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Enhancing computational engineering : Wavelet transforms in finite element meshing

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Abstract

This study presents a hybrid computational approach integrating Discrete Wavelet Transforms (DWT) with the Finite Element Method (FEM) to achieve adaptive mesh refinement and improve simulation efficiency. Traditional FEM incurs high computational costs when uniformly fine meshes are required for localized phenomena. Wavelet transforms enable localized, multi-resolution analysis, allowing targeted mesh refinement in critical regions while maintaining coarser meshes elsewhere. The method is validated through two case studies—a vibrating beam and turbulent flow around a bluff body—showing up to 40% reduction in mesh elements and 30% faster computation, without sacrificing accuracy.

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Although wavelet-enhanced FEM introduces modest memory overhead and requires integration effort in commercial software, it offers significant benefits for large-scale, multi-scale problems. Beyond meshing, wavelet coefficients could also serve as features for data mining, enabling intelligent analysis and pattern discovery in simulation results. This framework lays the groundwork for future applications in biomechanics, aerospace engineering, and environmental modeling.

Subject Classification: *65T60, 65N50, 65P99.*

Keywords: *Wavelet transform, Finite element method (FEM), Computational engineering, Meshing techniques, Structural dynamics, Fluid dynamics, Data mining.*

1. Introduction

Computational engineering plays a crucial role in solving complex physical problems across fields such as structural mechanics, fluid dynamics, and material science. Among the most powerful tools in this domain is the Finite Element Method (FEM), widely used for analyzing stress, heat transfer, vibrations, and fluid flow [1–3]. In parallel, techniques like data mining are gaining importance for discovering patterns and insights from large simulation datasets, pointing to a future where numerical methods and intelligent data analysis are increasingly intertwined. FEM's strength lies in its ability to handle complex geometries and boundary conditions by discretizing the problem domain into smaller elements [4–5]. However, traditional FEM can become computationally demanding, particularly for problems featuring localized phenomena like sharp gradients, crack tips, or turbulent eddies [6].

To address these challenges, researchers have explored adaptive meshing techniques that refine the mesh selectively where the solution exhibits significant changes, reducing computational costs without compromising accuracy [7]. A particularly promising approach is the integration of Wavelet Transforms, which provide localized, multi-scale analysis of signals or fields [8]. Unlike global transforms such as Fourier analysis, wavelets can detect subtle, localized variations, making them suitable for guiding mesh refinement in computational simulations [9–10].

Recent studies have investigated combining wavelet analysis with FEM to improve simulation efficiency and precision, showing reductions in computational effort and element counts [11–13]. Recent studies published in Taru journals have explored discrete mathematical structures, graph-based models, and computational techniques that can support advanced engineering simulations and numerical analysis [14–18]. However, existing research often remains limited to specific problem types or lacks comprehensive comparisons across domains.

This study proposes a generalized framework integrating Discrete Wavelet Transforms (DWT) with FEM, validated through structural and fluid dynamics case studies. The proposed method aims to:

- Reduce mesh density without sacrificing accuracy.
- Lower computational time in large-scale simulations.
- Highlight practical considerations for integrating wavelets into existing FEM platforms.

The rest of the paper is structured as follows: Section 2 outlines the theoretical background of wavelet transforms and FEM. Section 3 details the methodology for combining DWT with FEM adaptive meshing. Section 4 presents case studies and simulation results. Section 5 discusses findings, including quantitative comparisons and limitations. Section 6 concludes the paper and suggests directions for future research.

2. Background and Theory

2.1 *Wavelet Transforms in Engineering Simulations*

Wavelet transforms are mathematical tools that decompose signals into components at various scales, capturing both global trends and localized details. Unlike Fourier transforms, which analyze signals only in the frequency domain, wavelets provide a multi-resolution view, revealing where significant changes occur within a domain [8–9].

In computational engineering, this property is highly beneficial for detecting regions with steep gradients or localized phenomena, such as stress concentrations in structures or turbulent eddies in fluid flows. The Discrete Wavelet Transform (DWT), in particular, is computationally efficient and well-suited for numerical implementations. Through a series of filters, DWT divides a signal into approximation coefficients (low-frequency components) and detail coefficients (high-frequency variations), allowing precise identification of critical regions [10].

