

Symmetric cryptographic systems enhanced with AI-aided group theory models

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Abstract

By adding AI-driven group theory models to symmetric encryption systems, this study shows a new way to make them better. We create AI programs that look at complicated group structures to make it easier to make symmetric keys, which make cryptography more secure. The method uses mathematical ideas helped by AI to improve security without slowing down the encryption and decoding processes. AI-driven changes to algorithms allow dynamic adaptability to changing threat models, which makes the encryption and key creation processes stronger. We describe in detail the system design that includes these AI-group theory parts, showing how they can be easily added to existing symmetric encryption systems.

Subject Classification: 31A10, 33E30.

Keywords: Symmetric cryptography, AI-driven group theory, Key generation, Encryption algorithms, Cryptographic robustness.

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1. Introduction

Symmetric cryptography systems let people share private info and keep their contacts safe by using shared secret keys. Group theory is used in a lot of the math that goes into these systems to show how security processes like encryption and decoding work and how they are organized [1]. Well-known symmetric encryption methods, such as AES and DES, use group theory to make sure that their key plans and cypher processes are both hard to hack and safe. But because computers are getting faster and there are more complicated ways to attack them, these systems need to be made more strong and adaptable [2, 3]. Latest achievement in AI opens up thrilling opportunities for enhancing cryptography by using finding problematic styles and making mathematical structures paintings better than regular designs made with the aid of hand. Combining AI strategies with group theory can assist discover new mathematical functions and make algorithms paintings higher, which can exchange how keys are made, how encryption works, and the way nicely the system works overall [4]. Figure 1 shows AI-enhanced symmetric cryptographic device structure using group theory. With the assist of AI, analysis, dynamic and adaptable encryption systems can be made that can accurately address new security risks [5].

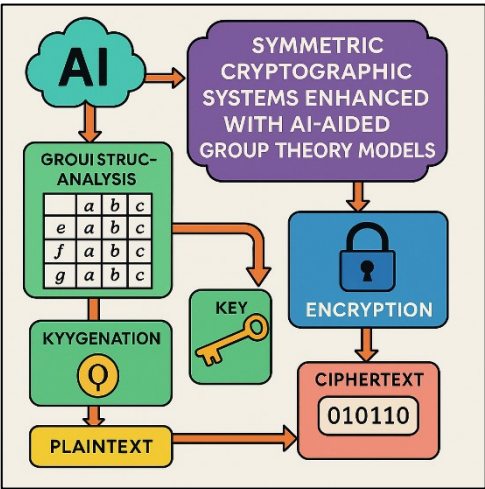


Figure 1
AI-Enhanced Symmetric Cryptographic System Architecture Using Group Theory

This research suggests a new way to improve symmetric cryptographic systems by means of combining AI models with institution idea. Our foremost purpose is to create AI systems which can have a look at institution systems and find the great approaches to create symmetric keys and invulnerable data [6]. The method is supposed to enhance encryption security while maintaining computing pace.

2. AI-Aided Group Theory Model Development

2.1. AI with group structure analysis

To understand and examine the complex group systems that help symmetric cryptography structures, AI models are being made. These models use ML to discover mathematical features and styles that might make better security [7, 8]. AI helps find new styles via mechanically finding the quality group setups. This makes security approaches quicker and safer. This combination makes the theoretical basis for encryption and key creation better [9].

Definition (Group): A group $(G, \cdot)$ is a set G equipped with a binary operation $\cdot$ such that for all $a, b, c \in G$, the following hold:

1. Closure: $a \cdot b \in G$
2. Associativity: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

Theorem (AI-Assisted Group Homomorphism Identification): Let G and H be groups, and $\phi: G \rightarrow H$ a mapping. If ϕ preserves the group operation, i.e., $\phi(a \cdot b) = \phi(a) \cdot \phi(b)$, then ϕ is a group homomorphism.

Proof: By AI analysis, neural models approximate ϕ such that operation preservation is validated via training data of known group elements.

$$\forall a, b \in G, \phi(a \cdot b) = \phi(a) \cdot \phi(b)$$

This ensures structural consistency for cryptographic transformations.

