Journal of Discrete Mathematical Sciences & Cryptography
ISSN 0972-0529 (Print), ISSN 2169-0065 (Online)
DOI : 10.47974/JDMSC-2384

On some fuzzy ideal of S_weakly Υ_regular of semi_groups

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Abstract

This article introduces some fuzzy ideal of S_weakly Υ_regular of semi_groups, and we have characterized of a S_weakly Υ_regular semi_groups using properties for fuzzy_left and fuzzy_right_ideals of some natural examples of semi_groups. Furthermore, a basic properties and characterizations of the fuzzy_ideals(right, left, two sided, quasi, interior, bi_generalized_bi, and (1, 2)) of a S_weakly Υ_semi_group have been investigated.

Subject Classification: *05C07, 05C09.*

Keywords: *Algebra, Graph, Cryptography and semi_groups.*

Introduction

Some concepts play a significant turn in algebra and cryptography see [1], [2], [4], [5] and [6]. The S_weakly regular semi_groups concept was first introduced by Akram. This article has been extended the idea for S_weakly regular semi_group $\mathcal{H}$ which is $\varkappa \in \mathcal{H}$, $\exists$q, ȶ $\in \mathcal{H}$, such that $\varkappa = \varkappa$q$\varkappa^2$ȶ. They named it as a left almost semi_group (LA_semi_group). An LA_semi_group be groupoid $\mathcal{H}$ which its elements a left invertive law ($\varkappa$ q) ȶ = (ȶq) $\varkappa$, $\forall$ $\varkappa$, q, ȶ $\in$ $\mathcal{H}$[1]. The fuzzy _subsets η for the semi_groups $\mathcal{H}$ named fuzzy_sub_

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semi_groups for $\mathcal{H}$ when $\eta(\varkappa \mathrm{q}) \geq \min\{\eta(\varkappa), \eta(\mathrm{q})\}$, whenever $\varkappa$, $\mathrm{q} \in \mathcal{H}$ [3, 6]. The fuzzy_sub_semi_groupq η for the semi_groups $\mathcal{H}$ called fuzzy_interior_ ideal for $\mathcal{H}$ where $\eta(\varkappa \mathrm{q} ŧ) \geq \eta(\mathrm{q})$, $\forall \varkappa$, q, ŧ $\in \mathcal{H}$ [7]. If η fuzzy_left _ideal, fuzzy_right_ideals for $\mathcal{H}$, so η named fuzzy_ideal for $\mathcal{H}$ [8]. if $\mathcal{H}$ ordered semi_groups, then fuzzy_subsets η for $\mathcal{H}$ named fuzzy_sub_semi_groups for $\mathcal{H}$ if $\eta(\varkappa$ q$\min\{\eta(\varkappa)$, $\eta(\mathrm{q})\}$, for every $\varkappa$, q $\in \mathcal{H}$ [9]. the fuzzy_subsets η for $\mathcal{H}$ named antifuzzy_left(right)_ideals for $\mathcal{H}$ if $\eta(\varkappa \mathrm{q}) \leq \eta(\mathrm{q}), (\eta(\varkappa \mathrm{q}) \leq \eta(\varkappa))$, forever $\varkappa, \mathrm{q} \in \mathcal{H}$ [11]. the fuzzy_subsets η for semi_groups $\mathcal{H}$ named fuzzy_generalized bi_ideals for $\mathcal{H}$ if $\eta((\varkappa \mathrm{q})ŧ) \geq \eta(\varkappa) \wedge \eta(ŧ)$, $\forall \varkappa$, q, ŧ $\in \mathcal{H}$ [12]. Each fuzzy_left_ideal (fuzzy_right_ideal) be fuzzy_bi_ideals for $\mathcal{H}$ [13]. if η is antifuzzy_right_ideals & μ is the antifuzzy_left_ideals for the semi_group $\mathcal{H}$, so $\eta * \mu \supseteq \eta \cup \mu$ [10].

