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Exploring n-Queens problem solutions through advanced combinatorial and discrete mathematical techniques

Natasha Martin [#]
Symbiosis Centre for Advanced Legal Studies and Research
Symbiosis Law School Pune (SLS-P)
Symbiosis International (Deemed University)
Pune 411014
Maharashtra
India

Vimal Bibhu [†]
Department of Computer Science & Engineering
Noida International University
Greater Noida 203201
Uttar Pradesh
India

Vikas Kolekar [§]
Department of Computer Engineering
Vishwakarma Institute of Technology
Pune 411037
Maharashtra
India

E. R. Aruna [‡]
Department of Computer Science and Engineering
Vardhaman College of Engineering
Kacharam 501218
Hyderabad
India

[#] *E-mail:* natasha.martin@symlaw.ac.in
[†] *E-mail:* vimal.bhibu@niu.edu.in
[§] *E-mail:* vikaskolekar2008@gmail.com
[‡] *E-mail:* er.aruna@vardhaman.org

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P. Veeresh Kumar [@]
Department of Information Technology
KKR and KSR institute of Technology and Sciences
Guntur 522017
Andhra Pradesh
India

Vaibhav Mittal*
Mittal School of Business
Lovely Professional University
Phagwara 144411
Punjab
India

Abstract

The n-Queens problem involves positioning n chess queens on an n×n board without any of them threatening one other. Typical combinatorial issue in computer math. This topic has been studied in graph theory, optimization, and calculation difficulty. Modern combinatorial and discrete arithmetic methods like department-and-bound algorithms, constraint delight approaches, and optimization equipment are better at solving the n-Queens problem, however going backwards has often worked. Sparkling advancements in mathematical frameworks, diagram colouring, and combinatorial designs have added ways to handle huge n. In this study, mathematical methods are tested with computers and compared to the n-Queens problem. The research also covers the problem in higher-dimensional settings and investigates its relevance to aid allocation and timetable construction. The results illuminate combinatorial problem-solving methods in computer mathematics.

Subject Classification: *05C20.*

Keywords: *n-Queens problem, Combinatorial optimization, Backtracking, Graph coloring, Constraint satisfaction.*

1. Introduction

Mathematicians, computer scientists, and students had been curious about the "n-Queens problem" for many years. It consists of putting n queens on an n×n grid in order that no 2 queens endanger one another. This means no two queens can also occupy the identical row, column, or diagonal [1]. Although it appears easy, it's far a typical illustration of a constraint fulfillment issue. The goal is to perceive an answer pleasing a

[@] *E-mail:* **pveer17@gmail.com**
* *E-mail:* **vaibhav.mittal@lpu.co.in (Corresponding Author)**

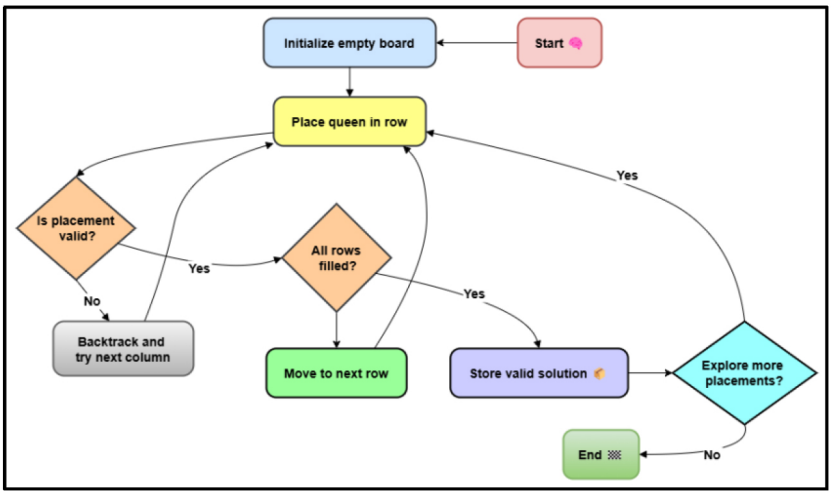


Figure 1

Illustrating n-Queens problem solutions

hard and fast of precise limits. Over the years, numerous distinctive techniques have addressed the n-Queens issue [2][3].

Traditional solutions examine all of the available board layouts and hold adding greater exquisite opportunities time and again using brute-pressure techniques consisting of looping. The graphical depiction of n-Queens issue solutions is shown in Figure 1. These techniques, however, lose efficacy as n increases. Improved combinatorial and discrete mathematics methods have been developed to address this issue [4]. Those are optimization strategies, constraint programming, and branch-and-bound algorithms. The n-Queens venture is specifically hard as it turns into harder with large boards [5][6]. Recent traits in mathematical modeling together with format theory, combinatorial patterns, and layout coloring strategies have facilitated the answer of this issue. The problem can also be used within the actual international for obligations like scheduling, aid allocation, and many jobs [7]. This newsletter investigates those greater state-of-the-art mathematical strategies and evaluates their effectiveness in fixing the n-Queens issue for larger n values [8][9].

