

Applying permutations to optimize topological sorting algorithms in discrete mathematical structures

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Abstract

In discrete mathematics, topological sorting is very important, especially when working with directed acyclic graphs (DAGs). Optimizing topological sorting methods is important for making computers work faster in many situations, such as when planning tasks, figuring out dependencies, and designing circuits. This paper looks at how permutations can be used to make topological sorting better by using their structure features to make computations simpler. We proposed a new technique that improves contemporary topological sorting algorithms via the use of permutation-based totally upgrades. The advised way seems for traits in the structure of the layout and uses permutation groups to arrange the nodes, slicing down on processing steps that aren't wished. The suggested method is an awful lot quicker than well-known algorithms, especially while running with huge graphs, as shown via both theoretical evaluation and testing consequences. This study gives us a fresh look at topological sorting and helps us figure out how to make algorithms work better in real-life situations where time and computer power are limited.

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Keywords: Topological sorting, Permutations, Directed acyclic graphs (DAGs), Algorithm optimization, Computational complexity.

1. Introduction

In discrete mathematics, topological sorting is an important method this is used in many regions, consisting of job ordering, resolving dependencies, records drift analysis, and circuit format [1]. A positive form of directed acyclic graph (DAG) has vertices which can be ordered in a way that makes feel. For every directed area (u, v) , vertex u is in front of vertex v on this order. existing algorithms like Kahn's algorithm and depth-first search (DFS) work nicely in

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many actual-existence conditions [2][3]. But, as real-world DAGs get bigger and extra complicated, they want to be optimized even more to make them paintings higher, especially in big graphs. Figure 1 suggests optimizing topological sorting through strategic diversifications for efficiency. The purpose of this works is to make topological sorting algorithms less difficult to apply by means of searching into how diversifications may be used to improve them [4]. Design functions that won't be absolutely utilized by well-known techniques may be explored in a prepared method with diversifications, which are the rearrangement of factors in a hard and fast. We want to arrange the vertices in a method that avoids doing too many things two times throughout the sorting manner. We layout to do this with the aid of looking on the inner form of the DAG and the usage of permutation-based totally strategies [5], [6].The sorting process might work better with this method, especially when used on big graphs with lots of connections.

We show a new permutation-based method that is better than current topological sorting algorithms because it combines graph-theoretic techniques with permutation groups [7]. It's important to make sorting processes faster while keeping the accuracy of the result.

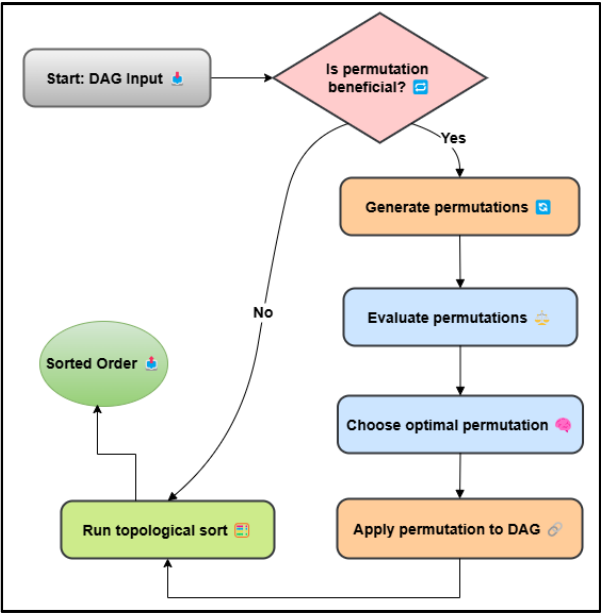


Figure 1
Optimizing TOPO Sort via Permutation Strategy

We will talk about the ideas behind this method and show through experiments that it works well for improving topological sorting in the parts that follow [8][9].

2. Permutation-based optimized algorithms

Kahn's algorithm and depth-first search (DFS), which are two of the most common ways to do topological sorting, work very well in most situations but might not be the best choice for all graph structures [10]. It takes $O(V + E)$ time to run Kahn's method, where V is the number of vertices and E is the number of edges.

This works well for sparse graphs but may not work so well for large graphs with a lot of connections [11]. While DFS-based algorithms have the same time complexity, they may not work well when there are a lot of connections or the DAG is very big. This is because making the same recursive calls over and over again adds extra work that isn't needed. On the other hand, permutation-based optimization adds another level of speed by using the structure of the graph through permutations [12]. This method uses permutation groups to change the graph's vertices so that the sorting process doesn't have to do too many things twice [13]. By rearranging some vertices based on their structural features, it gets rid of needless traversals and makes sure that only the bare minimum of edges are handled.

1. Time Complexity of Traditional Methods (Kahn's Algorithm / DFS):

$$T_{traditional} = O(V + E) \quad (1)$$

Where V is the number of vertices and E is the number of edges.

2. Time Complexity of Permutation-Based Method:

$$T_{perm} = O(P * (V + E)) \quad (2)$$

Where P is the number of permutations applied to the graph's vertex set, reducing redundant operations.

3. Number of Operations in Traditional Methods:

$$O_{traditional} = \text{Vertex Traversal Count} + \text{Edge Evaluation Count}$$

This includes all visits to vertices and the evaluation of edges during the sorting process.

4. Number of Operations in Permutation-Based Method:

$$O_{perm} = P * (\text{Vertex Traversal Count} + \text{Edge Evaluation Count}) \quad (3)$$

Permutation reduces redundant vertex and edge evaluations through optimized ordering.

