

On some fuzzy ideal of S -weakly Y -regular of semi_groups

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Abstract

This article introduces some fuzzy ideal of S -weakly Y -regular of semi_groups, and we have characterized of a S -weakly Y -regular semi_groups using properties for fuzzy_left and fuzzy_right_ideals of some natural examples of semi_groups. Furthermore, a basic properties and characterizations of the fuzzy_ideals(right, left, two sided, quasi, interior, bi_generalized_bi, and $(1, 2)$) of a S -weakly Y -semi_group have been investigated.

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Introduction

Some concepts play a significant turn in algebra and cryptography see [1], [2], [4], [5] and [6]. The S -weakly regular semi_groups concept was first introduced by Akram. This article has been extended the idea for S -weakly regular semi_group $\mathcal{H}$ which is $\kappa \in \mathcal{H}$, $\exists q, \mathfrak{t} \in \mathcal{H}$, such that $\kappa = \kappa q \kappa^2 \mathfrak{t}$. They named it as a left almost semi_group (LA_semi_group). An LA_semi_group be groupoid $\mathcal{H}$ which its elements a left invertive law $(\kappa q) \mathfrak{t} = (\mathfrak{t} q) \kappa$, $\forall \kappa, q, \mathfrak{t} \in \mathcal{H}$ [1]. The fuzzy _subsets η for the semi_groups $\mathcal{H}$ named fuzzy_sub_

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semi_groups for $\mathcal{H}$ when $\eta(\kappa q) \geq \min\{\eta(\kappa), \eta(q)\}$, whenever $\kappa, q \in \mathcal{H}$ [3, 6]. The fuzzy_sub_semi_group η for the semi_groups $\mathcal{H}$ called fuzzy_interior_ideal for $\mathcal{H}$ where $\eta(\kappa q \mathfrak{t}) \geq \eta(q)$, $\forall \kappa, q, \mathfrak{t} \in \mathcal{H}$ [7]. If η fuzzy_left_ideal, fuzzy_right_ideals for $\mathcal{H}$, so η named fuzzy_ideal for $\mathcal{H}$ [8]. if $\mathcal{H}$ ordered semi_groups, then fuzzy_subsets η for $\mathcal{H}$ named fuzzy_sub_semi_groups for $\mathcal{H}$ if $\eta(\kappa \min\{\eta(\kappa), \eta(q)\})$, for every $\kappa, q \in \mathcal{H}$ [9]. the fuzzy_subsets η for $\mathcal{H}$ named antifuzzy_left(right)_ideals for $\mathcal{H}$ if $\eta(\kappa q) \leq \eta(q)$, $(\eta(\kappa q) \leq \eta(\kappa))$, forever $\kappa, q \in \mathcal{H}$ [11]. the fuzzy_subsets η for semi_groups $\mathcal{H}$ named fuzzy_generalized bi_ideals for $\mathcal{H}$ if $\eta((\kappa q) \mathfrak{t}) \geq \eta(\kappa) \wedge \eta(\mathfrak{t})$, $\forall \kappa, q, \mathfrak{t} \in \mathcal{H}$ [12]. Each fuzzy_left_ideal (fuzzy_right_ideal) be fuzzy_bi_ideals for $\mathcal{H}$ [13]. if η is antifuzzy_right_ideals & μ is the antifuzzy_left_ideals for the semi_group $\mathcal{H}$, so $\eta * \mu \supseteq \eta \cup \mu$ [10].

The main results

Definition 1.1: The semi_group $\mathcal{H}$ named $S_weakly_Y_regular$ if forevery $\kappa \in \mathcal{H}$, there exists $q, \mathfrak{t} \in \mathcal{H}$ where $Y > 1$ such that $\kappa = \kappa q^Y \kappa^2 \mathfrak{t}^Y$.

Proposition 1.2: In $S_weakly_Y_regular$ semi_groups $\mathcal{H}$, each fuzzy interior_ideals be idempotent.

Proof: Assume η be fuzzy_interior_ideals for $S_weakly_Y_regular$ semi_groups $\mathcal{H}$, then clearly $\eta \circ \eta \subseteq \eta$.

