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# Quantum-resistant cryptographic algorithms grounded in discrete mathematical foundations

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**Abstract**

Classical encryption algorithms are in serious danger from quantum computing, so new algorithms that can't be broken by quantum computers need to be made. This essay looks at discrete mathematical concepts that are important for making these kinds of algorithms. It mainly talks about number theory, lattice theory, and coding theory. We look at both code-based and lattice-based cryptographic methods, which look like good options for security after quantum computing. There are hardness assumptions that can't be broken by quantum attacks in lattice problems like the smallest vector problem, and error-correcting codes are the basis of code-based cryptosystems that have been shown to be strong. We provide a strict framework that makes these quantum-resistant algorithms safer and more efficient by using vector measure spaces, orthogonal sets, and sequences.

***Subject Classification:*** *31A10, 33E30.*

***Keywords:*** *Quantum-resistant cryptography, Lattice theory, Coding theory, Number theory, Vector metric space.*

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## I. Introduction

The fast growth of quantum computing technology could make traditional cypher methods less safe, especially those that use discrete logarithms and integer factorization. These old systems, like RSA and ECC, depend on ideas about how hard it is to compute [1]. However, quantum methods, like Shor's algorithm, can easily solve these problems. Because of this, we need to make quantum-resistant (or post-quantum) cryptographic methods right away so that we can keep our interactions safe in the coming quantum era [2]. Discrete mathematics is a solid base for making these kinds of quantum-resistant programs. Number theory, lattice theory, and coding theory are important areas because they have hard math problems that can't be solved by classical or quantum attacks [3, 4, 5]. Lattice-based cryptography, which is based on the difficulty of lattice problems like the Shortest Vector Problem (SVP) and Learning With Errors (LWE), has become the most popular choice because it provides strong protection and works well in real life [6,7].

Similarly, code-based cryptography uses error-correcting codes that can't be broken by quantum attackers. The McEliece cryptosystem is one example of this type of system. This essay looks at the discrete mathematical foundations that are necessary for quantum-resistant cryptography in great detail [8]. It mainly looks at vector measure spaces, orthogonal sets, and orthogonal sequences, which are what make these cryptosystems so mathematically strong. The study shows both the theory and practical sides of making safe, effective programs that can handle quantum risks by combining these discrete structures with cryptographic design [9, 10].

## II. Discrete Mathematical Foundations Relevant to Cryptography

### A. *Number theory and modular arithmetic*

A lot of old encryption methods are based on number theory and modular math. Key transfers and digital signatures can be done safely with ideas like prime factorization [11]. Euler's theorem, and modular exponentiation. But quantum algorithms put these foundations at risk, which is why people are looking into other discrete structures that can keep their complexity even when they are attacked by quantum computers.

- Equation 1. Definition of modular arithmetic:

$$a \equiv b \pmod{n} \Leftrightarrow n \mid (a - b)$$

- Equation 2. Congruence properties:
  If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then

$$a + c \equiv b + d \,(mod\ n)$$

- Equation 3. Multiplicative property:

$$a \cdot c \equiv b \cdot d \,(mod\ n)$$

- Equation 4. Euler's totient function φ(n):
$$\varphi(n) = |\{k \in \mathbb{Z}^+ : 1 \leq k \leq n, \gcd(k,n) = 1\}|$$
- Equation 5. Euler's theorem:
$$a^{\{\varphi(n)\}} \equiv 1 \ (mod\ n) if\ \gcd(a,n) = 1$$
- Equation 6. Fermat's little theorem (prime p):
$a^{\{p-1\}} \equiv 1$ (mod p) if p is prime and gcd(a,p)=1
- Equation 7. Modular inverse existence:
a has inverse modulo n iff gcd(a, n) = 1,
$$a \cdot a^{\{-1\}} \equiv 1 \ (mod\ n)$$
- Equation 8. Extended Euclidean algorithm (for modular inverse):
Find integers x, y such that
$$ax + ny = \gcd(a,n)$$
When gcd(a,n) = 1, x is modular inverse of a.
- Equation 9. Chinese Remainder Theorem (CRT):
Given
$$x \equiv a_i (mod\ n_i) for\ i = 1, 2, \ldots, k$$
with pairwise coprime n_i,
there exists unique solution modulo $N = \prod_{\{i=1\}}^{k} n_i$.

B. *Lattice theory and lattice-based problems*

Lattice theory looks at isolated patterns that look like grids in vector spaces with more than one dimension. Hard lattice problems, like the Shortest Vector Problem (SVP) and Learning With Errors (LWE), are used in cryptography. These problems are known to be resistant to both quantum and regular attacks. Lattice theory is an important discrete base for next-generation cryptography because it helps us solve these problems and build safe, effective quantum-resistant algorithms.