The main results

Definition 1.1: The semi_group $\mathcal{H}$ named S_weakly Ɏ_regular if forevery $\varkappa \in \mathcal{H}$, there exists q, ŧ $\in \mathcal{H}$ where Ɏ > 1sach that $\varkappa = \varkappa \mathrm{q}^{Ɏ} \varkappa^2 ŧ^{Ɏ}$.

Proposition 1.2: In S_weakly Ɏ_regular semi_groups $\mathcal{H}$, each fuzzy interior_ ideals be idempotent.

Proof: Assume η be fuzzy_interior_ideals for S_weakly Ɏ_regular semi_ groups $\mathcal{H}$, then clearly $\eta \circ \eta \subseteq \eta$.

Let $\varkappa \in \mathcal{H}$ then $\exists$ q, ŧ $\in \mathcal{H}$, such that $\varkappa = \varkappa \mathrm{q}^{Ɏ} \varkappa^2 ŧ^{Ɏ}$ where Ɏ > 1.

$$\Rightarrow \varkappa = \varkappa \mathrm{q}^{Ɏ} \varkappa^2 ŧ^{Ɏ} = (\varkappa \mathrm{q}^{Ɏ})\varkappa(\varkappa)ŧ^{Ɏ}$$
$$= (\varkappa \mathrm{q}^{Ɏ})\varkappa(\varkappa \mathrm{q}^{Ɏ}\varkappa^2 ŧ^{Ɏ})ŧ^{Ɏ}$$
$$= (\varkappa \mathrm{q}^{Ɏ})\varkappa(\varkappa \mathrm{q}^{Ɏ})(\varkappa)\varkappa(ŧ^{2Ɏ})$$
$$(\eta \circ \eta)_{(\varkappa)} = \bigvee_{\varkappa = [(\varkappa \mathrm{q}^{Ɏ})\varkappa(\varkappa \mathrm{q}^{Ɏ})(\varkappa)\varkappa(ŧ^{2Ɏ})]} \{\eta[(\varkappa \mathrm{q}^{Ɏ})\varkappa(\varkappa \mathrm{q}^{Ɏ})] \wedge \eta[(\varkappa)\varkappa(ŧ^{2Ɏ})]\}$$
$$\geq \eta[(\varkappa \mathrm{q}^{Ɏ})\varkappa(\varkappa \mathrm{q}^{Ɏ})] \wedge \eta[(\varkappa)\varkappa(ŧ^{2Ɏ})]$$
$$\geq \eta(\varkappa) \wedge \eta(\varkappa) = \eta(\varkappa)$$

This is implying that $\eta \circ \eta \supseteq \eta$, hence $\eta \circ \eta = \eta$. Then η is idempotent.

Proposition 1.3: If η fuzzy_sub_set of S_weakly Ɏ_regular semi_groups $\mathcal{H}$, we get it is an antifuzzy_two side_ideals for $\mathcal{H}$ iff its antifuzzy_interior_ ideals for $\mathcal{H}$.

Proof: $\Rightarrow$ Since η is an antifuzzy_two sided_ideal of $\mathcal{H}$ and let $\varkappa$, q and ŧ $\in \mathcal{H}$,

$\eta(\varkappa \mathrm{q} ŧ) = \eta(\varkappa \mathrm{q})ŧ) \leq \eta(\varkappa \mathrm{q})$ (because η is an antifuzy_right_ideal)

$\leq \eta(\mathrm{q})$ (because η is an antifuzzy_left_ideal)

$\eta(\varkappa \mathrm{q} ŧ) \leq \eta(\mathrm{q})$ that is η is an antifuzzy_interior_ideal

$\Leftarrow$ assume η is antifuzzy_interior_ideal for η. Let $\varkappa_1, \varkappa_2 \in \mathcal{H}$, by hypothises so $\exists$ q_1, $ŧ_1 \mathrm{q}_2$ and $ŧ_2 \in \mathcal{H}$, Ɏ > 1 such that $\varkappa_1 = \varkappa_1 \mathrm{q}_1{}^{Ɏ} \varkappa_1{}^2 ŧ_1{}^{Ɏ}$, $\varkappa_2 = \varkappa_2 \mathrm{q}_2{}^{Ɏ} \varkappa_2{}^2 ŧ_2{}^{Ɏ}$.