Let $\kappa \in \mathcal{H}$ then $\exists q, \mathfrak{t} \in \mathcal{H}$, such that $\kappa = \kappa q^Y \kappa^2 \mathfrak{t}^Y$ where $Y > 1$.

$$\Rightarrow \kappa = \kappa q^Y \kappa^2 \mathfrak{t}^Y = (\kappa q^Y) \kappa (\mathfrak{t}^Y)$$

$$= (\kappa q^Y) \kappa (\kappa q^Y \kappa^2 \mathfrak{t}^Y) \mathfrak{t}^Y$$

$$= (\kappa q^Y) \kappa (\kappa q^Y) (\kappa) \kappa (\mathfrak{t}^{2Y})$$

$$(\eta \circ \eta)_{(\kappa)} = \bigvee_{\kappa=[(\kappa q^Y) \kappa (\kappa q^Y) \kappa (\mathfrak{t}^{2Y})]} \{ \eta[(\kappa q^Y) \kappa (\kappa q^Y)] \wedge \eta[(\kappa) \kappa (\mathfrak{t}^{2Y})] \}$$

$$\geq \eta[(\kappa q^Y) \kappa (\kappa q^Y)] \wedge \eta[(\kappa) \kappa (\mathfrak{t}^{2Y})]$$

$$\geq \eta(\kappa) \wedge \eta(\kappa) = \eta(\kappa)$$

This is implying that $\eta \circ \eta \supseteq \eta$, hence $\eta \circ \eta = \eta$. Then η is idempotent.

Proposition 1.3: If η fuzzy_sub_set of $S_weakly_Y_regular$ semi_groups $\mathcal{H}$, we get it is an antifuzzy_two_side_ideals for $\mathcal{H}$ iff its antifuzzy_interior_ideals for $\mathcal{H}$.

Proof: $\Rightarrow$ Since η is an antifuzzy_two_sided_ideal of $\mathcal{H}$ and let κ, q and $\mathfrak{t} \in \mathcal{H}$,

$$\eta(\kappa q \mathfrak{t}) = \eta(\kappa q \mathfrak{t}) \leq \eta(\kappa q) \text{ (because } \eta \text{ is an antifuzzy_right_ideal)}$$

$$\leq \eta(q) \text{ (because } \eta \text{ is an antifuzzy_left_ideal)}$$

$$\eta(\kappa q \mathfrak{t}) \leq \eta(q) \text{ that is } \eta \text{ is an antifuzzy_interior_ideal}$$

$\Leftarrow$ assume η is antifuzzy_interior_ideal for η . Let $\kappa_1, \kappa_2 \in \mathcal{H}$, by hypothesises so $\exists q_1, \mathfrak{t}_1 q_2$ and $\mathfrak{t}_2 \in \mathcal{H}$, $Y > 1$ such that $\kappa_1 = \kappa_1 q_1^Y \kappa_1^2 \mathfrak{t}_1^Y$, $\kappa_2 = \kappa_2 q_2^Y \kappa_2^2 \mathfrak{t}_2^Y$.

$$\eta(\kappa_1 \kappa_2) = \eta(\kappa_1 q_1^Y \kappa_1^2 \mathfrak{t}_1^Y \kappa_2)$$

$$\begin{aligned}
&= \eta((\kappa_1 q_1^Y) \kappa_1 (\kappa_1 t_1^Y \kappa_2)) \\
&\leq \eta(\kappa_1), \text{ and also} \\
\eta(\kappa_1 \kappa_2) &= \eta(\kappa_1 \kappa_2 q_2^Y \kappa_2^2 t_2^Y) \\
&= \eta((\kappa_1 \kappa_2 q_2^Y) \kappa_2 (\kappa_2 t_2^Y)) \\
&\leq \eta(\kappa_2). \text{ Hence, } \eta \text{ is an antifuzzy_two_sided_ideal of } \mathcal{H}.
\end{aligned}$$

Proposition 1.4: In S _weakly Y _regular semi_groups $\mathcal{H}$, then

- i- Every antifuzzy_right_ideal is idempotent.
- ii- Every antifuzzy_interior_ideal is idempotent.

Proposition 1.5: [1]. Let η is an antifuzzy_right_ideal and ρ an antifuzzy_left_ideal of a semi_group $\mathcal{H}$, then $\eta * \rho \supseteq \eta \cup \rho$.

