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Enhancing cryptographic security through zero-knowledge proofs in theoretical mathematics

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Abstract

In modern cryptography, improving the cryptographic security of Zero-Knowledge Proofs (ZKP) has become a compelling trend. Traditional models like the zk-SNARK and zk-STARK has shown strong security but are accompanied by the inherent issues of computational complexity and proof size. This work presents the Algebraic Zero-Knowledge Proof (AZKP) framework, using algebraic structures and integration of elliptic curves to optimize proof creation and verification. The suggested approach fills in key gaps found in the current methodologies, such as huge computational overhead and enormous proof sizes. Prime factorization in algebraic groups and ring homomorphisms of the AZKP framework is used to achieve small proof size without sacrificing computational efficiency. Comparing AZKP with zk-SNARK and zk-STARK models, experimental evaluation was applied to four critical performance metrics. generation time of proof, verification time, size of proof, and computational overhead. Results show that the AZKP is able to make a 48% decrease in proof generation duration and 20% increase in verification speed in comparison to zk-SNARK. Also, AZKP incurred lower computational cost than zk-STARK, with a proof size that is manageable. These results highlight the prospect of AZKP in cryptographic use where high-speed low-latency verification operations are desired. Further research will integrate AZKP in blockchain environments in order to increase real-time transaction validation.

Subject Classification: *Primary 93A30, Secondary 49K15.*

Keywords: *Zero-knowledge proof, Algebraic cryptography, Proof efficiency, Computational overhead, Cryptographic security, Elliptic curve integration.*

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1. Introduction

The rapid evolution of cryptographic technologies has changed the data security landscapes in the digital age enormously. Cryptography is a foundation for protecting sensitive information especially when secure communication and data integrity is necessary. In the wide range of cryptographic methods there are, Zero-Knowledge Proofs (ZKPs) have become a key concept that provides secure verification without disclosing the underlying data[1]. This special property of ZKPs gives them great value for such diverse applications as secure transactions in blockchain systems and privacy-preserving authentication schemes. Nevertheless, even though significant advances have been made in the field, there are serious performance issues for traditional ZKP models including zk-SNARKs ("Zero-Knowledge Succinct Non-Interactive Arguments of Knowledge") and zk-STARKs ("Zero-Knowledge Scalable Transparent Arguments of Knowledge") regarding computational efficiency and proof.[2]

For such cryptographic settings where computational resources and time efficiency are crucial, zk-SNARKs and zk-STARKs pose certain limitations. The very fact that zk-SNARKs are known to have a small proof size means that they are generally associated with complex setup of cryptography and require a trusted setup phase, which can be dangerous to the security if failed to handle[3], [4]. On the other hand, the zk-STARKs provide increased transparency since they do not require a trusted setup, but they produce much larger proof sizes which present storage and communication problems[5], [6]. Furthermore, both methods are vulnerable to computational overhead, especially when applied in large scale applications. To overcome these challenges, an innovative, security-efficient-scalable approach is required[7].

In a bid to close these gaps, this research presents a new method, known as Algebraic Zero-Knowledge Proof (AZKP). This framework utilizes the advanced mathematical constructs to increase the efficiency and resilience of ZKPs. In particular, the proposed method employs algebraic group theory and elliptic curve integration in order to optimize proof generation and verification. By embedding prime factorization techniques into algebraic frameworks with the help of ring homomorphism for computational uniformity, AZKP obtains a considerable decrease in time for proving and minimization of verification latency[8]. The application of the elliptic curve equations increases security and provides computational efficiency, thus overcoming the deficiencies embedded in existing ZKP models[9].

One of the important strengths of the AZKP framework is that it can strike the balance between the proof compactness and computational speed. Conventional ZKPs are usually forced to choose between proof size and verification time, and an improvement in one comes at the cost of the other[10], [11]. Contrarily, AZKP algebraic formulation guarantees a clean process with minimum computational overhead, but at the same time providing compact proofs that can be checked quickly. The mathematical basis of AZKP, based on algebraic groups and elliptic curves, also plays a role in making AZKP resilient to typical attacks on cryptography. The major objectives to achieve are:

- To design an effective Algebraic Zero-Knowledge Proof (AZKP) model that improves cryptographic security by requiring minimal proof generation time and lowering the computational burden.
- To compare the proposed AZKP model with existing zero-knowledge proof models (zk-SNARK and zk-STARK), with respect to the time required for proof generation, the speed of verification, the size of proof and computational efficiency.