$\eta(\varkappa_1 \varkappa_2) = \eta(\varkappa_1 \mathrm{q}_1{}^{Ɏ} \varkappa_1{}^2 ŧ_1{}^{Ɏ} \varkappa_2)$

$= \eta((\varkappa_1 q_1{}^{Y})\varkappa_1(\varkappa_1 t_1{}^{Y}\varkappa_2))$
$\leq \eta(\varkappa_1)$, and also
$\eta(\varkappa_1\varkappa_2) = \eta(\varkappa_1\varkappa_2 q_2{}^{Y}\varkappa_2{}^2 t_2{}^{Y})$
$= \eta((\varkappa_1\varkappa_2 q_2{}^{Y})\varkappa_2(\varkappa_2 t_2{}^{Y}))$
$\leq \eta(\varkappa_2)$. Hence, $\eta$is an antifuzzy_two_sided_ideal of $\mathcal{H}$.

Proposition 1.4: In S_weakly Y_regular semi_groups $\mathcal{H}$, then

i- Every antifuzzy_right_ideal is idempotent.

ii- Every antifuzzy_interior_ideal is idempotent.

Proposition 1.5: [1]. Let η is an antifuzzy_right_ideal and ρ an antifuzzy_left_ideal of a semi_group $\mathcal{H}$, then $\eta * \rho \supseteq \eta \cup \rho$.

To put reverse for (1.5), we need to strengthen a condition for semi_group $\mathcal{H}$.

Proposition 1.6: If η, ρ are any antifuzzy two−sided ideal in S_weakly Y_regular $\mathcal{H}$, then $\eta * \rho = \eta \cup \rho$.

Proof: Let η and ρ be any antifuzzy_two sided_ideal of $\mathcal{H}$, then obviously $\eta * \rho \supseteq \eta \cup \rho$.Since $\mathcal{H}$ is an S_weakly Y_regular so forever element $\varkappa \in \mathcal{H}$, ∃s, t $\in \mathcal{H}$, such that $\varkappa = \varkappa q^{Y}\varkappa^2 t^{Y}$, where $Y > 1$

$(\eta * \rho)_{(\varkappa)} = \wedge_{\varkappa=\varkappa q^{Y}\varkappa^2 t^{Y}=\varkappa q^{Y}\varkappa\varkappa t^{Y}}\{\eta(\varkappa q^{Y}) \vee \rho(\varkappa\varkappa t^{Y})\}$
$\leq \eta(\varkappa q^{Y}) \vee \rho(\varkappa(\varkappa t^{Y})) \leq \eta(\varkappa) \vee \rho(\varkappa) = (\eta \cup \rho)(\varkappa)$.

Then $(\eta * \rho) \subseteq \eta \cup \rho$. Hence, $\eta * \rho = \eta \cup \rho$.

Proposition 1.7: Let η be a fuzzy_sub_set of $\mathcal{H}$,where $\mathcal{H}$ is S_weakly Y_regular semi_group then η is anti fuzzy_bi_ideal for $\mathcal{H}$ iff its antifuzzy_generalization bi_ideal for $\mathcal{H}$.

Proof: $\Rightarrow$ assume η is antifuzzy bi_ideals for $\mathcal{H}$, so η be antifuzzy_ generalization bi_ideal of $\mathcal{H}$.

$\Leftarrow$ assume η is antifuzzy_generalization bi_ideal for $\mathcal{H}$, because $\mathcal{H}$ be S_weakly Y_regular semi_group, so forever $\varkappa_1$ and $\varkappa_2 \in \mathcal{H}$, ∃ q_1, t_1, q_2 and $t_2 \in \mathcal{H}$, where $Y > 1$, sach that $\varkappa_1 = \varkappa_1 q_1{}^{Y}\varkappa_1{}^2 t_1{}^{Y}$, $\varkappa_2 = \varkappa_2 q_2{}^{Y}\varkappa_2{}^2 t_2{}^{Y}$.