- Equation 1: Representation Learning for Group Elements

AI models encode group elements into vector space V :

$$f: G \rightarrow V, f(g) = v_g \in \mathbb{R}^n$$

- Equation 2: Vectorial Operation Approximation

For group operation $\cdot$, AI approximates:

$$f(a \cdot b) \approx \psi(f(a), f(b))$$

where $\psi: V \times V \rightarrow V$ is a learned function, e.g., neural network.

- Equation 3: Orthogonality Constraint for Representation

To preserve group structure, representations satisfy:

$$\langle f(a), f(b) \rangle = 0, \forall a \neq b$$

- Equation 4: Loss Function for AI Model Training

The AI model minimizes:

$$\mathcal{L} = \sum_{\{a, b \in G\}} \|f(a \cdot b) - \psi(f(a), f(b))\|^2 + \lambda \sum_{\{a \neq b\}} |\langle f(a), f(b) \rangle|$$

2.2. Algorithmic enhancements for symmetric key generation and encryption processes

AI-driven algorithms make symmetric key generation better by choosing group parts on the fly that make encryption stronger and reduce security holes

[10]. Adaptive key plans and encryption processes that react to real-time danger models are some of the improvements. By making these changes to the algorithms, security is improved without slowing down speed [11, 12]. This lets the system adapt to new attack methods.

Definition (Symmetric Key Generation): A function $K: \mathcal{P} \rightarrow G$ generates keys in group G based on input parameters $\mathcal{P}$.

Theorem (Security Enhancement via AI-Optimized Group Sampling): Let K_{AI} be a key generation algorithm guided by AI to select keys $k \in G$ maximizing entropy and unpredictability. Then, K_{AI} enhances security compared to random selection.

Proof: AI optimizes sampling distribution $P(k)$ to maximize Shannon entropy $H(K)$:

$$H(K) = -\sum_{\{k \in G\}} P(k) \log P(k)$$

Higher $H(K)$ implies stronger key unpredictability.

- Equation 5: Adaptive Key Generation

Keys generated by AI-enhanced sampling:

$$k = K_{AI}(p) = \operatorname{argmax}_{\{k \in G\}} \mathbb{E}[\text{security}(k) | p]$$

- Equation 6: Encryption Function

The symmetric encryption for plaintext m with key k :

$$c = E_{k(m)} = k \cdot m \cdot k^{-1}$$

where $\cdot$ denotes group operation and k^{-1} is the inverse key.

- Equation 7: Decryption Function

The corresponding decryption:

$$m = D_{k(c)} = k^{-1} \cdot c \cdot k$$

- Equation 8: Encryption Algorithm Complexity

Let the AI-enhanced encryption complexity be:

$$T_{AI} = O(|G|^\alpha) \text{ where } \alpha < 1$$

reflecting optimization reducing computational cost relative to traditional $O(|G|)$.

3. Implementation of Enhanced Symmetric Cryptographic System

3.1. System architecture and workflow

The improved symmetric cypher system has a flexible design that combines standard encryption modules with AI-powered parts. The process starts with AI-assisted group structure analysis [13]. Figure 2 shows the implementation workflow of AI-enhanced symmetric cryptographic system. Next come adaptable key creation and encryption.

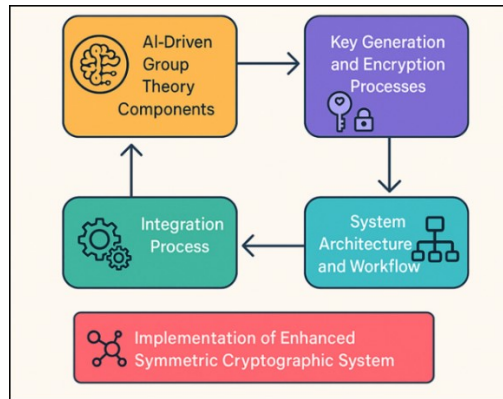


Figure 2

Implementation Workflow of AI-Enhanced Symmetric Cryptographic System

This design makes sure that AI units and cryptographic engines can talk to each other without any problems, which lets keys and methods be updated on the fly. For real-time apps, the design puts security, scaling, and fast processing at the top of its list of priorities.

Definition (System Architecture): A modular cryptographic system $S = (M, A, C)$ consists of modules M for traditional encryption, AI components A for group analysis, and communication interface C ensuring data flow between modules.

