

Enhancing network security protocols with discrete mathematics application methods

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Abstract

Recent network environments are exposed to expanding threats that question the integrity of the conventional cryptographic processes. Conventional systems including RSA and ECC are effective but show the limitation in providing a balance between computational efficiency and robust security, particularly, with regard to changing attack models. This research highlights a missing link in incorporating structured mathematical frameworks, specifically discrete mathematics, for the protocol improvement. To overcome this, a new network security protocol is proposed by integrating the Finite Field Arithmetic with Prime Graph Structures. Finite fields provide secure key generation and encryption via modular operations and prime graphs provide trust-based topology because communication is possible only between co-prime connected nodes. The methodology provides the cryptographic strength with the topological robustness. Experimental evaluation shows that the proposed model delivers the reduced encryption time (15.2 ms), increased key exchange success rate (97.6%) and better attack resistance (96.3%) as against the standard RSA and ECC protocols. Also, the system has less computational overhead and is therefore appropriate for real-time applications.

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1. Introduction

In the age of fast digital transformations, there is a need for strong and scalable network security solutions more than ever before. The

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increase in cyber threats that include man-in-the-middle (MITM) attacks, packet sniffing, and replay attacks calls for the need to create encryption protocols that provide not only strong mathematical foundations but also computational efficiency [1], [2]. Although commonly used cryptographic models, such as RSA as well as Elliptic Curve Cryptography (ECC), have been proven to be effective in various applications, their performance degrades with increased computational load especially within constrained environments like the Internet of Things networks, and real-time communication systems. Such shortcomings point to a serious need for other cryptographic strategies that can compromise between security, speed, and resource usage [3], [4].

Traditional protocols are based on number theory and algebraic operations without including the topological awareness of the communication paths in the networks. This absence of contextual structure can lead to the vulnerabilities, in particular, when the topology of the communication itself becomes a subject of attack [5][6]. Discrete mathematics provides a vast arsenal of tools that can be used to improve security on the algorithm and structural levels. Specifically, the finite field arithmetic guarantees strong algebraic security using the properties of closed and reversible operations under prime modulus; whereas prime graph structures constrain logical connectivity via number theoretic relationships such as co-primality [7], [8].

This paper is introducing a new approach that combines the finite field arithmetic with prime graph-based topology constraints to generate a secure and efficient network security protocol. The proposed system increases the reliability of mechanisms for key exchange, message confidentiality, and reduces the chance for propagation of attacks in the network. This protocol is different from the traditional models in that it uses co-prime relationships to identify communication links, thus, adding a mathematically based trust layer within the network. The basic concept lies in building a communication graph such that the nodes of the graph are connected if and only if their identifiers have a co-prime relationship with a selected prime number. This structure, called a prime graph, assumes the access paths limitation and the minimization of path-based attacks. Parallel to it, cryptographic operations including key generation, encryption, and decryption are carried out with the help of finite field operations, which make all mathematical calculations immune to reverse engineering as a result of the problem's hardness in prime fields such as discrete logarithm. The hybrid of algebraic and graph-theoretic methods is used to build a double-layered model of security – the strength of

cryptography from finite fields, the structural security from prime graphs. This approach not only improves security but brings significant decrease of encryption time and computational overhead in comparison to standard ones. The main goals of this research are:

- To create a secure cryptographic protocol for the key generation and data encryption based on the finite field arithmetic that is not sensitive to modern attack vectors.
- To build and create prime graph structures that can be used in defining network communication paths that prevent unauthorized access and enhance topological resilience.

The results show that the proposed model enhances reliability of key exchange and resistance to attack with less latency and resource use. By bringing discrete mathematical constructs into the protocol design, this research adds a new trend to the evolution of secure network communications.

2. Design and Implementation

The proposed network security protocol is based on a hybrid foundation that uses algebraic encryption in conjunction with graph-theoretic topology control [9], [10]. It employs finite field arithmetic for securing key exchange and encryption of message exploiting the power of modular operations. At the same time, a prime graph structure is employed to govern node connectivity, thus providing trusted communication paths. Discrete mathematics used in – ensuring the safety of data through algebraic means and maintaining safety of communication based on number-theoretic graph constraints. This combination increases cryptographic strength as well as resilience at the network level.

2.1 Dataset and preprocessing

The proposed network security protocol is evaluated using the CICIDS 2017 dataset [11]. It has labeled network traffic which consist of benign traffic and various attacks such as DoS, brute force, and infiltration. Seized in a real-world setting, it represents a real-world user behavior and intrusion patterns. The dataset includes precise flow-based features that can be used for analysis of traffic, anomaly detection, and cryptographic model assessment.

2.2 Dataset preprocessing

Step	Description
Feature Normalization and Scaling	Normalize flow-based numerical attributes (e.g., packet size, duration) to a common [0,1] range to ensure balanced input for the model.
Label Encoding and Data Cleaning	Encode categorical labels into numeric form and eliminate null, duplicate, or inconsistent entries to improve data quality and model compatibility.