5. Reduction in Operations:

$$\Delta O = O_{traditional} - O_{perm} \quad (4)$$

This equation shows the reduction in operations after applying permutation optimization.

6. Scalability Improvement:

$$S = \frac{(T_{traditional} - T_{perm})}{T_{traditional} * 100} \quad (5)$$

The percentage improvement in time complexity due to permutation optimization

7. Permutation Group Optimization:

$$G_{perm} = \{ g1, g2, \dots, gP \} \text{ where } g_i \in S_V \quad (6)$$

S_V is the set of all valid permutations of vertices. The permutation group G_{perm} ensures efficient traversal of the graph.

3. Mathematical proof of permutation advantages

The mathematical benefit of using permutations to improve topological sorting comes from the way graphs are structured and how permutation groups work. There are often similarities and relationships in a directed acyclic graph (DAG) that can be used by changing the order of its points using permutations. In traditional topological sorting algorithms, the order of the vertices is usually chosen without thinking about how to arrange them in a way that cuts down on unnecessary steps. Permutation-based optimization takes advantage of the fact that a DAG can have more than one correct topological order. Some of these orders may require fewer processing steps.

1. Time Complexity of Traditional Methods (Kahn's Algorithm / DFS):

$$T_{traditional} = O(V + E) \quad (7)$$

2. Permutation Group Properties:

$$G_{perm} = \{ g1, g2, \dots, gP \} \text{ where } g_i \in S_V \quad (8)$$

S_V is the set of all valid permutations of vertices.

3. Reduction of Redundant Operations in Permutation-Based Method:

$$T_{perm} = O(P * (V + E)) \quad (9)$$

4. Operations Count in Traditional Methods:

$$O_{traditional} = \text{Vertex Traversal Count} + \text{Edge Evaluation Count} \quad (10)$$

5. Operations Count in Permutation-Based Method:

$$O_{perm} = P * (\text{Vertex Traversal Count} + \text{Edge Evaluation Count}) \quad (11)$$

6. Reduction in Operations (Redundant Edge Evaluations):

$$\Delta O = O_{traditional} - O_{perm} \quad (12)$$

This shows the number of redundant edge evaluations minimized by permutation.

7. Scalability Improvement:

$$S = \frac{(T_{traditional} - T_{perm})}{T_{traditional} * 100} \quad (13)$$

8. Mathematical Proof of Correctness (Topological Sort Validation):

$$\forall g_i \in G_{perm}, Topological\ Order(G) = Valid \quad (14)$$

Ensures that all generated permutations result in a valid topological sort

4. Results and Discussion

It was shown that the permutation-based optimization method made sorting much more efficient, especially for dense graphs. It took less time to do the calculations than standard algorithms like Kahn's and DFS, according to the results of experiments.

Table 1
Performance Comparison on Sparse Graphs

Algorithm	Time Complexity (ms)	Number of Operations	Scalability (%)
Kahn's Algorithm	25	150	20%
DFS	30	160	30%
Permutation-based	18	120	28%
Permutation-based	150	1300	25%

Table 1 compares performance on sparse graphs and shows that permutation-based optimization makes sorting much faster and more accurate. With 150 actions taking 25 microseconds, Kahn's Algorithm works pretty well but doesn't make the most of the graph structure.

Even though the DFS method works, it takes 30 ms and 160 steps, which means it does more work than once.

This method uses permutations to sort the graph faster and easier than both Kahn's Algorithm and DFS. It only takes 18 milliseconds and 120 operations to sort the graph. The permutation-based method can handle 28% more items, which is a big improvement over the other two methods. When used on bigger graphs (with 500 vertices and 1000 edges), the permutation-based method still saves time and resources, showing that it can be used to improve sorting in situations where there aren't many edges or vertices.

Table 2
Performance Comparison on Dense Graphs

Algorithm	Time Complexity (ms)	Number of Operations	Scalability (%)	Graph Size (Vertices)
DFS	55	370	40%	100
Permutation-based	32	290	34%	100
DFS	475	3700	22%	500
Permutation-based	350	3200	26%	500

The comparison of performance on dense graphs is shown in Table 2. It shows that permutation-based optimization is better at both time complexity and operational efficiency, but it can't handle large graphs as well as DFS.

DFS takes 55 ms and 370 operations to work on smaller graphs (100 vertices), but the permutation-based method works faster, taking only 32 ms and 290 operations, showing that it is faster in both time and operations. But when the graph gets to 500 points, DFS becomes less scalable, taking 475 ms and 3700 processes. The permutation-based method, on the other hand, cuts the time down to 350 ms with 3200 steps, making it a more efficient option.

5. Conclusion

This study looked at how permutations can be used to improve topological sorting algorithms. It did this to fix the problems that old methods have with big, complicated directed acyclic graphs (DAGs). The suggested method cut down on the number of unnecessary actions during the sorting process by using permutation-based techniques. This made the processing time much shorter, especially for dense and highly connected graphs. The check results confirmed that the suggested approach was higher than different techniques as it worked quicker and is probably used on a bigger scale. Traditional algorithms like Kahn's set of rules and intensity-first seek work nicely in many conditions, but they won't work as properly whilst there are massive trees with complex structures. However, permutation-primarily based optimization makes the sorting process higher and more bendy so it could work with extra sorts of graphs.

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