5 * 0.6\}] = [0.08, 0.30]$$

$$[0.4, 0.5][0.3, 0.9] = [\min\{0.4 * 0.3, 0.4 * 0.9, 0.5 * 0.3, 0.5 * 0.9\},$$

$$\max\{0.4 * 0.3, 0.4 * 0.9, 0.5 * 0.3, 0.5 * 0.9\}] = [0.12, 0.45]$$

$$C_1 = [0.08, 0.30] + [0.12, 0.45] = [0.18, 0.75]$$

$$C_2 = [0.04, 0.36] + [0.09, 0.36] = [0.13, 0.72]$$

$$C_3 = [0.12, 0.54]$$

$$C_4 = [0.08, 0.30] + [0.06, 0.36] = [0.14, 0.66]$$

$$C_5 = [0.16, 0.24] + [0.09, 0.45] = [0.25, 0.69]$$

$$\text{HOM: } P_{IT2FR} \circ Q_{IT2FR} = \left\langle \begin{array}{ccc} & [0.18, 0.75] & \\ [0.13, 0.72] & [0.12, 0.54] & [0.14, 0.66] \\ & [0.25, 0.69] & \end{array} \right\rangle$$

$$\text{Similarly by RCM: } P_{IT2FR} \circ Q_{IT2FR} =$$

$$\left\langle \begin{array}{ccc} & [0.14, 0.45] & \\ [0.13, 0.63] & [0.12, 0.54] & [0.10, 0.45] \\ & [0.14, 0.69] & \end{array} \right\rangle$$

Definition 3.5: The Transpose of IT2FR is calculated by interchanging the rows and columns and is denoted by $IT2FR^T$.

Example 3.5: If

$$P_{IT2FR} = \left\langle \begin{array}{cccc} & & [0.5, 0.7] & \\ & [0.1, 0.9] & [0.3, 0.5] & [0.1, 0.5] \\ [0.3, 0.7] & [0.4, 0.7] & [0, 0.5] & [0.4, 0.7] & [0.3, 0.6] \\ & [0.5, 0.6] & [0.2, 0.9] & [0.5, 0.9] \\ & & [0.5, 0.6] & \end{array} \right\rangle \text{ then}$$

$$P_{IT2FR}^T = \left\langle \begin{array}{ccccc} & & [0.5, 0.7] & & \\ & [0.1, 0.5] & [0.3, 0.5] & [0.1, 0.5] & \\ [0.3, 0.6] & [0.4, 0.7] & [0, 0.5] & [0.4, 0.7] & [0.3, 0.7] \\ & [0.5, 0.9] & [0.2, 0.9] & [0.5, 0.6] & \\ & & [0.5, 0.6] & & \end{array} \right\rangle$$

Definition 3.6: An IT2FR is said to be Null IT2FR if all its elements are of the form $[0, 0]$.

Definition 3.7: An IT2FR of order n is said to be Identity IT2FR or unit IT2FR if the heart of IT2FR is $[1, 1]$ and all other elements are $[0, 0]$ under the HOM. In the case of ROM all the elements in the diagonal are $[1, 1]$, the other elements are $[0, 0]$. Q_{IT2FR} is said to be the identity of P_{IT2FR} if $P_{IT2FR} \circ Q_{IT2FR} = P_{IT2FR}[10]$.

$$P_{IT2FR} \circ Q_{IT2FR} = \left\langle \begin{array}{ccc} & [\alpha_{1L}, \alpha_{1U}] & \\ [\alpha_{2L}, \alpha_{2U}] & [\alpha_{3L}, \alpha_{3U}] & [\alpha_{4L}, \alpha_{4U}] \\ & [\alpha_{5L}, \alpha_{5U}] & \end{array} \right\rangle \circ$$

$$\left\langle \begin{array}{ccc} & [\beta_{1L}, \beta_{1U}] & \\ [\beta_{2L}, \beta_{2U}] & [\beta_{3L}, \beta_{3U}] & [\beta_{4L}, \beta_{4U}] \\ & [\beta_{5L}, \beta_{5U}] & \end{array} \right\rangle$$

