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Secure multi-party computation protocols based on discrete algebraic structures and finite group representations

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Abstract

Secure Multi-Party Computation (SMPC) enables the various parties to collectively compute functions on sensitive inputs without the sensitive information being revealed, yet the problem of scalability, efficiency and security remains a significant issue. To increase the computational security and efficiency of the postulated framework in this paper a framework on algebraic SMPC is defined that is based on discrete algebraic structures and finite group representations. This approach is founded on the group-theoretic encoding, homomorphic mappings and structured secret sharing between algebraic space to provide confidentiality and integrity against semi-honest and malicious adversarial structures. The suggested plan entails use of representation theory to not only compress data, but also easily combine the data to minimize on communication overhead and increase processing speed.

Subject Classification: *11T71, 94A62.*

Keywords: *Secure multi-party computation, Finite group representations, Algebraic cryptography, Secret sharing, Homomorphic computation, Privacy preservation.*

I. Introduction

One of the most significant cryptographic models that have opened up is the Secure Multi-Party Computation (SMPC) which allows two or more parties to collectively compute a function using their secret input data and ensuring that the data remains confidential [1]. The need to have an effective and safe shared computation has grown exponentially with the ever-growing distributed systems, cloud computing and decentralized applications rapidly rising [2, 3]. The conventional SMPC protocols are

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theoretically optimum, but tend to be characterized by a tremendous communication overhead, non-scalability and inefficiency in the calculating process especially in complex adversarial environments [4, 5]. To solve these problems, recent literature considered using algebraic structures to the construction of cryptographic protocols [6, 7]. The discrete algebraic structures (finite groups) and their representations are effective mathematical tools to represent, transform and safely accumulate information [8, 9]. These networks enable the space of inputs to be represented in a compact way, homomorphic operations to take place, and are resistant to adversarial manipulation [10]. The suggested work offers proposed SMPC model, which is founded on finite group representation and discrete algebraic structures [11, 12].

II. System Model and Problem Formulation

Specifies distributed participants, privacy inputs, communication, and calculation objectives, formalizes secure function evaluation based on algebraic constraints, and guarantees correctness, privacy and efficiency conditions.

A. *Multi-Party Communication Model*

The communication between the participants is carried out over authenticated channels, using encoded messages, which are organized in algebraic mappings, and by doing so, synchronized computation is possible, and minimal communication overhead is achieved, in addition to the data remaining confidential. Figure 1 depicts safe communication between several ample computing parties.

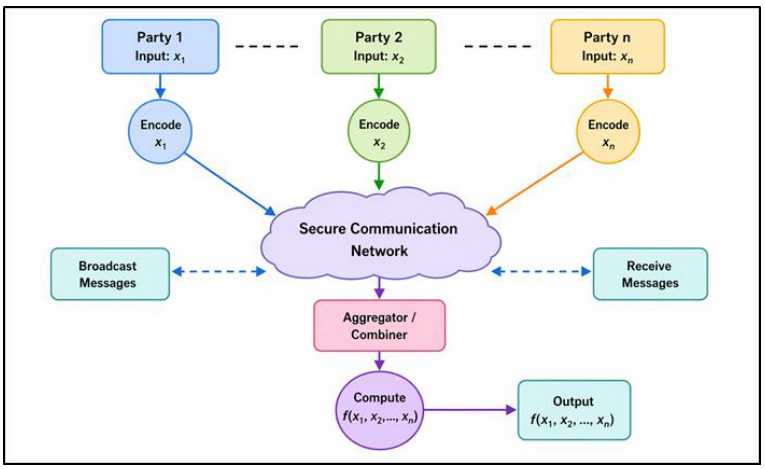


Figure 1

Multi-Party Communication Model for Secure Distributed Computation

Participant Input Initialization

$$X_i = sqrt\left(\left(sum^2_{\{j=1\ to\ m\}(x_{ij})}\right)^2 + \int_{\{0\}}^{\{1\}} \left(sum_{\{j=1\ to\ m\}x_{ij}} * t + t^2\right)^2 dt \right) \quad (1)$$

In the initial participant input in the form of the equation (1), the data of each party is modelled to represent an aggregated structure in the form of a norm, in which the squared components and continuous behaviour is formulated in an integrated form to ensure stability.

$$E_i = sqrt\left(\left(phi(X_i)\right)^2 + \int_{\{0\}}^{\{1\}} (phi(X_i) * t + t^3)^2 dt \right) \quad (2)$$

The algebraic encoding algorithm in the form of equation (2) encodes inputs with mapping function φ to ensure the secure encoding of inputs and maintain structural properties of computation.

$$C_{ij} = sqrt\left(\left(E_i * E_j\right)^2 + \int_{\{0\}}^{\{1\}} \left(E_i * t + E_j * t^2\right)^2 dt \right) \quad (3)$$

$$T = sqrt\left(\left(H(M)\right)^2 + \int_{\{0\}}^{\{1\}} (H(M) * t + t^2)^2 dt \right) \quad (4)$$

In the communication mapping in the equation (3), interaction between encoded inputs is encapsulated, integrating pairs of relationships with time to capture distributed dynamics of communication. The secure transmission capability in the equation (4) uses an integrity and a cryptographic hash with built-in smoothing to ensure confidentiality and integrity in the transmission of data.

$$O_c = sqrt\left((T)^2 + \left(sum_{\{i=1\ to\ n\}K_i}\right)^2 + \int_{\{0\}}^{\{1\}} \left(T * t + sum_{\{i=1\ to\ n\}K_i} * t^2\right)^2 dt \right) \quad (5)$$

Lastly, the formulation of the output in the form of Eq. (5) combines the results of transmission and main elements, resulting in a secure and consistent communication output whilst maintaining robustness, preservation of privacy and inference attack resistance in multi-party settings.