$\eta(\varkappa_1\varkappa_2) = \eta(\varkappa_1 q_1{}^{Y}\varkappa_1{}^2 t_1{}^{Y}\varkappa_2) = \eta(\varkappa_1(q_1{}^{Y}\varkappa_1{}^2 t_1{}^{Y})\varkappa_2) \leq \eta(\varkappa_1) \vee \eta(\varkappa_2)$.

Therefore, η is an antifuzzy sub_semi_group of $\mathcal{H}$. Hence, η is an antifuzzy_generalization bi_ideal of $\mathcal{H}$.

Proposition 1.8: For antifuzzy_generalization bi_ideal η and ρ antifuzzy_left_ ideal of S_weakly Y_regular semi_group $\mathcal{H}$, then $\eta * \rho \leq \eta \vee \rho$.

Proof: Let η and ρ be any antifuzzy_generalization bi_ideal and antifuzzy_ right_ideal of $\mathcal{H}$, respectively, then forever $\varkappa \in \mathcal{H}$, ∃ s, t $\in \mathcal{H}$, such that

$\varkappa = \varkappa \mathrm{q}^{\Upsilon}\varkappa^2 \mathrm{t}^{\Upsilon}$ where $\Upsilon > 1$, so. Then $\varkappa = \varkappa \mathrm{q}^{\Upsilon}\varkappa^2 \mathrm{t}^{\Upsilon} = \varkappa = \varkappa \mathrm{q}^{\Upsilon}\varkappa\varkappa \mathrm{t}^{\Upsilon} = (\varkappa \mathrm{q}^{\Upsilon}\varkappa)(\varkappa \mathrm{t}^{\Upsilon})$

$(\eta * \rho)_{(\varkappa)} = \wedge_{\varkappa = (\varkappa \mathrm{q}^{\Upsilon}\varkappa)(\varkappa \mathrm{t}^{\Upsilon})}\{\eta(\varkappa \mathrm{q}^{\Upsilon}\varkappa) \vee \rho(\varkappa \mathrm{t}^{\Upsilon})\}$
$\leq \eta(\varkappa \mathrm{q}^{\Upsilon}\varkappa) \vee \rho(\varkappa \mathrm{t}^{\Upsilon}) \leq \eta(\varkappa) \vee \rho(\varkappa) = (\eta \vee \rho)(\varkappa)$

And so, we have $\eta * \rho \leq \eta \cup \rho$.

Proposition 1.9: If η and ρ be any antifuzzy_interior_ideals of S_weakly Y_regular semi_group $\mathcal{H}$, then $(\eta * \rho) \vee (\rho * \eta) \leq \eta \vee \rho$.

Proof: Let η, ρ are antifuzzy interior_ideal for $\mathcal{H}$, & $\varkappa \in \mathcal{H}$. Then since $\mathcal{H}$ is an S_weakly Y_regular semi_group $\mathcal{H}$, then $\exists$ q, $\mathrm{t} \in \mathcal{H}$ and $\Upsilon > 1$, such that $\varkappa = \varkappa \mathrm{q}^{\Upsilon}\varkappa^2 \mathrm{t}^{\Upsilon}$.

$\varkappa = \varkappa \mathrm{q}^{\Upsilon}\varkappa^2 \mathrm{t}^{\Upsilon} = \varkappa \mathrm{q}^{\Upsilon}\varkappa\varkappa \mathrm{q}^{\Upsilon}\varkappa^2 \mathrm{t}^{\Upsilon}\mathrm{t}^{\Upsilon} = (\varkappa \mathrm{q}^{\Upsilon})\varkappa(\varkappa \mathrm{q}^{\Upsilon})\varkappa\varkappa \mathrm{t}^{2\Upsilon}$.