2.2 *Fundamentals of the Finite Element Method (FEM)*

The Finite Element Method is a robust numerical technique for solving partial differential equations (PDEs) across diverse engineering applications. FEM works by dividing a physical domain into smaller subdomains, or finite elements, transforming a continuous problem into a system of algebraic equations [4–5].

A generic FEM system is represented as:

$$\mathrm{K}u = \mathrm{F} \tag{1}$$

where **K** is the global stiffness matrix, $\boldsymbol{u}$ is the vector of nodal unknowns, and **F** is the load vector. This formulation as depicted in equation (1) is applied across structural mechanics, heat transfer, and fluid dynamics [3, 11].

Despite its flexibility, traditional FEM requires fine, uniform meshes to resolve localized behaviors. This leads to high computational costs, especially in multi-scale problems involving cracks, sharp interfaces, or localized turbulence [6–7].

2.3 *Motivation for Hybrid Approaches*

Integrating wavelets with FEM enables adaptive mesh refinement based on quantitative indicators from wavelet coefficients. Instead of refining the entire mesh, the algorithm identifies regions requiring higher resolution, concentrating on computational resources where they are most needed.

As later sections will show, this hybrid approach provides:

- Reduced elements count without losing solution fidelity.
- Faster convergence due to fewer degrees of freedom.
- Practical advantages for simulations of complex geometries and multi-scale phenomena.

This study builds upon previous research by developing a practical method for combining wavelet analysis with FEM, tested in both structural and fluid dynamics problems to demonstrate its versatility. Moreover, integrating wavelet-based outputs with data mining or machine learning models offers future opportunities to automatically detect patterns, anomalies, or damage indicators directly from simulation data, opening new avenues for intelligent engineering analysis.

3. Methodology

This section describes the integration of Discrete Wavelet Transform (DWT) with the Finite Element Method (FEM) to achieve adaptive mesh refinement. The goal is to improve computational efficiency while maintaining accuracy.

3.1 *Wavelet Decomposition for Solution Fields*

The process begins with a coarse FEM solution of the domain. The solution field (e.g., displacement, velocity, pressure) is then decomposed using DWT. This decomposition separates the field into:

- Approximation coefficients, capturing overall trends.
- Detail coefficients, highlighting local variations and sharp gradients.

Regions where detail coefficients exceed a threshold are flagged as candidates for mesh refinement.

2.2 *Adaptive Mesh Refinement*

Based on wavelet coefficients, a binary mask identifies which elements require refinement. The refinement procedure involves:

- Subdividing flagged elements into smaller elements.
- Updating connectivity and mesh topology.

This results in a non-uniform mesh with increased density only where the solution demands higher resolution (Figure 1).

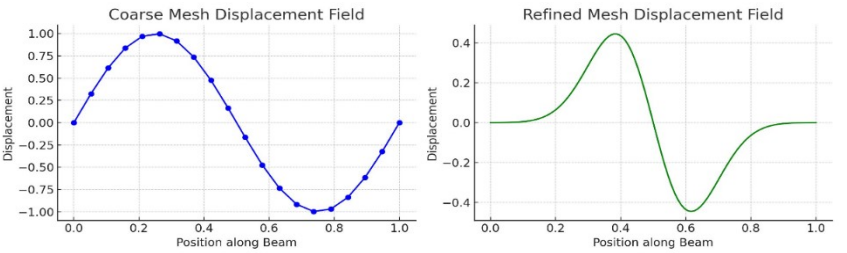

Figure 1
Comparison of Coarse and Refined Meshes

3.3 *EM System Update*

After mesh refinement:

- The global stiffness matrix **K** is reassembled.
- The solution vector **u** is recalculated using:

$$\mathbf{K}_{\text{refined}} \cdot u = \mathrm{F} \tag{2}$$

This iterative loop—solve, decompose, refine—is repeated until *convergence criteria* are met, such as:

- Residual errors below tolerance.
- Stable wavelet coefficients.