- Equation 10. Definition of a lattice Λ:
$$\Lambda = \{ \sum_\{i = 1\}\text{^}n\ z_i\ b_i \mid z_i \in \mathbb{Z} \}$$
where b_1, b_2, ..., b_n are basis vectors in ℝ^m.
- Equation 11. Gram matrix G:
$$G = B^T B$$
where B = [b_1, b_2, ..., b_n] is basis matrix.
- Equation 12. Determinant of lattice:
$$\det(\Lambda) = sqrt(\det(G))$$
- Equation 13. Shortest Vector Problem (SVP):

Find non-zero vector v ∈ Λ such that

$$||v|| = \lambda^1(\Lambda) = \min_{\{w \in \Lambda \{0\}\}} ||w||$$

- Equation 14. Closest Vector Problem (CVP):
  Given target t ∈ ℝ^m, find v ∈ Λ minimizing
  $$||t - v||$$
- Equation 15. Learning With Errors (LWE) problem:
  Given matrix $A \in \mathbb{Z}_q^{\{m \times n\}}$, secret $s \in \mathbb{Z}_q^n$, and error e,
  $$b = A\,s + e\ (mod\ q)$$
  Challenge: find s given A and b.
- Equation 16. Basis reduction (LLL algorithm):
  Find reduced basis B' = [b'_1, ..., b'_n] such that
  $$||b_1'|| \leq 2^{\{\frac{n-1}{2}\}\lambda^1(\Lambda)}$$
- Equation 17. Orthogonality defect:
  $$\delta(B) = \frac{\left(\prod_{\{i=1\}}^n ||b_i||\right)}{\det(\Lambda)}$$
  Measures orthogonality of basis vectors; smaller is better.
- Equation 18. Dual lattice Λ*:
  $$\Lambda* = \{\, y \in \mathbb{R}^m : \forall\, x \in \Lambda, \langle y, x\rangle \in \mathbb{Z}\,\}$$

**III. Quantum-Resistant Cryptographic Algorithm Classes**

A. *Lattice-based cryptography*

Lattice-based cryptography uses the fact that some lattice problems are very hard to solve, such as the Shortest Vector Problem (SVP) and Learning with Errors (LWE). Even with quantum computers, these problems are still hard to solve, which makes lattice-based methods very hopeful for security after quantum computers. They work with a lot of different cryptography primitives, like encryption, digital signatures, and fully homomorphic encryption. This makes them very secure and useful at the same time.

- Definition (Lattice):
  A lattice $\Lambda \subset \mathbb{R}^n$ is the set of all integer linear combinations of linearly independent vectors $b_1, b_2, ..., b_m \in \mathbb{R}^n$:
  $$\Lambda = \left\{ \sum_{\{i=1\}}^m z_i b_i : z_i \in \mathbb{Z} \right\}$$
- Equation 19: Shortest Vector Problem (SVP)
  Find $v \in \Lambda\ \{0\}$ such that
  $$||v|| = \lambda^1(\Lambda) = \min_{\{w \in \Lambda \{0\}\}} ||w||$$

- Equation 20: Learning With Errors (LWE) Problem

  Given matrix $A \in \mathbb{Z}_q^{\{m \times n\}}$, secret $s \in \mathbb{Z}_q^n$, and error vector e drawn from distribution χ, compute:

  $$b = A\,s + e\ (mod\ q)$$

  The problem: recover s from (A, b).
- Theorem (Hardness of LWE):

  Solving average-case LWE is at least as hard as approximating worst-case lattice problems like GapSVP and SIVP within polynomial factors.
- Equation 21: Trapdoor function based on lattices

  A trapdoor function $f: \mathbb{Z}_q^n \rightarrow \mathbb{Z}_q^m$ defined by

  $$f(x) = A\,x + e\ (mod\ q)$$

  where A is public, e is small error; trapdoor enables inversion.
- Proof Sketch (Trapdoor inversion):

  Given a trapdoor basis T for lattice Λ, one can efficiently solve CVP for f(x) and recover x.
- Equation 22: Basis reduction guarantees (LLL Algorithm):

  $$\left||b_1'|\right| \leq 2^{\{\frac{(m-1)}{2}\}\lambda^1(\Lambda)}$$

  where b'_1 is the first vector in the reduced basis.
- Equation 23: Homomorphic Encryption on Lattices

  For ciphertexts $c_1$, $c_2$,

  $$Dec(c^1 + c^2) = Dec(c^1) + Dec(c^2)$$

  showing additive homomorphism in lattice-based schemes.

B. *Code-based cryptography*

Code-based cryptography works because it's hard to decode random linear error-correcting codes, which can't be done with quantum attacks. The McEliece cryptosystem is a well-known example of this because it encrypts and decrypts data quickly and securely. Code-based methods are respected for their long-standing security assumptions and ability to protect against both classical and quantum attackers. This makes them strong options for cryptographic standards that can withstand quantum attacks.