Hence $(\eta * \rho)_{(\varkappa)} = \wedge_{\varkappa = [(\varkappa \mathrm{q}^{\Upsilon})\varkappa(\varkappa \mathrm{q}^{\Upsilon})](\varkappa\varkappa \mathrm{t}^{2\Upsilon})}\{\eta(\varkappa \mathrm{q}^{\Upsilon})\varkappa(\varkappa \mathrm{q}^{\Upsilon}) \vee \rho(\varkappa\varkappa \mathrm{t}^{2\Upsilon})\}$
$\leq \eta((\varkappa \mathrm{q}^{\Upsilon})\varkappa(\varkappa \mathrm{q}^{\Upsilon})) \vee \rho(\varkappa\varkappa \mathrm{t}^{2\Upsilon})$
$\leq \eta(\varkappa) \vee \rho(\varkappa) = (\eta \vee \rho)(\varkappa)$, and so, we have $\eta * \rho \leq \eta \vee \rho$.

To proof $(\rho * \eta) \leq \eta \vee \rho$.

$(\rho * \eta)_{(\varkappa)} = \wedge_{\varkappa = \varkappa \mathrm{q}^{\Upsilon}\varkappa^2 \mathrm{t}^{\Upsilon} = (\varkappa \mathrm{q}^{\Upsilon})\varkappa(\varkappa \mathrm{q}^{\Upsilon})\varkappa\varkappa \mathrm{t}^{2\Upsilon}}\{\rho(\varkappa \mathrm{q}^{\Upsilon})\varkappa(\varkappa \mathrm{q}^{\Upsilon}) \vee \eta(\varkappa\,\varkappa(\mathrm{t}^{2\Upsilon}))\}$
$\leq \rho((\varkappa \mathrm{q}^{\Upsilon})\varkappa(\varkappa \mathrm{q}^{\Upsilon})) \vee \eta(\varkappa\varkappa(\mathrm{t}^{2\Upsilon}))$
$\leq \rho(\varkappa) \vee \eta(\varkappa) = (\rho \vee \eta)_{(\varkappa)} = (\eta \vee \rho)_{(\varkappa)}$.

And so, we have $\eta * \rho \leq \eta \vee \rho$. Therefore $(\eta * \rho) \vee (\rho * \eta) \leq \eta \vee \rho$.

Proposition 1.10: Let η be an antifuzzy_left_ideal, ρ be an antifuzzy_ generalization bi_ideal, and ξ be an antifuzzy interior_ideal in S_weakly Y_regular semi_group $\mathcal{H}$, then $\rho * \eta * \xi \subseteq \rho \cup \eta \cup \xi$.

Proof: Let η, ρ and ξ be any antifuzzy_right_ideal, any antifuzzy_generalization bi_ideal and antifuzzy_interior_ideal ξ of S_weakly Y_regular semi_group $\mathcal{H}$, respectively, forever $\varkappa \in \mathcal{H}$, $\exists$ q, $\mathrm{t} \in \mathcal{H}$, and $\Upsilon > 1$, such that $\varkappa = \varkappa \mathrm{q}^{\Upsilon}\varkappa^2 \mathrm{t}^{\Upsilon}$.

$\varkappa = \varkappa \mathrm{q}^{\Upsilon}\varkappa^2 \mathrm{t}^{\Upsilon} = (\varkappa \mathrm{q}^{\Upsilon}\varkappa)\varkappa \mathrm{t}^{\Upsilon}$

$(\rho * \eta * \xi)_{(\varkappa)} = \wedge_{\varkappa = ((\varkappa \mathrm{q}^{\Upsilon}\varkappa)\varkappa \mathrm{t}^{\Upsilon})}\{\rho(\varkappa \mathrm{q}^{\Upsilon}\varkappa) \vee (\eta * \xi)(\varkappa \mathrm{t}^{\Upsilon})\}$
$\leq \rho(\varkappa) \vee \{\wedge_{\varkappa \mathrm{t}^{\Upsilon} = \varkappa \mathrm{q}^{\Upsilon}\varkappa^2 \mathrm{t}^{\Upsilon}\mathrm{t}^{\Upsilon} = \varkappa \mathrm{q}^{\Upsilon}\varkappa\varkappa \mathrm{t}^{2\Upsilon}}\{\eta(\varkappa \mathrm{q}^{\Upsilon}) \vee \xi(\varkappa\varkappa \mathrm{t}^{2\Upsilon})\}$
$\leq \rho(\varkappa) \vee \eta(\varkappa) \vee \xi(\varkappa)$
$= (\rho \cup \eta \cup \xi)(\varkappa)$, and so, we have $\rho * \eta * \xi \subseteq \rho \cup \eta \cup \xi$.