$$\left\langle \begin{array}{ccc} & [\alpha_{1L}, \alpha_{1U}][\beta_{3L}, \beta_{3U}] + [\beta_{1L}, \beta_{1U}][\alpha_{3L}, \alpha_{3U}] & \\ [\alpha_{2L}, \alpha_{2U}][\beta_{3L}, \beta_{3U}] + & [\alpha_{3L}, \alpha_{3U}][\beta_{3L}, \beta_{3U}] & [\alpha_{4L}, \alpha_{4U}][\beta_{3L}, \beta_{3U}] + \\ [\beta_{2L}, \beta_{2U}][\alpha_{3L}, \alpha_{3U}] & [\alpha_{5L}, \alpha_{5U}][\beta_{3L}, \beta_{3U}] + [\beta_{5L}, \beta_{5U}][\alpha_{3L}, \alpha_{3U}] & [\beta_{4L}, \beta_{4U}][\alpha_{3L}, \alpha_{3U}] \end{array} \right\rangle$$

$$= \begin{pmatrix} & [\alpha_{1L}, \alpha_{1U}] & \\ [\alpha_{2L}, \alpha_{2U}] & [\alpha_{3L}, \alpha_{3U}] & [\alpha_{4L}, \alpha_{4U}] \\ & [\alpha_{5L}, \alpha_{5U}] & \end{pmatrix}$$

By equating

$$[\alpha_{1L}, \alpha_{1U}][\beta_{3L}, \beta_{3U}] + [\beta_{1L}, \beta_{1U}][\alpha_{3L}, \alpha_{3U}] = [\alpha_{1L}, \alpha_{1U}]$$

$$[\alpha_{2L}, \alpha_{2U}][\beta_{3L}, \beta_{3U}] + [\beta_{2L}, \beta_{2U}][\alpha_{3L}, \alpha_{3U}] = [\alpha_{2L}, \alpha_{2U}]$$

$$[\alpha_{3L}, \alpha_{3U}][\beta_{3L}, \beta_{3U}] = [\alpha_{3L}, \alpha_{3U}]$$

$$[\alpha_{4L}, \alpha_{4U}][\beta_{3L}, \beta_{3U}] + [\beta_{4L}, \beta_{4U}][\alpha_{3L}, \alpha_{3U}] = [\alpha_{4L}, \alpha_{4U}]$$

$$[\alpha_{5L}, \alpha_{5U}][\beta_{3L}, \beta_{3U}] + [\beta_{5L}, \beta_{5U}][\alpha_{3L}, \alpha_{3U}] = [\alpha_{5L}, \alpha_{5U}]$$

Solving this equation, we get that

$$[\alpha_{1L}, \alpha_{1U}] = [\alpha_{2L}, \alpha_{2U}] = [\alpha_{4L}, \alpha_{4U}] = [\alpha_{5L}, \alpha_{5U}] = [0,0] \text{ \& } [\alpha_{3L}, \alpha_{3U}] = [1,1]$$

$$\text{Therefore } \begin{pmatrix} & [0,0] & \\ [0,0] & [1,1] & [0,0] \\ & [0,0] & \end{pmatrix} \text{ is the identity element under HOM.}$$

$$\text{Similarly, } \begin{pmatrix} & [1,1] & \\ [0,0] & [1,1] & [0,0] \\ & [1,1] & \end{pmatrix} \text{ is the identity element under RCM.}$$

Definition 3.8: Let $P_{IT2FR} = \begin{pmatrix} & [\alpha_{1L}, \alpha_{1U}] & \\ [\alpha_{2L}, \alpha_{2U}] & [\alpha_{3L}, \alpha_{3U}] & [\alpha_{4L}, \alpha_{4U}] \\ & [\alpha_{5L}, \alpha_{5U}] & \end{pmatrix}$ be the given

IT2FR and the $\det(P_{IT2FR}) = [\alpha_{3L}, \alpha_{3U}] [[\alpha_{1L}, \alpha_{1U}][\alpha_{5L}, \alpha_{5U}] - [\alpha_{2L}, \alpha_{2U}][\alpha_{4L}, \alpha_{4U}]]$.

