

Graph-driven side-channel attack detection using AI and discrete math

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Abstract

By combining graph-theoretic modelling and artificial intelligence methods, this study shows a new way to find side-channel attacks. Data from the side channels is shown as graphs that show the connections and patterns that could be used in an attack. Feature extraction techniques are used on these graph models to show off unique attack patterns. Graph Neural Networks (GNNs) and Support Vector Machines (SVMs) are two advanced AI algorithms that are designed and trained to accurately find side-channel intrusions. Discrete mathematics ideas like vector measure spaces and orthogonal sequences are used in the suggested method to make identification more reliable. Evaluation measures and validation methods are used to check the accuracy and dependability of detection, showing that the framework works well at finding both simple and complex side-channel dangers.

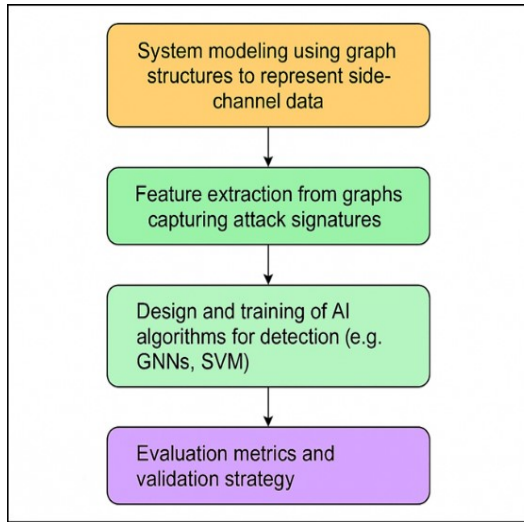
Subject Classification: 68R01, 68M25.

Keywords: Side-channel attack detection, Graph neural networks, Vector metric space, Orthogonal sequences, Feature extraction.

1. Introduction

Side-channel attacks are a big problem for cryptographic systems and safe hardware because they take advantage of physical signals that get leaked by accident, like power use, electromagnetic waves, or time data. By looking at secondary data linked to secret keys or private processes, these hacks get around normal encryption defences [1]. It's still hard to find these kinds of threats because side-channel messages are often hidden by noise and changes in the surroundings. Recently made progress in artificial intelligence (AI) suggests useful ways to find complex trends in side-channel data [2, 3]. Normal AI methods, on the other hand, often use raw or already processed data, which might not fully capture the structural and relational properties of side-channel signals. Graph-theoretic approaches are being looked into because of this limitation [4]. Figure 1 illustrates AI-based graph analysis detecting side-channel attack patterns. In these approaches, side-channel information is modelled as graphs that show how observed signals depend on, correlate with, or change over time [5, 6].

This study uses discrete mathematics, especially vector metric spaces and orthogonal sequences, to create a system that improves the way side-channel data is represented and how features are extracted. Graph structures make it possible to group together complicated relationships [7, 8]. This helps AI programs like Graph Neural Networks (GNNs) and Support Vector Machines (SVMs) learn how to recognize different types of attacks. This article carefully creates the graph-driven detection method by explaining system modelling, feature extraction, AI design, and strict evaluation measures [9]. Adding discrete mathematics tools not only makes detection more accurate, but it also gives us theoretical information about how well the system can defend itself against complex side-channel attacks.

**Figure 1****Graph-Driven Side-Channel Attack Detection Using AI and Discrete Mathematics****2. Method Work****2.1 System modeling using graph structures to represent side-channel data**

Side-channel signals are represented as graphs, with nodes being single measurement points or time samples and edges being connections between them, like how they are related or when they happen next in time [10]. This graph-based model keeps structure and environmental details, which lets us better understand signal relationships that normal vectorised data might miss. The model uses graph theory to include both local and global exchanges in the side-channel data [11].

Definition: Graph

A graph G is an ordered pair $G = (V, E)$, where V is a finite set of vertices (nodes), and $E \subseteq V \times V$ is a set of edges connecting the vertices.

Definition: Weighted Graph

A weighted graph is a graph where each edge $e \in E$ has an associated weight $w(e) \in \mathbb{R}^+$, representing the strength of the relationship.

Theorem: Adjacency Matrix Representation

Every graph $G = (V, E)$ with $|V| = n$ can be represented by an adjacency matrix $A \in \mathbb{R}^{n \times n}$, where

$$A_{ij} = \{ w(v_i, v_j), \text{ if } (v_i, v_j) \in E, 0, \text{ otherwise } \}$$

Proof: By definition of adjacency matrix, each element corresponds to the presence and weight of an edge between nodes.