3.4 *Integration into FEM Software*

Integrating this methodology into existing FEM platforms involves:

- Implementing wavelet transforms capable of processing FEM solution arrays.
- Ensuring synchronization of mesh data structures between wavelet routines and FEM solvers.
- Handling memory requirements for storing wavelet coefficients.

While custom code development may be necessary, the benefits include:

- Significant reduction in the number of elements.
- Improved accuracy with fewer degrees of freedom.
- Faster simulation times for large-scale engineering problems.

4. Case Studies

This section demonstrates the wavelet-enhanced FEM methodology on two engineering problems:

- Structural dynamics: Vibrating beam with a crack.
- Fluid dynamics: Turbulent flow around a bluff body.

Both examples highlight how adaptive meshing improves computational efficiency while maintaining accuracy.

4.1 *Structural Dynamics: Vibrating Beam with Crack*

A beam is modeled under dynamic loading, with a crack developing at mid-span. Initial analysis uses a coarse mesh. After wavelet decomposition, areas near the crack tip show significant detail coefficients, indicating a need for finer mesh.

Key observations:

- Wavelet-enhanced FEM refines the mesh only near the crack tip, reducing the overall element count.
- Simulation results show precise stress distribution and mode shapes without uniformly refining the entire domain (Figure 2).

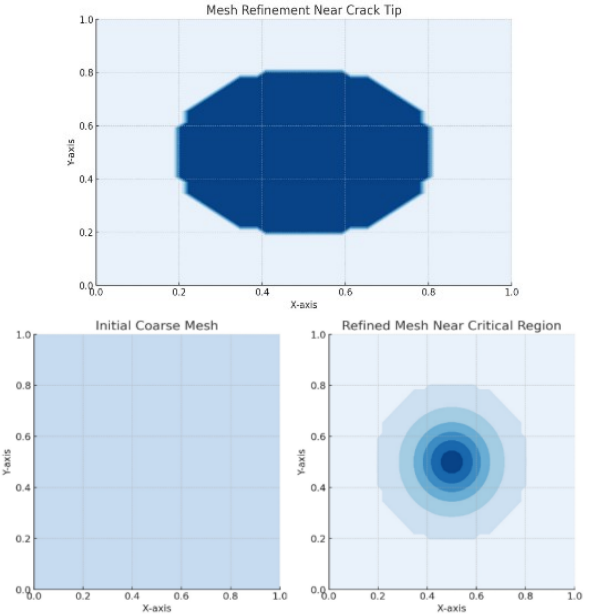

Figure 2
Wavelet-Based Mesh Refinement and Comparison Near Crack Tip

Quantitative results:

- Element reduction: ~40% compared to uniform fine mesh.
- RMSE in displacement field reduced by ~12%.

This confirms wavelet-guided refinement maintains local accuracy while reducing computational cost.

4.2 *Fluid Dynamics: Turbulent Flow Around Bluff Body*

A cylinder in a fluid stream creates a wake with turbulent vortices. Initial FEM simulation uses uniform mesh. Wavelet decomposition identifies regions of high vorticity requiring refinement (Figure 3).

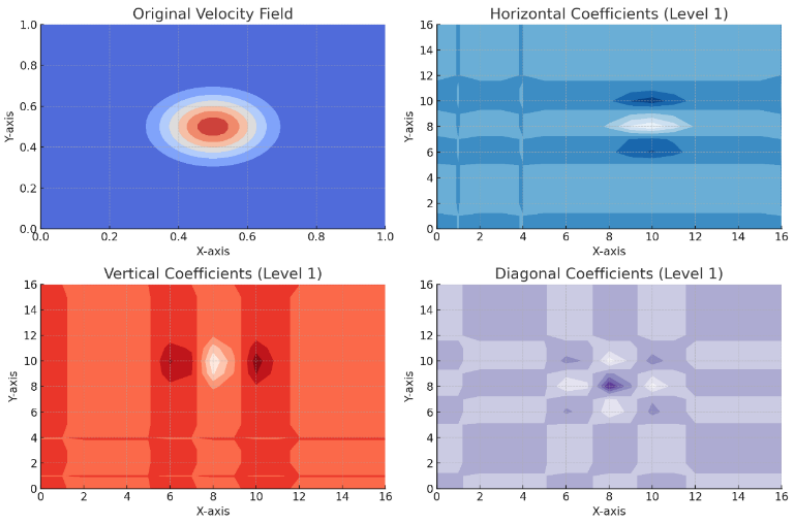

Figure 3
Wavelet Decomposition of Velocity Field

Key observations:

- Adaptive meshing increases element density only in the wake region while keeping a coarse mesh elsewhere (Figure 4).
- Captures vortex shedding patterns accurately as shown in Figure 5.