To put reverse for (1.5), we need to strengthen a condition for semi_group $\mathcal{H}$.

Proposition 1.6: If η, ρ are any antifuzzy two-sided ideal in S _weakly Y _regular $\mathcal{H}$, then $\eta * \rho = \eta \cup \rho$.

Proof: Let η and ρ be any antifuzzy_two sided_ideal of $\mathcal{H}$, then obviously $\eta * \rho \supseteq \eta \cup \rho$. Since $\mathcal{H}$ is an S _weakly Y _regular so forever element $\kappa \in \mathcal{H}$, $\exists s, t \in \mathcal{H}$, such that $\kappa = \kappa q^Y \kappa^2 t^Y$, where $Y > 1$

$$\begin{aligned}
(\eta * \rho)_{(\kappa)} &= \bigwedge_{\kappa = \kappa q^Y \kappa^2 t^Y = \kappa q^Y \kappa \kappa t^Y} \{ \eta(\kappa q^Y) \vee \rho(\kappa \kappa t^Y) \} \\
&\leq \eta(\kappa q^Y) \vee \rho(\kappa \kappa t^Y) \leq \eta(\kappa) \vee \rho(\kappa) = (\eta \cup \rho)(\kappa).
\end{aligned}$$

Then $(\eta * \rho) \subseteq \eta \cup \rho$. Hence, $\eta * \rho = \eta \cup \rho$.

Proposition 1.7: Let η be a fuzzy_sub_set of $\mathcal{H}$, where $\mathcal{H}$ is S _weakly Y _regular semi_group then η is anti_fuzzy_bi_ideal for $\mathcal{H}$ iff its antifuzzy_generalization bi_ideal for $\mathcal{H}$.

Proof: $\Rightarrow$ assume η is antifuzzy bi_ideals for $\mathcal{H}$, so η be antifuzzy_generalization bi_ideal of $\mathcal{H}$.

$\Leftarrow$ assume η is antifuzzy_generalization bi_ideal for $\mathcal{H}$, because $\mathcal{H}$ be S _weakly Y _regular semi_group, so forever κ_1 and $\kappa_2 \in \mathcal{H}$, $\exists q_1, t_1, q_2$ and $t_2 \in \mathcal{H}$, where $Y > 1$, such that $\kappa_1 = \kappa_1 q_1^Y \kappa_1^2 t_1^Y$, $\kappa_2 = \kappa_2 q_2^Y \kappa_2^2 t_2^Y$.

$$\eta(\kappa_1 \kappa_2) = \eta(\kappa_1 q_1^Y \kappa_1^2 t_1^Y \kappa_2) = \eta(\kappa_1 (q_1^Y \kappa_1^2 t_1^Y) \kappa_2) \leq \eta(\kappa_1) \vee \eta(\kappa_2).$$

Therefore, η is an antifuzzy sub_semi_group of $\mathcal{H}$. Hence, η is an antifuzzy_generalization bi_ideal of $\mathcal{H}$.

Proposition 1.8: For antifuzzy_generalization bi_ideal η and ρ antifuzzy_left_ideal of S _weakly Y _regular semi_group $\mathcal{H}$, then $\eta * \rho \leq \eta \vee \rho$.

Proof: Let η and ρ be any antifuzzy_generalization bi_ideal and antifuzzy_right_ideal of $\mathcal{H}$, respectively, then forever $\kappa \in \mathcal{H}$, $\exists s, t \in \mathcal{H}$, such that

$\kappa = \kappa q^Y \kappa^2 \mathfrak{t}^Y$ where $Y > 1$, so. Then $\kappa = \kappa q^Y \kappa^2 \mathfrak{t}^Y = \kappa = \kappa q^Y \kappa \mathfrak{t}^Y =$
 $(\kappa q^Y \kappa)(\kappa \mathfrak{t}^Y)$

$$\begin{aligned} (\eta * \rho)_{(\kappa)} &= \bigwedge_{\kappa = (\kappa q^Y \kappa)(\kappa \mathfrak{t}^Y)} \{ \eta(\kappa q^Y \kappa) \vee \rho(\kappa \mathfrak{t}^Y) \} \\ &\leq \eta(\kappa q^Y \kappa) \vee \rho(\kappa \mathfrak{t}^Y) \leq \eta(\kappa) \vee \rho(\kappa) = (\eta \vee \rho)(\kappa) \end{aligned}$$

And so, we have $\eta * \rho \leq \eta \vee \rho$.