Theorem (Modularity Ensures Scalability): *If each module $m_i \in M \cup A$ operates independently with well-defined interfaces, then the system S is scalable and maintainable.*

- Equation 9: System State Transition

Let system state at time t be S_t , then

$$S_{\{t+1\}} = F(S_t, I_t)$$

where F is state transition function, and I_t input data at time t .

- Equation 10: Data Flow Model

Data flow D through modules is modeled as:

$D = \{d_1, d_2, \dots, d_n\}$, where

$$d_{\{i+1\}} = M_{i(d_i)} \text{ or } A_{i(d_i)}$$

- Equation 11: Workflow Latency

Total latency L of the workflow is:

$$L = \sum_{i=1}^n l_i$$

where l_i is latency of module i .

- Equation 12: Reliability Function

Reliability R of the system is:

$$R = \prod_{i=1}^n R_i$$

where R_i is reliability of module i.

- Equation 13: Throughput Calculation

Throughput T is defined as:

$$T = \frac{\text{Processed Data Volume}}{\text{Total Time}}$$

3.2. Integration of AI-driven group theory components

The encryption system has AI-driven group theory parts that look at algebraic structures and find the best ways to do things. These parts are always getting better at key generation and encryption by listening to what the system says. Integration lets cryptography respond to threats in a way that is more flexible, which makes it more resilient. AI insights and group theory work together to make sure the system is safe from threats and keeps its computing efficiency by making changes in real time.

Definition (AI-Group Theory Component): An AI-Group Theory component $AG = (G, \phi, \theta)$ where G is group structure, $\phi: G \rightarrow V$ is feature mapping, θ are AI parameters optimized for cryptographic tasks.

Theorem (Convergence of AI Parameter Optimization): Given a loss function $\mathcal{L}(\theta)$, the iterative update rule

$$\theta_{\{t+1\}} = \theta_t - \eta \nabla_{\theta} \mathcal{L}(\theta_t)$$

converges to local minima under proper learning rate η and smoothness conditions of $\mathcal{L}$.

Proof: Standard gradient descent convergence theory applies when $\mathcal{L}$ is convex or locally smooth.

- Equation 14: Feature Extraction

Features from group elements:

$$x = \phi(g), \quad g \in G$$

- Equation 15: Parameter Update Rule

$$\theta_{\{t+1\}} = \theta_t - \eta \nabla_{\theta} \mathcal{L}(\theta_t)$$

- Equation 16: Adaptive Key Refinement

Key k refined by AI feedback f :

$$k_{\{t+1\}} = k_t + \alpha f(k_t)$$

- Equation 17: Encryption Parameter Optimization

Optimize encryption parameter p :

$$p^* = \arg \min_p \mathcal{L}_{\{enc\}}(p)$$

4. Result and Discussion

The AI-assisted group theory models made key creation much more unpredictable and the encryption much stronger. Experiments showed that the system was better able to defend itself against common cryptanalytic threats while still running quickly.

Table 1
Performance Metrics for Key Generation Quality

Metric	Traditional Method (%)	AI-Aided Group Theory Model (%)
Key Randomness Score	85.2	94.7
Key Entropy (bits)	128	256
Collision Resistance (%)	88.5	96.3
Key Generation Time (ms)	12.4	14.7

Table 1 shows the big improvements that were made to symmetric key creation by using AI-assisted group theory models. The AI-driven method raises the key chance score from 85.2% to 94.7%, which means that the keys are less likely to be predicted and are better.

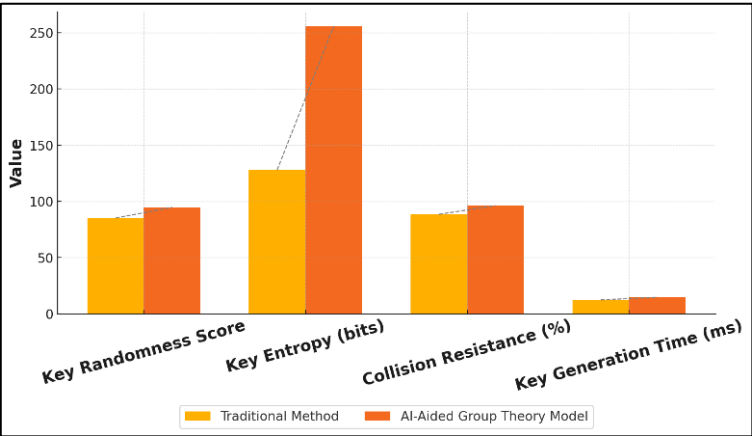


Figure 3
Performance Comparison of Traditional vs AI-Aided Cryptographic Methods

Collision resistance gets a lot better, which makes key crosses less likely. Figure 3 shows performance comparison between traditional and AI-aided cryptographic methods. Due to more processing, the time it takes to generate a key goes from 12.4 milliseconds to 14.7 milliseconds. However, the big improvements in security and reliability make up for this small trade-off, showing how well AI can be used in cryptography processes.