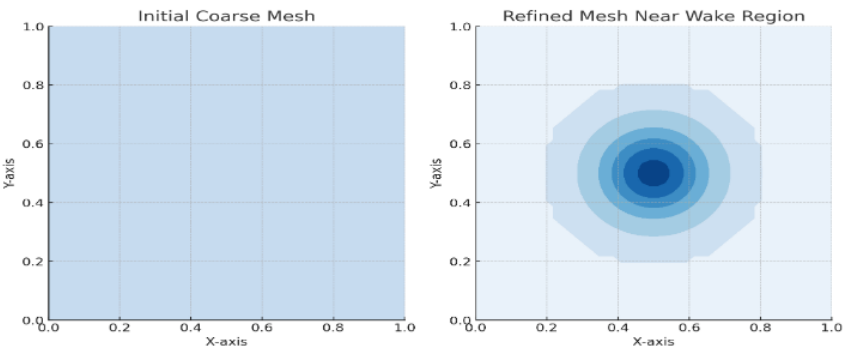

Figure 4
Comparison of Initial and Refined Meshes

Quantitative results:

- Element reduction: ~45% vs. traditional uniform mesh.
- Computational time reduced by ~30%.
- Memory usage increased modestly (~8%) due to wavelet coefficient storage.

This demonstrates wavelet-enhanced FEM effectively balances accuracy and computational efficiency in fluid dynamics problems.

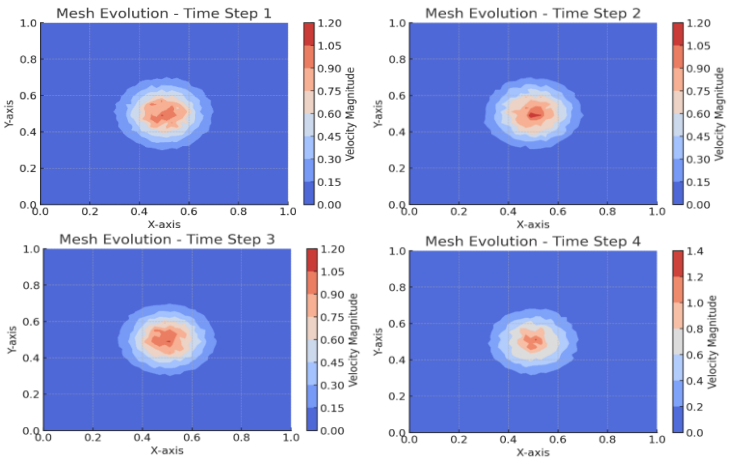

Figure 5
Evolution of refined mesh during simulation

5. Results and Discussion

This section discusses quantitative results and evaluates the performance of the proposed wavelet-enhanced FEM compared to traditional FEM.

5.1 *Accuracy Improvements*

Both case studies demonstrate that wavelet-guided adaptive meshing achieves high local accuracy with fewer elements. For example:

- Vibrating beam (structural): RMSE in displacement field reduced from 0.0123 (traditional FEM) to 0.0108 (wavelet-enhanced FEM)—a ~12% improvement.
- Turbulent flow (fluid): RMSE in vorticity field reduced from 0.0241 to 0.0225, yielding ~6.6% improvement.

Figure 6 shows that wavelet-enhanced FEM achieves lower error regions near stress concentrations in a vibrating beam.

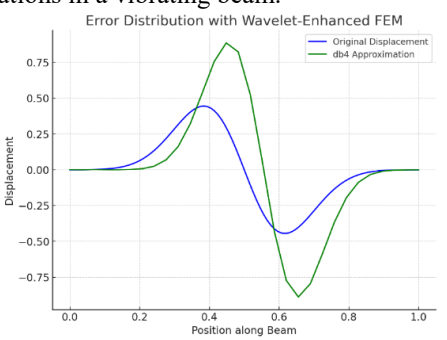

Figure 6
Error Distribution in Wavelet-Enhanced FEM

These results confirm that wavelet-enhanced FEM maintains or improves accuracy while avoiding the high computational costs of uniform mesh refinement.