Proposition 1.11: A fuzzy_sub_set η of S_weakly Y_regular LA_semi_group $\mathcal{H}$ be fuzzy_right_ideals iff its fuzzy_left_ideals.

Proof: $\Longrightarrow$ assume η be fuzzy_right_ideals for S_weakly Y_regular LA_semi_ group $\mathcal{H}$, since $\mathcal{H}$ is S_weakly Y_regular so forever $\varkappa_1, \varkappa_2 \in \mathcal{H}$, $\exists$ q_1, t_1, q_2 and t_2 $\in \boldsymbol{\mathcal{H}}$, where $\Upsilon > 1$, sach that $\varkappa_1 = \varkappa_1 {\mathrm{q}_1}^{\Upsilon}{\varkappa_1}^2{\mathrm{t}_1}^{\Upsilon}$, $\varkappa_2 = \varkappa_2 {\mathrm{q}_2}^{\Upsilon}{\varkappa_2}^2{\mathrm{t}_2}^{\Upsilon}$.

$\eta(\varkappa_1\varkappa_2) = \eta([(\varkappa_1 {\mathrm{q}_1}^{\Upsilon}{\varkappa_1}^2){\mathrm{t}_1}^{\Upsilon}]\varkappa_2) = \eta((\varkappa_2 {\mathrm{t}_1}^{\Upsilon})(\varkappa_1 {\mathrm{q}_1}^{\Upsilon}{\varkappa_1}^2))$

$\geq \eta(\varkappa_2 ҍ_1{}^{\Upsilon})$
$\geq \eta(\varkappa_2)$

That is η fuzzy_left_ideal

$\Longleftarrow$ Suppose η fuzzy_left_ideal of of S_weakly Υ_regular LA_semi_group $\mathcal{H}$, then

$\eta(\varkappa_1\varkappa_2) = \eta((\varkappa_1(\varkappa_2 q_2{}^{\Upsilon}))(\varkappa\varkappa ҍ_2{}^{\Upsilon})) = \eta([(\varkappa\varkappa ҍ_2{}^{\Upsilon})(\varkappa_2 q_2{}^{\Upsilon})]\varkappa_1)$
$= \eta((\varkappa_1))$(because η fuzzy_left_ideal)
$\geq \eta(\varkappa_1)$). That is ξ fuzzy_left_ideal

Proposition 1.12: Each fuzzy_two side_ideals for S_weakly Υ_regular LA_semi_group $\mathcal{H}$, and is left identity be idempotent.

Proof: Assume η be fuzzy_two sided_ideals for $\mathcal{H}$, so $\eta \circ \eta \subseteq \eta \circ \mathcal{H} \subseteq \eta$.

Hence, forever $\varkappa \in \mathcal{H}$, $\exists$ q, ҍ $\in \mathcal{H}$, where $\Upsilon > 1$, s. t

$\varkappa = \varkappa q^{\Upsilon}\varkappa^2 ҍ^{\Upsilon} = \varkappa q^{\Upsilon}\varkappa\varkappa ҍ^{\Upsilon} = (\varkappa q^{\Upsilon}\varkappa)(\varkappa ҍ^{\Upsilon})$

$(\eta \circ \eta)_{(\varkappa)} = \bigvee_{\varkappa=((\varkappa q^{\Upsilon})\varkappa)(\varkappa ҍ^{\Upsilon})} \eta((\varkappa q^{\Upsilon})\varkappa) \wedge \eta(\varkappa ҍ^{\Upsilon})$
$\geq \eta((\varkappa q^{\Upsilon})\varkappa) \wedge \eta(\varkappa ҍ^{\Upsilon})$
$\geq \eta(\varkappa) \wedge \eta(\varkappa) = \eta(\varkappa)$.