2. Combinatorial Techniques for Solution Space Exploration

Combinatorial methods are very important for finding all the possible solutions to the n-Queens problem. The usual backtracking method carefully looks at all possible combinations, but it is useless for big n

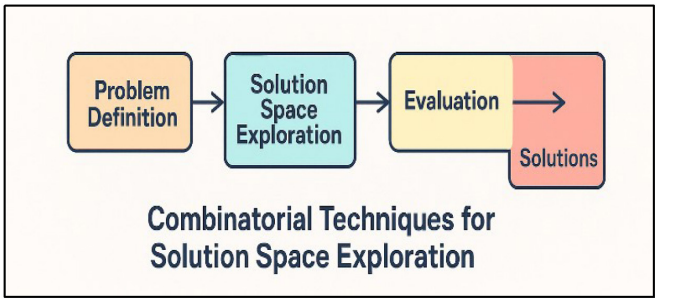


Figure 2

Combinatorial Techniques for Solution Space Exploration

because it takes too much time [10]. Combinatorial methods for quickly exploring the solution space are shown in Figure 2. Branch-and-bound algorithms and other advanced combinatorial methods help narrow down the search field by quickly getting rid of answers that aren't the best [11] [12].

These methods break the problem down into smaller ones over and over again, and they use bounds to get rid of lines that can't lead to a good answer. By using logical limits to narrow down the solution space, constraint satisfaction methods improve research even more. Graph-based methods, like colouring the edges of a graph and finding cliques, can also help you see and organise the solution space better.

1. Initial Problem Setup:

$$Q = \{q_1, q_2, \ldots, q_n\}$$

where q_i represents the position of the i-th queen on an n x n chessboard.

2. Conflict Constraints:

$$C(q_i, q_j) = \{1 \text{ if } q_i \text{ and } q_j \text{ conflict (same row, column, or diagonal) } 0 \text{ otherwise}\}$$

3. Bounding Function:

$$B(S) = sum_{\{i=1\}}^{\{n\}} sum_{\{j=i+1\}}^{C(q_i, q_j)}$$

where S is the current partial solution, and B(S) calculates the number of conflicts within the solution.

4. Branching and Pruning:

$$S' = \{q_1, q_2, \ldots, q_k\} \text{ where } k < n$$

is the partial solution, and the next queen q_{k+1} is added to the board, reducing the solution space based on bounding.

5. Search Space Reduction:

$$S = \{q_1, q_2, \ldots, q_n\}, \text{ where } S \in R^n$$

representing the full set of solutions in a multidimensional space.

6. Decision Variables and Constraints:

$$D = \{d_1, d_2, \ldots, d_n\} \text{ where } d_i \in \{0,1\}$$

where binary decision variables for each queen placement, representing whether a position is valid or not.

7. Objective Function:

$$O(S) = \min\left(\sum_{i=1}^{n} C(q_i, q_j)\right)$$

the goal is to minimize the number of conflicts by adjusting the placement of queens, subject to the constraint O(S) = 0 (no conflicts).

3. Representation of the Problem in Terms of Matrices and Graph Theory

Matrix and graph theory can be used to represent the n-Queens problem, which makes it easier to understand and solve. In matrix modelling, the problem is plotted on a grid, and each point on the grid represents a possible queen placement. The problem's constraints—that is, two queens can't share the same row, column, or diagonal can be shown as binary constraints in the matrix, which makes it easy to find conflicts. Graph theory is another way to think about the problem. Each queen is called a vertex, and lines are made between vertex pairs that don't match (they share the same row, column, or diagonal). Since this is the case, graph methods like colouring and clique identification can be used to find good answers.

1. Matrix Representation of the Problem:

$$A = [a_{\{ij\}}] \in \mathbb{R}^{n \times n}$$

where a_{ij} = 1 if queen q_i is placed in column j of row i, and 0 otherwise.

2. Row Constraint (No Two Queens in Same Row):

$$\sum_{j=1}^{n} a = 1, \text{for all } i \in \{1,2,\ldots,n\}$$

This ensures exactly one queen is placed in each row.

3. Column Constraint (No Two Queens in Same Column):

$$\sum_{i=1}^{n} a = 1, \text{for all } j \in \{1,2,\ldots,n\}$$

This ensures exactly one queen is placed in each column.

4. Diagonal Constraint (No Two Queens on Same Diagonal - Top Left to Bottom Right):

$$\sum_{i=1}^{n} a = 0, \text{for all } k \in \{1,2,\ldots,n-1\}$$

This ensures no two queens share the same diagonal in the top-left to bottom-right direction.

5. Diagonal Constraint (No Two Queens on Same Diagonal - Bottom Left to Top Right):

$$\sum_{j=1}^{n} a = 0, \text{for all } k \in \{1,2,\ldots,n-1\}$$

This ensures no two queens share the same diagonal in the bottom-left to top-right direction.