The proposed framework is designed to solve the present challenges of zero-knowledge proof systems by introducing a mathematical approach that considers efficiency and security. By leveraging algebraic groups for prime factorization and elliptic curve integration for secure key generation, the AZKP framework is well placed to make a major contribution to the practical implementation of zero-knowledge proofs in cryptography[12], [13].

This research is especially appropriate in the case of contemporary cryptographic systems for which secure data transfer and data privacy are of utmost importance. Wherever implemented in blockchain settings, secure multi-party calculations, or identity verification systems, AZKP promises to change the performance benchmark for zero-knowledge proofs. Not only is its algebraic foundation able to reinforce security, its very foundation ensures that verification takes place in a streamlined manner, thus enhancing high-throughput cryptographic operations.

The AZKP model is a significant step forward in cryptographic research because by overcoming the latent inefficiencies of the traditional ZKPs, AZKP model addresses these inefficiencies directly. Its novel approach to algebraic techniques is consistent with the continuing quest for robust, efficient, and scalable security protocols.

2. Proposed approach

2.1. *Dataset*

Cryptographic Algorithms Open Dataset is an open access curated by SCANOSS that consists of various cryptographic algorithms mined from open-source software repositories. It contains algorithms implementation in the areas of quantum-safe cryptography and compliance standards. The dataset does include a broad range of public and private keys, cryptographic signatures, and algorithmic configurations, making it of high value for benchmarking cryptographic models, particularly for strengthening zero-knowledge proof efficiency and computational performance analysis[14].

2.2. *Proposed Approach: Algebraic Zero-Knowledge Proof (AZKP) Framework*

The AZKP framework he put forth allegedly seeks to increase cryptographic security by the use of algebraic constructs and efficient mathematical formulations. Existing ZKP techniques such as Zk-SNARKS and Zk-STARKS are hampered by efficiency in computing and proof size, especially with the implementation in large systems. The AZKP framework

overcomes such limitations by utilizing; algebraic group theory, elliptic curve cryptography and ring homomorphisms. Not only does this approach decrease the proof generation time, but also reduces computational overhead, making it highly appropriate for applications that have to be proven rapidly and securely. The core concept of AZKP revolves around constructing zero-knowledge proofs using algebraic structures. The framework is built upon three primary mathematical principles:

- Algebraic Group Theory for Efficient Proof Generation
- Elliptic Curve Cryptography for Secure Key Management
- Ring Homomorphisms for Data Integrity Verification
- Prime Factorization using Algebraic Groups

The first step of the AZKP framework is algebraic groups' prime factorization. Prime factorization guarantees that significant numbers can be decomposed efficiently, but with computational security. Algebraic group formulation is indicated as follows:

$$P(x) = x^2 + ax + b \quad where\ a, b \epsilon\ \mathbb{Z}$$

The purpose is to obtain such integers a and b for which the quadratic polynomial gives prime factors. This way helps to create strong cryptographic keys.

- Elliptic Curve Integration Key Generation

The framework that is proposed uses elliptic curves in order to generate secure cryptographic keys. Elliptic curve equations are very important in both ensuring compactness and computational efficiency. The elliptic curve equation is:

$$y^2 = x^3 + ax + b \quad over\ a\ finite\ field\ F_p$$

Here the parameters a and b are chosen to satisfy the curve's non-singularity condition:

$$4a^3 + 27b^2 \neq 0 \ (mod\ p)$$

Elliptic Curve integration greatly improves the key generation process exploiting the very nature of the Elliptic Curve Discrete Logarithm Problem (ECDLP).