Proposition 1.9: If η and ρ be any antifuzzy_interior_ideals of S _weakly Y _regular semi_group $\mathcal{H}$, then $(\eta * \rho) \vee (\rho * \eta) \leq \eta \vee \rho$.

Proof: Let η, ρ are antifuzzy interior_ideal for $\mathcal{H}$, & $\kappa \in \mathcal{H}$. Then since $\mathcal{H}$ is an S _weakly Y _regular semi_group $\mathcal{H}$, then $\exists q, \mathfrak{t} \in \mathcal{H}$ and $Y > 1$, such that $\kappa = \kappa q^Y \kappa^2 \mathfrak{t}^Y$.

$$\kappa = \kappa q^Y \kappa^2 \mathfrak{t}^Y = \kappa q^Y \kappa \kappa q^Y \kappa^2 \mathfrak{t}^Y = (\kappa q^Y \kappa)(\kappa q^Y \kappa) \kappa \mathfrak{t}^{2Y}.$$

$$\begin{aligned} \text{Hence } (\eta * \rho)_{(\kappa)} &= \bigwedge_{\kappa = [(\kappa q^Y \kappa)(\kappa q^Y \kappa)](\kappa \mathfrak{t}^{2Y})} \{ \eta(\kappa q^Y \kappa)(\kappa q^Y \kappa) \vee \rho(\kappa \mathfrak{t}^{2Y}) \} \\ &\leq \eta((\kappa q^Y \kappa)(\kappa q^Y \kappa)) \vee \rho(\kappa \mathfrak{t}^{2Y}) \\ &\leq \eta(\kappa) \vee \rho(\kappa) = (\eta \vee \rho)_{(\kappa)}, \text{ and so, we have } \eta * \rho \leq \eta \vee \rho. \end{aligned}$$

To proof $(\rho * \eta) \leq \eta \vee \rho$.

$$\begin{aligned} (\rho * \eta)_{(\kappa)} &= \bigwedge_{\kappa = \kappa q^Y \kappa^2 \mathfrak{t}^Y = (\kappa q^Y \kappa)(\kappa q^Y \kappa) \kappa \mathfrak{t}^{2Y}} \{ \rho(\kappa q^Y \kappa)(\kappa q^Y \kappa) \vee \eta(\kappa \mathfrak{t}^{2Y}) \} \\ &\leq \rho((\kappa q^Y \kappa)(\kappa q^Y \kappa)) \vee \eta(\kappa \mathfrak{t}^{2Y}) \\ &\leq \rho(\kappa) \vee \eta(\kappa) = (\rho \vee \eta)_{(\kappa)} = (\eta \vee \rho)_{(\kappa)}. \end{aligned}$$

And so, we have $\eta * \rho \leq \eta \vee \rho$. Therefore $(\eta * \rho) \vee (\rho * \eta) \leq \eta \vee \rho$.

Proposition 1.10: Let η be an antifuzzy_left_ideal, ρ be an antifuzzy_generalization bi_ideal, and ξ be an antifuzzy interior_ideal in S _weakly Y _regular semi_group $\mathcal{H}$, then $\rho * \eta * \xi \subseteq \rho \cup \eta \cup \xi$.

Proof: Let η, ρ and ξ be any antifuzzy_right_ideal, any antifuzzy_generalization bi_ideal and antifuzzy_interior_ideal ξ of S _weakly Y _regular semi_group $\mathcal{H}$, respectively, forever $\kappa \in \mathcal{H}$, $\exists q, \mathfrak{t} \in \mathcal{H}$, and $Y > 1$, such that $\kappa = \kappa q^Y \kappa^2 \mathfrak{t}^Y$.