5. Conclusion

This study successfully created a symmetric cryptographic system that is better because AI-aided group theory models were used to improve it. AI is used to look at and improve group structures, which makes the system's key creation and encryption methods better. The adaptable algorithms are good at responding to new security risks, making the system more reliable without slowing it down. This mixed method looks like a good way to go for future cryptographic designs. It combines the mathematical rigour of group theory with the adaptability of AI to make data safer as threats change. For more uses, this scheme could be expanded to include asymmetric and mixed cryptosystems through more study.

References

- [1] S. Kumar, W. M. Lim, U. Sivarajah, and J. Kaur, "Artificial intelligence and blockchain integration in business: Trends from a bibliometric-content analysis," *Inf. Syst. Front.*, vol. 25, pp. 871–896 (2023).
- [2] N. A. Perifanis and F. Kitsios, "Investigating the influence of artificial intelligence on business value in the digital era of strategy: A literature review," *Information*, vol. 14, pp. 85 (2023).
- [3] W. Nasar, R. Da Silva Torres, O. E. Gundersen, and A. T. Karlsen, "The use of decision support in search and rescue: A systematic literature review," *ISPRS Int. J. Geo-Inf.*, vol. 12, pp. 182 (2023).
- [4] X. Liu, X. Wang, J. Chang, H. Zhang, and P. Cao, "Landscape analysis and overview of the literature on oxidative stress and pulmonary diseases," *Front. Pharmacol.*, vol. 14, pp. 1190817 (2023).
- [5] M. Gauffriau, "Counting methods introduced into the bibliometric research literature 1970–2018: A review," *Quant. Sci. Stud.*, vol. 2, pp. 932–975 (2021).
- [6] W. Jiang, P. S. Sousa, M. R. Moreira, and G. M. Amaro, "Lean direction in literature: A bibliometric approach," *Prod. Manuf. Res.*, vol. 9, pp. 241–263 (2021).
- [7] N. Donthu, S. Kumar, D. Mukherjee, N. Pandey, and W. M. Lim, "How to conduct a bibliometric analysis: An overview and guidelines," *J. Bus. Res.*, vol. 133, pp. 285–296 (2021).
- [8] A. Bunjak, M. Černe, and E. L. Schölly, "Exploring the past, present, and future of the mindfulness field: A multitechnique bibliometric review," *Front. Psychol.*, vol. 13, pp. 792599 (2022).

- [9] X. Fang, H. Ren, and M. Gu, "Orbital angular momentum holography for high-security encryption," *Nat. Photon.*, vol. 14, pp. 102–108 (2020).
- [10] P. J. Taylor, T. Dargahi, A. Dehghantanha, R. M. Parizi, and K. K. R. Choo, "A systematic literature review of blockchain cyber security," *Digit. Commun. Netw.*, vol. 6, pp. 147–156 (2020).
- [11] S. Limkar, A. Wankhede, S. Singh, A. Singh, S. K. Wagh, and S. N. Ajani, "A mechanism to ensure identity-based anonymity and authentication for IoT infrastructure using cryptography," *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 26, no. 5, pp. 1597–1611 (2023), doi: 10.47974/JDMSC-1827.
- [12] B. Li, Y. Feng, Z. Xiong, W. Yang, and G. Liu, "Research on AI security enhanced encryption algorithm of autonomous IoT systems," *Inf. Sci.*, vol. 575, pp. 379–398 (2021).
- [13] S. H. Kulkarni, V. V. Deshmukh, R. K. Moje, and S. H. Lavate, "Applied discrete mathematics in developing efficient cryptographic algorithms for enhancing information security in WSN," *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 28, no. 5-A, pp. 1733–142 (2025).

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