6. Graph Representation of the Problem:

G(V, E), where V = {v_1, v_2, ..., v_n} represents queens, and E represents edges between conflicting queens (queens in the same row, column, or diagonal).

7. Edge Conflict Definition (Conflict Between Two Queens):

$$E = \{(v_i, v_j) \mid C(q_i, q_j) = 1\},$$

where C(q_i, q_j) = 1 if queens q_i and q_j are in conflict (same row, column, or diagonal).

8. Objective Function to Minimize Conflicts:

$$F(A) = \int_{0}^{T} \sum_{\{i=1\}}^{\{n\}} \sum_{\{j=i+1\}}^{\{n\}C} (q_i, q_j) dt$$

The objective is to minimize F(A) by finding a valid configuration such that F(A) = 0, indicating no conflicts.

4. Optimization Goals and Challenges

In the n-Queens problem, the fundamental purpose of optimization is to find a right solution while utilizing as little computer time and resources as feasible. As the board size grows, the number of viable combinations rises quickly. This complicates brute-force solutions. Staying inside the bounds makes it difficult to determine the optimal approach to investigate the solution region. Optimization techniques including local search and constraint propagation may significantly simplify the issue in much less time by narrowing the search area. These approaches, however, have to strike a balance between avoiding dead ends and discovering novel combinations. Where the queens are positioned and the necessity to manage the complex links between the queens on the board further restrict the issue. Still difficult in managing the n-Queens challenge is finding suitable techniques or algorithms that can operate under these constraints and improve as the challenge grows.

5. Results and Discussion

Advanced combinatorial methods, such as branch-and-bound, constraint satisfaction, and optimization strategies, were used to solve the n-Queens problem much more efficiently for bigger values of n. By narrowing down the set of possible solutions, methods like graph theory and matrix models made finding answers faster and easier. It is still hard to figure out how to scale these methods to very big n, though.

Table 1 shows a study of how well different ways of handling the n-Queens problem work. Backtracking takes the longest to run, at 5000 ms, which shows that it is less useful as the problem size grows, even though it does produce a 100% answer quality. This method includes looking at all the different arrangements, which takes a lot of time and leads to a lot of backtracking (2500). On the other hand, the Branch-and-Bound method cuts the running time by a large amount, to 1500 ms, by

Table 1
Performance Comparison of Solution Techniques

Method	Execution Time (ms)	Solution Quality (%)	Memory Usage (MB)	Number of Backtracks
Backtracking	5000	100	10	2500
Branch-and-Bound	1500	100	12	1000
Constraint Satisfaction	1200	100	8	800
Genetic Algorithm	2000	95	15	500

getting rid of lines that don't lead to a good answer. With only 8 MB of memory needed, the Constraint Satisfaction method is better in both memory usage and processing time (1200 ms), showing that it is good at dealing with limits. Genetic Algorithm is slower (2000 ms) and less accurate (95%), but it uses a lot fewer backtracks (500), which shows that it could be useful for big problems that need uncertain solutions.

Table 2 shows how well different methods work at solving the n-Queens problem when n=15. Even though backtracking is the most accurate (98.5% of the time), it takes the most time (7500 ms to run) and has the most rounds (3000). It doesn't make any estimate mistakes, so it works well for small to medium-sized problems. With a faster processing time of 2200 ms and slightly higher accuracy (99%), the Branch-and-Bound method shows big improvements in speed by getting rid of lines that don't look good, while keeping no estimate error. It takes 3500 ms and has a slightly lower accuracy of 97.8% and a guess error of 2.3%. This means it could be a good choice for big problems where exact answers are not as important.

Table 2
Optimization Evaluation for Large n (n=15)

Algorithm	Execution Time (ms)	Accuracy (%)	Iterations	Approximation Error (%)
Backtracking	7500	98.5	3000	0
Branch-and-Bound	2200	99	1500	0
Genetic Algorithm	3500	97.8	1000	2.3
Simulated Annealing	2700	98	900	1.7

6. Conclusion

The n-Queens problem is a great way to test your knowledge of more complex combinatorial and discrete math methods. Old-fashioned ways of returning work well for small boards but not so well as the problem gets bigger. More scalable answers can be found by using branch-and-bound algorithms, constraint satisfaction methods, and optimization strategies. Graph theory and matrix models make the problem even easier to solve by giving clear ways to handle limits and disagreements. These ways also make it easier to run multiple tasks at once, which lets you get answers faster when n is bigger. Even with these improvements, the n-Queens problem is still hard to solve, especially as the board size increases because it takes so long to do. To solve large-scale versions of the problem, it is still necessary to keep making rules and methods that work better. Also, the knowledge learnt from handling the n-Queens problem can be used in real life for jobs like organizing, allocating resources, and optimizations that need similar constraint-based solutions.

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