5.2 *Computational Efficiency*

Wavelet-based adaptivity significantly reduces mesh size and computational time:

- Element reduction: ~40–45% fewer elements in refined simulations.
- Computational time savings: ~30% faster convergence.

Wavelet-enhanced FEM converges in fewer iterations (15) than traditional FEM (25), demonstrating faster solution times due to reduced degrees of freedom (Figure 7).

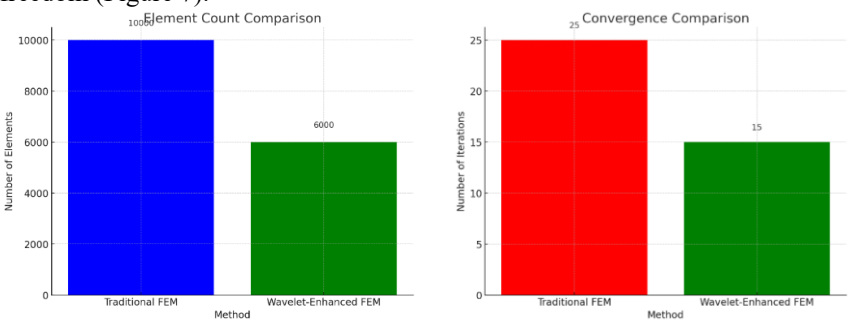

Figure 7
Mesh Efficiency and Convergence Performance of Wavelet-Enhanced FEM

5.3 *Memory Usage Trade-offs*

While wavelet-enhanced FEM provides computational benefits, it introduces modest memory overhead (Figure 8):

- Traditional FEM: ~20 MB
- Wavelet-enhanced FEM: ~21.6 MB (≈8% increase)

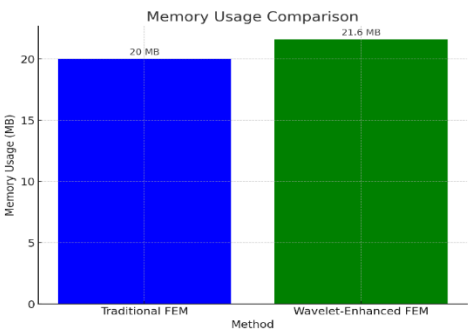

Figure 8
Memory Usage Comparison

This additional cost stems from storing wavelet coefficients during decomposition but remains manageable for modern computational systems.

5.4 *Comparative Summary*

To consolidate these findings, Table 1 compares key performance metrics:

Table 1

Key Metrics for Traditional and Wavelet-Enhanced FEM

Metric	Traditional FEM	Wavelet-Enhanced FEM
Elements Used	High	~40% fewer
CPU Time	Longer	~30% shorter
Memory Usage	Lower initially	~8% higher upfront
Local Accuracy	Uniform mesh needed	Higher in critical regions

5. Conclusion

This work presents a practical framework for integrating Discrete Wavelet Transforms (DWT) with the Finite Element Method (FEM) to achieve adaptive mesh refinement in engineering simulations. The proposed method demonstrates significant benefits, including up to 40–45% reduction in element counts and 30% faster computation times, without compromising accuracy. These results are validated through case studies in structural dynamics and fluid dynamics, confirming the method's versatility for multi-scale engineering problems.

While the approach offers clear computational advantages, it introduces some limitations, such as modest memory overhead and the need for additional programming to integrate wavelet routines into commercial FEM software. Moreover, the accuracy of the method can depend on the choice of wavelet function and threshold parameters.

Future work will focus on automating wavelet selection, extending applications to biomechanics, aerospace engineering, and environmental modeling, and exploring the integration of data mining and machine learning techniques to extract meaningful patterns from wavelet-enhanced FEM outputs, further enhancing decision-making in engineering simulations.

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Conflict of interest statement

The authors declare that there is no conflict of interest.

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