And this implies that $\eta \circ \eta \supseteq \eta$, hence $\eta \circ \eta = \eta$.

Conclusion.

The main results are obtained are:

1. If η, ρ are any antifuzzy two$-$sided ideal in S_weakly Υ_regular $\mathcal{H}$, then $\eta * \rho = \eta \cup \rho$.
2. For antifuzzy_generalization bi_ideal η and ρ antifuzzy_left_ideal of S_weakly Υ_regular semi_group $\mathcal{H}$, then $\eta * \rho \leq \eta \vee \rho$.
3. If η and ρ be any antifuzzy_interior_ideals of S_weakly Υ_regular semi_group $\mathcal{H}$, then $(\eta * \rho) \vee (\rho * \eta) \leq \eta \vee \rho$.

References

[1] A. S. Mohammed, S. H. Asaad, and I. S. Ahmed, "New properties of strongly regular left almost semigroups," *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 25, no. 2, pp. 615–622 (2022), doi: 10.1080/09720529.2021.1982491.

[2] I. S. Ahmed, A. Al-Fayadh, H. H. Ebrahim, and L. S. Abdalbaqi, "Implementation of the neutrosophic sets in measurable space with respect to neutrosophic ring," *International Journal of Neutrosophic Science*, vol. 25, no. 3, pp. 187–193 (2025), doi: 10.54216/IJNS.250317.

[3] S. H. Asaad, I. S. Ahmed, and H. H. Ebrahim, "On finitely null-additive and finitely weakly null-additive relative to the σ-ring,"

Baghdad Science Journal, vol. 19, no. 5, Art. no. 3 (2022), doi: 10.21123/bsj.2022.5771.

[4] I. S. Ahmed, A. Al-Fayadh, and H. H. Ebrahim, "Fuzzy σ-algebra and some related concepts," in AIP Conference Proceedings, vol. 2834, no. 1 (2023), doi: 10.1063/5.0162037.

[5] A. R. S. Juma, A. Al-Fayadh, S. P. Vijayalakshmi, and T. V. Sudharsan, "Upper bound on the third Hankel determinant for the class of univalent functions using an integral operator," *Afrika Matematika*, vol. 33, no. 2, Art. no. 56 (2022).

[6] S. H. Assad and A. S. Mohammed, "New properties of Υ-regular LA-semigroups and related concepts," in AIP Conference Proceedings, vol. 2845, no. 1, Art. no. 060023 (2023).

[7] S. M. Hong, Y. B. Jun, and J. Meng, "Intuitionistic fuzzy interior ideals in semigroups," *Indian Journal of Pure and Applied Mathematics,* vol. 26, no. 9, pp. 859–863 (1995).

[8] N. Kehayopulu and M. Tsingelis, "Fuzzy sets in ordered groupoids and semigroups," *Information Sciences*, vol. 65, pp. 128–132 (2002).

[9] N. Kehayopulu and M. Tsingelis, "Fuzzy bi-ideals in ordered semigroups," *Information Sciences,* vol. 171, pp. 13–28 (2005).

[10] M. Khan and T. Asif, "Characterizations of semigroups by their anti-fuzzy ideals," *Journal of Mathematics Research,* vol. 2, no. 3, pp. 134–143 (2010).

[11] J. N. Mordeson, D. S. Malik, and N. Kuroki, Fuzzy Semigroups. Berlin, Germany: Springer-Verlag (2003).

[12] N. Kuroki, "On fuzzy ideals and fuzzy bi-ideals in semigroups," *Fuzzy Sets and Systems*, vol. 5, pp. 203–215 (1981).

[13] M. Shabir and Y. Nawaz, "Semigroups characterized by the properties of their anti-fuzzy ideals," *Journal of Advanced Research in Pure Mathematics*, vol. 1, no. 3, pp. 42–59 (2009).

Received June, 2025