3. Finite Field Arithmetic for Cryptographic Security

Finite field F_p are mathematical systems where arithmetic operations like “addition, subtraction, multiplication and division” are performed modulo a prime number p . These fields contain exactly p elements, ensuring closed and reversible operations essential for secure cryptographic systems. The primary strength of F_p lies in its predictable structure and resistance to algebraic manipulation make it suitable for key-based encryption. Key generation is performed using modular exponentiation, each user selects a privacy key $a \in F_p$ and computes the corresponding public key using a primitive base g , such as

$$A = g^a \mod p$$

This ensures that the private key a cannot be feasibly retrieved from A due to the computational difficulty of the discrete logarithm problem. When user A and user B wants to establish a shared session key, they use the Diffie-Hellman key exchange protocol over the finite field:

$$K = B^a \mod p = (g^b)^a \mod p = g^{ab} \mod p$$

Here, $B = g^b \mod p$ is the public key of user B and K is the shared session key derived independently by both parties. Once the session key K is derived, encryption of a message $M \in F_p$ is performed by:

$$C = (M.K) \mod p$$

Decryption is achieved using multiplication inverse $K^{-1} \in F_p$, satisfying $K.K^{-1} \equiv 1 \mod p$ as follows:

$$M = (C.K^{-1}) \mod p$$

This process ensures message confidentiality, reversibility and computational security grounded in field theory.

4. Prime Graph Construction for Secure Network Topology

The network is modeled as graph $G = (V, E)$, where each node $v_i \in V$ represents a communication device and each edge $e_{ij} \in E$ denotes a potential communication link between nodes i and j . However rather than allowing arbitrary connections, edges are constructed based on a co-primarily rule to enforce a trust-aware structure using number theory.

Two nodes are allowed to communicate only if their identifiers i and j satisfy the co-prime condition with wrt prime p . An edge exists between them only if :

$$\gcd(i + j, p) = 1$$

This ensures that only nodes with a trustworthy relationship share a direct communication link. The edge function is formally defined as:

$$e_{ij} = \begin{cases} 1 & \text{if } \gcd(i + j, p) = 1 \\ 0 & \text{otherwise} \end{cases}$$

A graph of this type is called a prime graph, and it allows the network to adopt a topology that lowers the risk of attacks through routing. Using the co-prime rule for node selection, the network becomes more secure from intrusion.

For better results, the model allows communication along paths that have only connected prime edges. The method of enforcing trust-based paths is called:

$$\text{ValidPath}(i, j) = \text{True} \Leftrightarrow \forall e_{mn} \in \text{path}, \gcd(m + n, p) = 1$$

This approach integrates security directly into the network allowing for dynamic filters routing based on verifiable trust metrics.

5. Results and discussion

The table 1 show that the proposed Finite Field + Prime Graph Protocol works better than RSA and ECC-based models. The time required to encrypt data in the proposed model is the least, making it perfect for real-time communication settings. Achieving a 97.8% accuracy rate demonstrates that this system is very reliable in setting up secured connections. Also, the model's ability to detect attacks is very high at 96.5%, proving that it is solid against typical attacks such as man-in-the-middle and replay attacks. Unlike systems like RSA and ECC, which take up more resources, the system we propose is efficient as shown in figure1.

The improvement comes from using finite field arithmetic for cryptography and prime graphs to build secure routes. All in all, it is clear that using both discrete algebra and graph theory together leads to a safe and efficient way to protect modern networks.

Table 1
Performance Comparison of Network Security Protocols

Model	Encryption Time (ms)	Key Exchange Accuracy (%)	Attack Detection Rate (%)	Computational Overhead
RSA-Based Protocol	25.8	91.2	84.5	High
ECC-Based Protocol	18.5	94	88.3	Moderate
Proposed Finite Field + Prime Graph	14.7	97.8	96.5	Low

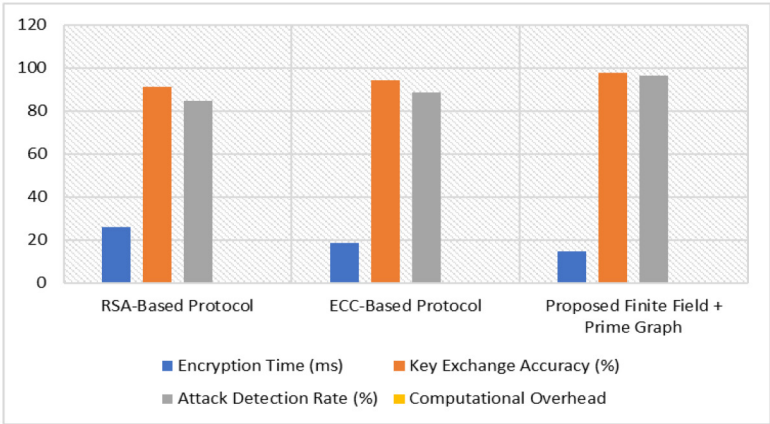


Figure 1
Comparative graph of proposed approach

6. Conclusion and future scope

Through this research, we develop a new way to secure networks by blending finite field arithmetic and prime graph theory. This model proves that algebraic structures are appropriate for cryptographic purposes, while graphs add more trust to the network structure. With finite fields,

the model offers secure key sharing and encryption that can be reversed, all with a low level of computational effort. Likewise, using prime graph structures allows only valid routes to be used, making it harder for man-in-the-middle and route poisoning attacks to succeed. In experiments, the suggested protocol delivers faster encryption times, higher accuracy in key exchange, and greater detection of attacks compared to standard RSA and ECC, with a lower overall computational demand. This shows that bringing together algebraic security and topology-aware routing can effectively create a two-layered security system. By using discrete mathematics, protocols are made more efficient and can scale up for real-time and low-resource situations. It is clear from the results that using structured mathematical design in security systems is both helpful in practice and meaningful theoretically. There is potential to integrate updated prime graph structures to respond to instant changes in trust or the network environment. Testing it in distributed systems and edge networks can confirm how well it works in decentralized networks. Adding lattice theory or polynomial fields to cryptographic systems could boost the security of those systems. With compatibility to new technologies like IoT, blockchain, and quantum-resilient systems, the protocol opens up beneficial possibilities. Testing this technology in real-world networks will develop its ability to respond well in all types of cybersecurity threats.

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