Equations:

1. Graph Representation:

$$G = (V, E, w) \quad (1)$$

2. Adjacency Matrix:

$$A = [a_{ij}], a_{ij} = w(v_i, v_j) \quad (2)$$

3. Degree of Node
- v_i
- :

$$d(v_i) = \sum_{j=1}^n a_{ij} \quad (3)$$

4. Graph Laplacian
- L
- :

$$L = D - A, \text{ where } D = \text{diag}(d(v_1), \dots, d(v_n)) \quad (4)$$

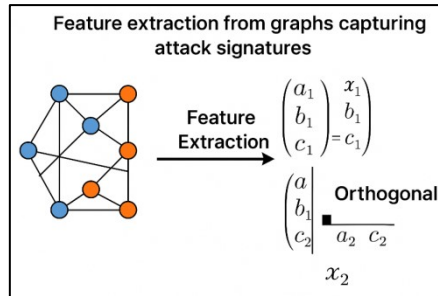
5. Spectral Decomposition of
- L
- :

$$L = U \Lambda U^T \quad (5)$$

Where U is a matrix of eigenvectors and Λ is a diagonal matrix of eigenvalues.

3. Feature extraction from graphs capturing attack signatures

Key features are taken from the built graphs to show features that are unique to side-channel attacks. To figure out how big structure patterns are, things like node centrality, clustering factors, and spectral features are calculated [12]. Figure 2 shows graph feature extraction and orthogonal signatures for attack detection.

**Figure 2****Graph-Based Feature Extraction and Orthogonal Signature Representation for Side-Channel Attack Detection**

Vector measure space traits and orthogonal sequences are also used to show how signals are orthogonal and how they converge. All of these characteristics work together like unique fingerprints, helping AI models tell the difference between good and bad side-channel activities.

Definition: Vector Metric Space

A vector metric space is a vector space V over a field $\mathbb{F}$ equipped with a metric $d: V \times V \rightarrow \mathbb{R}^+$ satisfying positivity, symmetry, and triangle inequality.

Definition: Orthogonal Sequence

A sequence $\{x_n\} \subseteq V$ is orthogonal if for $m \neq n$,

$$\langle x_m, x_n \rangle = 0 \quad (6)$$

where $\langle \cdot, \cdot \rangle$ is an inner product.

Theorem: Convergence in Vector Metric Space

A sequence $\{x_n\}$ converges to $x \in V$ if

$$\lim_{n \rightarrow \infty} d(x_n, x) = 0$$

Equations:

1. Inner Product:

$$\langle x, y \rangle = x^T y \quad (7)$$

2. Norm from Inner Product:

$$\|x\| = \sqrt{\langle x, x \rangle} \quad (8)$$

3. Orthogonality Condition:

$$\langle x_i, x_j \rangle = 0, \quad i \neq j$$

4. Clustering Coefficient C_i :

$$C_i = \frac{(2 T_i)}{(d(v_i)(d(v_i) - 1))} \quad (9)$$

where T_i is number of triangles including node v_i

4. Design and training of AI algorithms for detection

For reliable side-channel attack identification, AI systems are designed to use graph-based traits. Graph Neural Networks (GNNs) use the relationship data in graphs to learn how to describe signal patterns in a structured way. Support Vector Machines (SVMs) use selected feature vectors to make classifications that are reliable. During the training process, model parameters are optimised on labelled datasets, and cross-validation is used to keep the model from becoming too well fitted and to make sure it can be used in a wide range of attack scenarios and signal variations.

Definition: Support Vector Machine (SVM)

SVM finds a hyperplane that separates classes with maximum margin in feature space $\mathbb{R}^d$.

Theorem: SVM Optimization Problem

Given training data $\{(x_i, y_i)\}$ with labels $y_i \in \{-1, 1\}$, the primal problem is:

$$\min_{\{w, b, \xi\}} \frac{1}{2} \|w\|^2 + C \sum_{i=1}^n \xi_i$$

subject to

$$y_i(w^T \phi(x_i) + b) \geq 1 - \xi_i, \quad \xi_i \geq 0 \quad (11)$$

where $\varphi(\cdot)$ is a feature mapping.