Example 3.6: Find the determinant of the

$$P_{IT2FR} = \begin{pmatrix} & [0.2, 0.5] & \\ [0.1, 0.6] & [0.3, 0.9] & [0.2, 0.5] \\ & [0.4, 0.5] & \end{pmatrix}$$

Solution:

$$\begin{aligned} \det(P_{IT2FR}) &= [0.3, 0.9] [[0.2, 0.5] [0.4, 0.5] - [0.1, 0.6] [0.2, 0.5]] \\ &= [0.3, 0.9] [0.08, 0.25] - [0.02, 0.20] = [0.3, 0.9] [0.06, 0.05] = [0.15, 0.54]. \end{aligned}$$

Properties 3.1: Let P_{IT2FR} , Q_{IT2FR} be the two IT2FR and it is given by are two IT2FR of dimension n then we can verify the following results

1. $\det(P_{IT2FR} Q_{IT2FR}) = \det P_{IT2FR} \det Q_{IT2FR}$
2. $\det(P_{IT2FR} + Q_{IT2FR}) = \det P_{IT2FR} + \det Q_{IT2FR}$
3. $\det(k P_{IT2FR}) = k \det P_{IT2FR}$
4. $\det(P_{IT2FR}) = \det(P_{IT2FR}^T)$

Definition 3.9: The Coupled IT2F matrix is obtained by rotating the given IT2FR through 45° (represented by half transpose $\frac{T}{2}$). For a 5-dimensional IT2FR, we have

$$P_{IT2FR\ 5}^{\frac{T}{2}} = \left(\begin{array}{ccccc} & & [\alpha_{1L}, \alpha_{1U}] & & \\ & & [\alpha_{3L}, \alpha_{3U}] & & \\ [\alpha_{5L}, \alpha_{5U}] & [\alpha_{2L}, \alpha_{2U}] & [\alpha_{4L}, \alpha_{4U}] & & \\ & [\alpha_{6L}, \alpha_{6U}] & [\alpha_{7L}, \alpha_{7U}] & [\alpha_{8L}, \alpha_{8U}] & [\alpha_{9L}, \alpha_{9U}] \\ & [\alpha_{10L}, \alpha_{10U}] & [\alpha_{11L}, \alpha_{11U}] & [\alpha_{12L}, \alpha_{12U}] & \\ & & [\alpha_{13L}, \alpha_{13U}] & & \end{array} \right)^{\frac{T}{2}}$$

$$= [BC]_5, \text{ coupled matrix is made of two square matrices with orders of } t$$

and $t-1$ where $t = \frac{n+1}{2}$.

$$B = \left(\begin{array}{ccc} [\alpha_{1L}, \alpha_{1U}] & [\alpha_{4L}, \alpha_{4U}] & [\alpha_{9L}, \alpha_{9U}] \\ [\alpha_{2L}, \alpha_{2U}] & [\alpha_{7L}, \alpha_{7U}] & [\alpha_{12L}, \alpha_{12U}] \\ [\alpha_{5L}, \alpha_{5U}] & [\alpha_{10L}, \alpha_{10U}] & [\alpha_{13L}, \alpha_{13U}] \end{array} \right) \text{ and}$$

$$C = \left(\begin{array}{cc} [\alpha_{3L}, \alpha_{3U}] & [\alpha_{8L}, \alpha_{8U}] \\ [\alpha_{6L}, \alpha_{6U}] & [\alpha_{11L}, \alpha_{11U}] \end{array} \right)$$

where B is the complementary Component matrix of the coupled matrix $[BC]_5$. If we fill zeros at the missing places, the resulting $n \times n$ matrix is filled coupled matrix. The filled coupled matrix of above is given by

$$\left(\begin{array}{ccccc} [\alpha_{1L}, \alpha_{1U}] & [0,0] & [\alpha_{4L}, \alpha_{4U}] & [0,0] & [\alpha_{9L}, \alpha_{9U}] \\ [0,0] & [\alpha_{3L}, \alpha_{3U}] & [0,0] & [\alpha_{8L}, \alpha_{8U}] & [0,0] \\ [\alpha_{2L}, \alpha_{2U}] & [0,0] & [\alpha_{7L}, \alpha_{7U}] & [0,0] & [\alpha_{12L}, \alpha_{12U}] \\ [0,0] & [\alpha_{6L}, \alpha_{6U}] & [0,0] & [\alpha_{11L}, \alpha_{11U}] & [0,0] \\ [\alpha_{5L}, \alpha_{5U}] & [0,0] & [\alpha_{10L}, \alpha_{10U}] & [0,0] & [\alpha_{13L}, \alpha_{13U}] \end{array} \right)$$