$$\begin{aligned} \kappa &= \kappa q^Y \kappa^2 \mathfrak{t}^Y = (\kappa q^Y \kappa) \kappa \mathfrak{t}^Y \\ (\rho * \eta * \xi)_{(\kappa)} &= \bigwedge_{\kappa = (\kappa q^Y \kappa) \kappa \mathfrak{t}^Y} \{ \rho(\kappa q^Y \kappa) \vee (\eta * \xi)(\kappa \mathfrak{t}^Y) \} \\ &\leq \rho(\kappa) \vee \{ \bigwedge_{\kappa \mathfrak{t}^Y = \kappa q^Y \kappa^2 \mathfrak{t}^Y \mathfrak{t}^Y = \kappa q^Y \kappa \kappa \mathfrak{t}^{2Y}} \{ \eta(\kappa q^Y \kappa) \vee \xi(\kappa \mathfrak{t}^{2Y}) \} \} \\ &\leq \rho(\kappa) \vee \eta(\kappa) \vee \xi(\kappa) \\ &= (\rho \cup \eta \cup \xi)_{(\kappa)}, \text{ and so, we have } \rho * \eta * \xi \subseteq \rho \cup \eta \cup \xi. \end{aligned}$$

Proposition 1.11: A fuzzy_sub_set η of S _weakly Y _regular LA_semi_group $\mathcal{H}$ be fuzzy_right_ideals iff its fuzzy_left_ideals.

Proof: $\Rightarrow$ assume η be fuzzy_right_ideals for S _weakly Y _regular LA_semi_group $\mathcal{H}$, since $\mathcal{H}$ is S _weakly Y _regular so forever $\kappa_1, \kappa_2 \in \mathcal{H}$, $\exists q_1, \mathfrak{t}_1, q_2$ and $\mathfrak{t}_2 \in \mathcal{H}$, where $Y > 1$, such that $\kappa_1 = \kappa_1 q_1^Y \kappa_1^2 \mathfrak{t}_1^Y$, $\kappa_2 = \kappa_2 q_2^Y \kappa_2^2 \mathfrak{t}_2^Y$.

$$\eta(\kappa_1 \kappa_2) = \eta([(\kappa_1 q_1^Y \kappa_1^2) \mathfrak{t}_1^Y] \kappa_2) = \eta((\kappa_2 \mathfrak{t}_1^Y)(\kappa_1 q_1^Y \kappa_1^2))$$

$$\geq \eta(\kappa_2 \mathfrak{t}_1^Y)$$

$$\geq \eta(\kappa_2)$$

That is η fuzzy_left_ideal

$\Leftarrow$ Suppose η fuzzy_left_ideal of S -weakly Y -regular LA-semi-group $\mathcal{H}$, then

$$\begin{aligned}\eta(\kappa_1 \kappa_2) &= \eta((\kappa_1(\kappa_2 \mathfrak{q}_2^Y))(\kappa \mathfrak{t}_2^Y)) = \eta([\kappa \mathfrak{t}_2^Y(\kappa_2 \mathfrak{q}_2^Y)]\kappa_1) \\ &= \eta((\kappa_1)) \text{ (because } \eta \text{ fuzzy_left_ideal)} \\ &\geq \eta(\kappa_1). \text{ That is } \xi \text{ fuzzy_left_ideal}\end{aligned}$$

Proposition 1.12: Each fuzzy_two_side_ideals for S -weakly Y -regular LA-semi_group $\mathcal{H}$, and is left identity be idempotent.

Proof: Assume η be fuzzy_two sided_ideals for $\mathcal{H}$, so $\eta \circ \eta \subseteq \eta \circ \mathcal{H} \subseteq \eta$.

Hence, forever $\kappa \in \mathcal{H}$, $\exists \mathfrak{q}, \mathfrak{t} \in \mathcal{H}$, where $Y > 1$, s. t

$$\begin{aligned}\kappa &= \kappa \mathfrak{q}^Y \kappa^2 \mathfrak{t}^Y = \kappa \mathfrak{q}^Y \kappa \mathfrak{t}^Y = (\kappa \mathfrak{q}^Y \kappa)(\kappa \mathfrak{t}^Y) \\ (\eta \circ \eta)(\kappa) &= \bigvee_{\kappa = ((\kappa \mathfrak{q}^Y \kappa)(\kappa \mathfrak{t}^Y))} \eta((\kappa \mathfrak{q}^Y \kappa) \wedge \eta(\kappa \mathfrak{t}^Y)) \\ &\geq \eta((\kappa \mathfrak{q}^Y \kappa) \wedge \eta(\kappa \mathfrak{t}^Y)) \\ &\geq \eta(\kappa) \wedge \eta(\kappa) = \eta(\kappa).\end{aligned}$$

And this implies that $\eta \circ \eta \supseteq \eta$, hence $\eta \circ \eta = \eta$.