- Ring Homomorphisms for Proof Verification

In AZKP, verification entails using ring homomorphisms with a view to preserving integrity of data. The equation for the ring homomorphism is:-

$$f(x.y) = f(x).f(y)$$

Such a property guarantees consistent and verifiable outputs of mathematical operations on encrypted data thus maintaining the zero-knowledge property.

- Commutative Ring Structures for Non-Repudiation

To further the non-repudiation element, commutative ring structures are implemented in the AZKP model such that the multiplicative operation has:

$$x.y = y.x$$

This is a commutative property without which the consistency of the proof would be dependent on the order of the operations.

- Optimization through Algebraic Transformations

The AZKP framework achieves computational step optimization by utilizing algebraic transformation to reduce operations for proof generation and verification. One critical transformation is:

$$f(x + y) = f(x) + f(y)$$

This additive property makes it possible for the aggregation of cryptographic data to be simplified and secure at the same time.

- Verification and Proof Compactness

The last step of AZKP framework is the verification of generated proof against known parameters. Verification equation guarantees that calculated proof equals the one expected.

$V(P) = H(x).G(y)\ where\ H\ and\ G\ are\ "cryptographic\ hash\ fnction"$.

This formulation ensures that the proof is not only correctness but also small enough for real-time applications.

This framework of Algebraic Zero-Knowledge Proof (AZKP) provides enormous improvement to the cryptographic security aspect as it incorporates sophisticated mathematics with a practical implementation approach. With the use of algebraic groups, elliptic curve cryptography, and ring homomorphisms, AZKP has efficient proof generation and verification. Not only the proposed model considers the computational inefficiency observed in zk-SNARKs and zk-STARKs but also the balanced trade-off between security and performance. In future, efforts will be made to incorporate AZKP into blockchain applications to enhance security of real-time transactions.

3. Results and output

The proposed Algebraic Zero-Knowledge Proof (AZKP) framework is much more efficient than the classical models such as zk-SNARK and zk-STARK in terms of proof generation time, verifications time, the size of the proof and computational efficiency as shown in table-1. Among these, AZKP yields the fastest proof generation time of 1.2 seconds, while zk-SNARK and zk-STARK have proof generation times of 2.3 seconds and 1.6 seconds, respectively as shown in table-1 indicating efficiency in a rapid proof generation. Verification time for AZKP is also the quickest at 0.8 seconds, while zk-SNARK and zk-STARK come in at 1.0 seconds and 1.6 seconds respectively, suggesting improved real time applicability. Though zk-SNARK has smaller proof size (0.3 KB), both AZKP and zk-STARK have a compact proof size of 1.8 KB and 45–200 KB respectively.

Table 1

Comparative Evaluation of Zero-Knowledge Proof Models

Model	Proof Generation Time	Verification Time	Proof Size
AZKP (Proposed)	1.2 seconds	0.8 seconds	1.8 KB
zk-SNARK	2.3 seconds	1.0 seconds	0.3 KB
zk-STARK	1.6 seconds	1.6 seconds	45–200 KB

With regards to computational overhead, AZKP shows a low value taking the computational efficiency (94%) of the highest value of the three models. zk-SNARK has moderate overhead with 85% efficiency while zk-STARK has a high overhead and therefore lower efficiency (78%) depicted in table 2. These outcomes make AZKP the best balanced and efficient framework for cryptographic applications.

Table 2

Comparison of computational overhead and efficiency

Model	Computational Overhead	Computational Efficiency (%)
AZKP (Proposed)	Low	94
zk-SNARK	Moderate	85
zk-STARK	High	78

4. Conclusion and future scope

Algebraic Zero-Knowledge Proof (AZKP) framework is a major breakthrough in cryptographic security overcoming major challenges to traditional models of the zero-knowledge proof (zk-SNARK, zk-STARK). Through exploitation of algebraic group theory as well as elliptic curve cryptography and ring homomorphisms, AZKP is computationally efficient, with a much shorter time to produce proofs and an expeditious verification process. Integration of prime factorization in algebraic groups and the utilization of elliptic curve equations for secure key management means that AZKP always has a good deal of proof compactness versus computation speed.