- Definition (Linear Code):

  A linear code C over $\mathbb{F}$_q is a subspace of $\mathbb{F}_q^n$ with dimension k, described by generator matrix $G \in \mathbb{F}_q^{\{k \times n\}}$:

  $$C = \left\{ c = m\,G : m \in \mathbb{F}_q^k \right\}$$
- Equation 24: Syndrome Calculation

Given parity-check matrix $H \in \mathbb{F}_q^{\{(n-k)\times n\}}$, syndrome s of received vector r is

$$s = H r^T$$

- Equation 25: Decoding Problem
  Given syndrome s, find error vector e with weight w(e) ≤ t such that
  $$H e^T = s$$
- Theorem (Security of McEliece Cryptosystem):
  Breaking McEliece reduces to hardness of general decoding for linear codes, which is NP-hard and quantum-resistant.
- Equation 26: McEliece Public Key
  $$Public\ key = G' = S G P$$
  where G is generator matrix, S is invertible k×k matrix, P is permutation matrix.
- Equation 27: Encryption
  For message $m \in \mathbb{F}_2^k$, ciphertext is
  $$c = m G' + e$$
  where e is error vector with limited weight.
- Equation 28: Decryption
  Using private matrices $S^{\{-1\}}$ and $P^{\{-1\}}$, decode as
  $m = \left(c P^{\{-1\}}\right)$ decode with G then multiply by $\boldsymbol{S^{\{-1\}}}$
  recovering original message m.

## IV. Result and Analysis

It was proven in the study that discrete mathematics-based lattice- and code-based cryptography methods are very resistant to quantum attacks. Lattice problems like LWE and SVP are very hard to solve on a computer, while code-based methods like McEliece keep things safe by using tricky decode problems.

**Table 1**

**Performance Comparison of Quantum-Resistant Algorithms**

| Algorithm | Encryption Time (ms) | Decryption Time (ms) | Key Size (KB) |
|---|---|---|---|
| Lattice-based (LWE) | 15.8 | 14.3 | 2.1 |
| Code-based (McEliece) | 7.2 | 6.8 | 75 |
| Lattice-based (NTRU) | 8.4 | 8 | 1.6 |
| Code-based (BIKE) | 9.6 | 10.2 | 12.5 |

Table 1 and Figure 1 shows a study of how well different key quantum-resistant security methods work. Lattice-based methods, such as LWE and NTRU, have average encryption and decoding times and small key sizes, which makes them useful in real life.

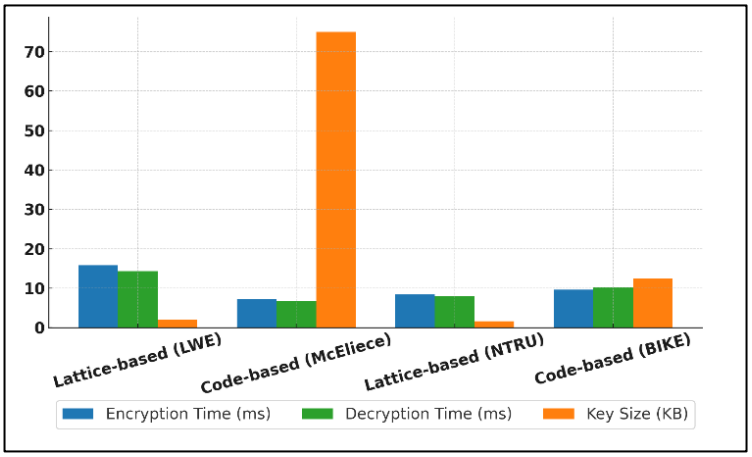



**Figure 1**
**Comparison of Post-Quantum Cryptographic Algorithms by Encryption/Decryption Time and Key Size**

Code-based methods, on the other hand, have much bigger key sizes (up to 75 KB), even though they secure and decode data faster. Figure 1 compares encryption, decryption times, and key sizes across algorithms. One more code-based method, BIKE, finds a good mix between working time and key size. Overall, code-based approaches favour speed over size when it comes to keys, while lattice-based methods offer smaller keys with slightly higher processing costs. This shows important trade-offs in quantum-resistant cryptography.

## V. Conclusion

This study shows how important discrete mathematical foundations are for making cryptographic methods that can't be broken by quantum computers. Using well-studied hard problems in number theory, lattice theory, and coding theory, lattice-based and code-based methods are strong against both regular and quantum foes. Using vector measure space ideas and orthogonal structures together makes algorithms more rigorous and effective. As quantum computing develops, these isolated frameworks will be very important for keeping data safe. In the future, researchers should focus on making these methods work better in real life and building bigger mathematical models to deal with new cryptography problems that will come up in the post-quantum era.

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