Definition 3.10: The number of three-dimensional IT2FR than can be drawn either along the major row or major column is said to be the index of IT2FR which is denoted by ρ . The Index of IT2FR of dimension 1, 3, 5, 7, 9 are 0, 1, 2, 3, 4 respectively.

Example 3.7:

$$\text{If } P_{IT2FR} = \left(\begin{array}{ccccc} & & [0.5, 0.7] & & \\ & [0.1, 0.9] & [0.3, 0.5] & [0.1, 0.5] & \\ [0.3, 0.7] & [0.4, 0.7] & [0, 0.5] & [0.4, 0.7] & [0.3, 0.6] \\ & [0.5, 0.6] & [0.2, 0.9] & [0.5, 0.9] & \\ & & [0.5, 0.6] & & \end{array} \right)$$

Then $\rho(P_{IT2FR}) = 2$.

Solution: Along Major Row the 3-dimensional IT2FR are

$$\left\langle \begin{array}{ccc} & [0.1,0.9] & \\ [0.3,0.7] & [0.4,0.7] & [0,0.5] \\ & [0.5,0.6] & \end{array} \right\rangle \& \left\langle \begin{array}{ccc} & [0.1,0.5] & \\ [0,0.5] & [0.4,0.7] & [0.3,0.6] \\ & [0.5,0.6] & \end{array} \right\rangle$$

Therefore $\rho(P_{IT2FR}) = 2$

Along Major Column the 3-dimensional IT2FR are

$$\left\langle \begin{array}{ccc} & [0.5,0.7] & \\ [0.1,0.9] & [0.3,0.5] & [0.1,0.5] \\ & [0,0.5] & \end{array} \right\rangle \& \left\langle \begin{array}{ccc} & [0,0.5] & \\ [0.5,0.6] & [0.2,0.9] & [0.5,0.9] \\ & [0.5,0.6] & \end{array} \right\rangle$$

Therefore $\rho(P_{IT2FR}) = 2$. Hence $\rho(P_{IT2FR}) = 2$ for 5- dimensional odd IT2FR.

Definition 3.11: The Codeterminant function of the higher dimensional IT2FR is calculated by reducing into IT2FR of dimension 3 and the determinant of each of them is calculated and summing up and is denoted by Codet.

Example 3.8: Let

$$P_{IT2FR} = \left\langle \begin{array}{cccc} & & [0.5,0.7] & \\ & [0.1,0.9] & [0.3,0.5] & [0.1, 0.5] \\ [0.3,0.7] & [0.4, 0.7] & [0,0.5] & [0.4, 0.7] & [0.3,0.6] \\ & [0.5,0.6] & [0.2,0.9] & [0.5, 0.9] \\ & & [0.5,0.6] & \end{array} \right\rangle$$

then

$$\left\langle \begin{array}{ccc} & [0.1,0.9] & \\ [0.3,0.7] & [0.4,0.7] & [0,0.5] \\ & [0.5,0.6] & \end{array} \right\rangle \& \left\langle \begin{array}{ccc} & [0.1,0.5] & \\ [0,0.5] & [0.4,0.7] & [0.3,0.6] \\ & [0.5,0.6] & \end{array} \right\rangle$$

are two 3-dimensional IT2FR that can be reduced from P_{IT2FR} and let it be Q_{IT2FR} and R_{IT2FR}

Then $\det(Q_{IT2FR}) = [0.02, 0.1]$ and $\det(R_{IT2FR}) = [0.02, 0.42]$.

Hence Codet $(P_{IT2FR}) = [0.04, 0.52]$.

Conclusion

This paper gives a brief idea about IT2FR and the basic definitions like determinant, transpose, Codeterminant function and Index of IT2FR. IT2FR has vast application in spam detection, FHAP, and the energy vehicle industry. We can extend this application for IT2FR.

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