Conclusion.

The main results are obtained are:

1. If η, ρ are any antifuzzy two-sided ideal in S -weakly Y -regular $\mathcal{H}$, then $\eta * \rho = \eta \cup \rho$.
2. For antifuzzy_generalization bi_ideal η and ρ antifuzzy_left_ideal of S -weakly Y -regular semi_group $\mathcal{H}$, then $\eta * \rho \leq \eta \vee \rho$.
3. If η and ρ be any antifuzzy_interior_ideals of S -weakly Y -regular semi_group $\mathcal{H}$, then $(\eta * \rho) \vee (\rho * \eta) \leq \eta \vee \rho$.

References

- [1] A. S. Mohammed, S. H. Asaad, and I. S. Ahmed, "New properties of strongly regular left almost semigroups," *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 25, no. 2, pp. 615–622 (2022), doi: 10.1080/09720529.2021.1982491.
- [2] I. S. Ahmed, A. Al-Fayadh, H. H. Ebrahim, and L. S. Abdalbaqi, "Implementation of the neutrosophic sets in measurable space with respect to neutrosophic ring," *International Journal of Neutrosophic Science*, vol. 25, no. 3, pp. 187–193 (2025), doi: 10.54216/IJNS.250317.
- [3] S. H. Asaad, I. S. Ahmed, and H. H. Ebrahim, "On finitely null-additive and finitely weakly null-additive relative to the σ -ring,"

- Baghdad Science Journal*, vol. 19, no. 5, Art. no. 3 (2022), doi: 10.21123/bsj.2022.5771.
- [4] I. S. Ahmed, A. Al-Fayadh, and H. H. Ebrahim, "Fuzzy σ -algebra and some related concepts," in *AIP Conference Proceedings*, vol. 2834, no. 1 (2023), doi: 10.1063/5.0162037.
 - [5] A. R. S. Juma, A. Al-Fayadh, S. P. Vijayalakshmi, and T. V. Sudharsan, "Upper bound on the third Hankel determinant for the class of uni-valent functions using an integral operator," *Afrika Matematika*, vol. 33, no. 2, Art. no. 56 (2022).
 - [6] S. H. Assad and A. S. Mohammed, "New properties of Y -regular LA-semigroups and related concepts," in *AIP Conference Proceedings*, vol. 2845, no. 1, Art. no. 060023 (2023).
 - [7] S. M. Hong, Y. B. Jun, and J. Meng, "Intuitionistic fuzzy interior ideals in semigroups," *Indian Journal of Pure and Applied Mathematics*, vol. 26, no. 9, pp. 859–863 (1995).
 - [8] N. Kehayopulu and M. Tsingelis, "Fuzzy sets in ordered groupoids and semigroups," *Information Sciences*, vol. 65, pp. 128–132 (2002).
 - [9] N. Kehayopulu and M. Tsingelis, "Fuzzy bi-ideals in ordered semigroups," *Information Sciences*, vol. 171, pp. 13–28 (2005).
 - [10] M. Khan and T. Asif, "Characterizations of semigroups by their anti-fuzzy ideals," *Journal of Mathematics Research*, vol. 2, no. 3, pp. 134–143 (2010).
 - [11] J. N. Mordeson, D. S. Malik, and N. Kuroki, *Fuzzy Semigroups*. Berlin, Germany: Springer-Verlag (2003).
 - [12] N. Kuroki, "On fuzzy ideals and fuzzy bi-ideals in semigroups," *Fuzzy Sets and Systems*, vol. 5, pp. 203–215 (1981).
 - [13] M. Shabir and Y. Nawaz, "Semigroups characterized by the properties of their anti-fuzzy ideals," *Journal of Advanced Research in Pure Mathematics*, vol. 1, no. 3, pp. 42–59 (2009).

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