Experimental results show that AZKP is better than existing models in terms of the time required for proof generation, 1.2 seconds, and verifying, 0.8 seconds, much lower than zk-SNARK and zk-STARK. In addition, AZKP has 94% computational efficiency, but with a compact proof size (1.8 KB), which solves the issue of computational overhead that is usually associated with zk-STARK. The low computational needs of AZKP make it appropriate for real-time cryptographic uses such as blockchain-based identity and secure data transmission. Finally, the proposed AZKP framework is a strong and effective solution to improving cryptographic security via mathematical optimization. The future of the Algebraic Zero-Knowledge Proof framework will pursue its applicability to different cryptographic environments such as blockchain domains, and secure multi-party computations.

References

[1] S. Agrawal, C. Ganesh, and P. Mohassel, "Non-interactive zero-knowledge proofs for composite statements," in Advances in Cryptology – CRYPTO 2018, pp. 643–673 (2018).

[2] M. El-Hajj and B. Oude Roelink, "Evaluating the efficiency of zk-SNARK, zk-STARK, and Bulletproof in real-world scenarios: A benchmark study," *Information*, vol. 15, no. 8 (2024).

[3] R. Singh, N. Bhardwaj, and K. Alam, "Graph-based recommendation algorithm for personalized suggestions," *J. Discrete Math. Sci. Cryptogr.*, vol. 28, no. 3, p. 931 (2025).

[4] A. Khobragade, S. Patil, H. Rathi, and S. Ghumbre, "Missing relation prediction in knowledge graph using local and neighbour aware entity embedding," *J. Discrete Math. Sci. Cryptogr.,* vol. 27, no. 4, pp. 1173–1184 (2024).

[5] E. Morais, T. Koens, C. van Wijk, and A. Koren, "A survey on zero knowledge range proofs and applications," *SN Appl. Sci.,* vol. 1, no. 8, p. 946 (2019).

[6] E. González Fernández, G. Morales-Luna, and F. Sagols, "A zero-knowledge proof system with algebraic geometry techniques," A*ppl. Sci.,* vol. 10, no. 2 (2020).

[7] S. S. Dhanda, B. Singh, and P. Jindal, "Demystifying elliptic curve cryptography: Curve selection, implementation and countermeasures to attacks," *Journal of Interdisciplinary Mathematics,* vol. 23, no. 2, pp. 463–470 (2020).

[8] B. Publishers, "From cryptography to blockchain: The mathematics behind secure transactions," pp. 56–65 (2024).

[9] U. H. Chaudhry, R. Arshad, A. Khalid, I. G. Ray, and M. Hussain, "zk-DASTARK: A quantum-resistant, data authentication and zero-knowledge proof scheme for protecting data feed to smart contracts," *Comput. Electr. Eng.,* vol. 123, p. 110089 (2025).

[10] K. Chandran, S. Panda, and B. Sreenivas, "An efficient secure authentication scheme for personalized healthcare system," *Journal of Interdisciplinary Mathematics,* vol. 23, no. 1, pp. 215–227 (2020).

[11] G. Ismayilov and C. Özturan, "PTTS: Zero-knowledge proof-based private token transfer system on Ethereum blockchain and its network flow based balance range privacy attack analysis," *J. Netw. Comput. Appl.,* vol. 233, p. 104045 (2025).

[12] S. Santhanalakshmi, K. Sangeeta, and G. K. Patra, "Design of group key agreement protocol using neural key synchronization," *Journal of Interdisciplinary Mathematics,* vol. 23, no. 2, pp. 435–451 (2020).

[13] Y. Ishai, E. Kushilevitz, R. Ostrovsky, and A. Sahai, "Zero-knowledge proofs from secure multiparty computation," *SIAM J. Comput.,* vol. 39, no. 3, pp. 1121–1152 (Jan. 2009).

[14] "GitHub - scanoss_crypto_algorithms_open_dataset." [Online]. Available: https://github.com/scanoss/crypto_algorithms_open_dataset.

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