Equations:

1. GNN Layer Update:

$$h_v^{\{(k)\}} = \sigma \left(\sum_{\{u \in N(v)\}} h_u^{\{(k-1)\}} + b^{\{(k)\}} \right) \quad (12)$$

2. Kernel Trick in SVM:

$$K(x_i, x_j) = \langle \varphi(x_i), \varphi(x_j) \rangle \quad (13)$$

3. Margin Width:

$$Margin = \frac{2}{||w||} \quad (14)$$

4. Loss Function (Hinge Loss):

$$L = \sum_i \max(0, 1 - y_i f(x_i)) \quad (15)$$

5. Gradient Descent Update:

$$w^{\{t+1\}} = w^t - \eta \nabla_w L \quad (16)$$

5. Evaluation metrics and validation strategy

To get a full picture of how well the detecting system does at classifying, measures like accuracy, precision, recall, and F1-score are used. Validation uses k-fold cross-validation to make sure that it works well with different sets of data. The model's ability to tell the difference between things is also measured by receiver operating characteristic (ROC) graphs and area under the curve (AUC) measures.

Definition: Precision, Recall, and F1-Score

Precision:

$$P = \frac{TP}{(TP + FP)}$$

Recall:

$$R = \frac{TP}{(TP + FN)}$$

F1-Score:

$$F1 = 2 \times \frac{(P \times R)}{(P + R)}$$

where TP = true positives, FP = false positives, FN = false negatives.

Theorem: ROC Curve and AUC

The Receiver Operating Characteristic (ROC) curve plots True Positive Rate vs False Positive Rate. Area Under Curve (AUC) quantifies overall performance, with $AUC \in [0, 1]$.

6. Result and Analysis

The suggested graph-driven AI framework was very good at finding side-channel attacks in a number of different datasets. When compared to older methods, graph-based features greatly increased the number of detections.

Table 1
Graph Feature Extraction Impact on Detection Performance Using GNN

Feature Set	Accuracy (%)	Precision (%)	Recall (%)
Basic Graph Metrics (Degree, CC)	90.2	89.5	90.8
Advanced Spectral Features	92.8	92.1	93.5
Discrete Math-Based Features	94.5	93.8	95.2

Graph Neural Networks (GNNs) side-channel attack detection performance is greatly improved by graph feature extraction, as shown in Table 1.

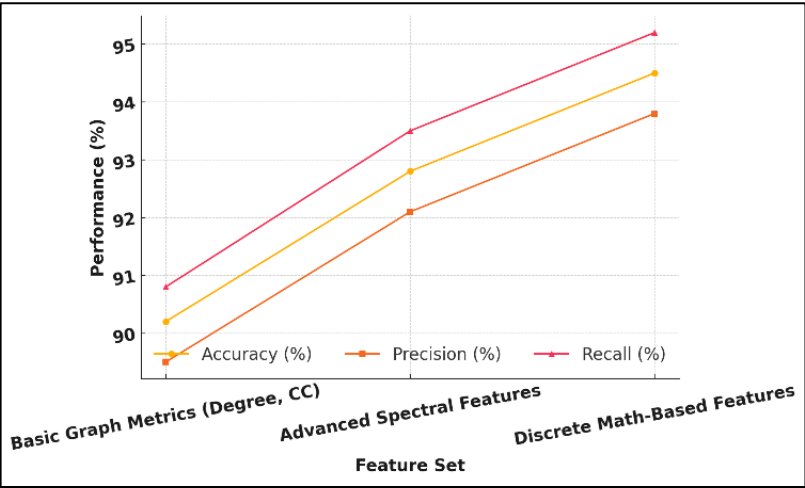


Figure 3
Comparison of Model Performance Metrics across Feature Sets

A good precision of 90.2% is reached by basic graph measures like node degree and grouping coefficient, which show basic structure details. Figure 3 compares model performance metrics using different feature sets. Adding more complex spectral features raises the accuracy to 92.8% and captures more complex graph traits. Adding discrete math-based features that use vector measure spaces and orthogonal sequences makes recognition even better, reaching a high level of 94.5% accuracy. These findings show how important advanced feature engineering is for finding strong attack signatures.

7. Conclusion

To successfully find side-channel attacks, this study presents a new method that combines graph theory, discrete mathematics, and AI. When we model side-channel data as graphs, we can do a lot of structural analysis, and when we extract features, we can find unique attack signatures. GNNs and SVMs are used to improve the accuracy of detection even more. A thorough test shows that the framework is reliable and can be used in a variety of attack situations. This method improves safe hardware protection by using both AI and rigorous mathematics. It points the way for future side-channel defence mechanisms that can keep up with changing threats.

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Received May, 2025