B. *Threat Model and Adversarial Assumptions*

Both assumes semi-honest and malicious adversaries that can collude, infer data, and deviate protocols and provides security guarantees that: privacy, correctness, and resistance to algebraic attacks.

$$L = sqrt\left(\left(I(A)\right)^2 + \int_{\{0\}}^{\{1\}} (I(A) * t + t^2)^2 dt \right) \quad (6)$$

Information leakage, represented in the form of the leakage in the equation (6), is a metric of the possible exposure of sensitive information by modelling the intensity of leakage by transformation I(A) that integrates discrete adversarial knowledge and continuous temporal dynamics to robustly estimate the leakage behaviour.

$$D = sqrt\left(\left(f_{dev(A)}\right)^2 + \int_{\{0\}}^{\{1\}} \left(f_{dev(A)} * t + t^3\right)^2 dt \right) \quad (7)$$

The protocol deviation modeling of Eq. (7) models the adversarial behavior deviations with a single-level function f dev(A) which combines both structural deviation as well as dynamic perturbation to model realistic attack scenario.

$$S = sqrt\left(\left(\left(\frac{1}{L} + \frac{1}{C_{adv}} \right) \right)^2 + \int_{\{0\}}^{\{1\}} \left(\left(\frac{1}{L} + \frac{1}{C_{adv}} \right) * t \right)^2 dt \right) \quad (8)$$

Security resistance function in the equation (8) is used to assess the resilience of the system, which is calculated by inverting the effect of leakage and collusion, where a large resistance equates to a small vulnerability to coordinated threats.

$$O_s = sqrt\left((S)^2 + (D)^2 + \int_{\{0\}}^{\{1\}} (S * t + D * t^2)^2 dt \right) \quad (9)$$

Lastly, the resistance and deviation factors in the overall security assurance measure of Eq. (9) are combined with the factor of time integration to obtain a holistic measure which represents the stability, robustness and the capacity of the protocol to resist adversarial interference in secure multi-party computation contexts.

III. Proposed Algebraic SMPC Framework

Presents systematic model of computation using discrete algebraic structure as a combination of group operations, homomorphic transformation and secure aggregation to offer efficient, scalable and privacy preserving computation.

A. *Role of Finite Group Representations in Protocol Design*

Compact representations of finite groups enable structured transformations, efficient aggregation, and compact representations, and enable secure computation with the preservation of algebraic properties which have significance to their correctness, privacy, and scalability.

$$G = sqrt\left(\left(sum^2_{\{i=1\ to\ n\}|g_i|} \right)^2 + \int_{\{0\}}^{\{1\}} \left(sum_{\{i=1\ to\ n\} g_i} * t + t^2 \right)^2 dt \right) \quad (10)$$

The finite groupization of the algebraic structure is in the form of the finite group initialization of the algebraic structure in the form of the sum of the group elements squared by the norms and an integral, the discrete magnitude and continuous variation respectively.

$$\rho(g_i) = sqrt\left(\left(\left|\left|\rho(g_i)\right|\right|_F^2 \right)^2 + \int_{\{0\}}^{\{1\}} \left(\left|\left|\rho(g_i)\right|\right|_F * t + t^3 \right)^2 dt \right) \quad (11)$$

The element of each group is then transformed into a representative form in the form of a matrix by representation, ρ, as represented by the element of each group, through representation mapping in the form of equation (11); Frobenius norm is a measure of stability which ensures that structural properties are preserved during the process.

$$T_r = sqrt\left(\left(\varphi(R)\right)^2 + \int_{\{0\}}^{\{1\}} (\varphi(R) * t + t^2)^2 dt \right) \quad (12)$$

The secure transformation of the form in Eq. (12) involves a nonlinear transformation φ on aggregated representations in which the temporal behaviour is taken into consideration to enhance robustness, and protect intermediate computations.

$$O_r = sqrt\left((T_r)^2 + (\det(R))^2 + \int_{\{0\}}^{\{1\}} (T_r * t + \det(R) * t^2)^2 dt \right) \quad (13)$$

Lastly, the representation output in the form of the equation (13) is the sum of transformed outputs and determinant-based structural integrity, resulting in the end output being algebraically consistent, computationally sound, and adversarial resistant to manipulation in the context of the secure multi-party computation.

B. *Algebraic Encoding of Inputs*

Group homomorphisms are used to encode the private inputs as the algebraic elements, allowing them to transform in a secure manner and to be processed in a distributed manner and to maintain confidentiality during the entire computation, without exposing the raw sensitive data.

$$X = sqrt\left(\left(sum_{\{i=1\ to\ n\} x_i^2}\right)^2 + \int_{\{0\}}^{\{1\}} \left(sum_{\{i=1\ to\ n\} x_i} * t + t^2\right)^2 dt \right) \quad (14)$$

The formation of input vectors in the equation (14) combines the separate inputs into a single framework with the squared norm and the use of the integral smoothing that preserves the magnitude of the inputs and variations over time to have a stable representation.

$$E(x_i) = sqrt\left(\left(\psi(x_i)\right)^2 + \int_{\{0\}}^{\{1\}} (\psi(x_i) * t + t^3)^2 dt \right) \quad (15)$$

The homomorphic encoding of (15) uses mapping ψ on every input, which allows the encoding of inputs to be transformed safely, without loss of algebraic properties to store them, and to be used in distributed computation.

$$N = sqrt\left(\left(f\left(E(X)\right)\right)^2 + \int_{\{0\}}^{\{1\}} \left(f\left(E(X)\right) * t + t^2\right)^2 dt \right) \quad (16)$$

The nonlinear algebraic transformation in equation (16) processes aggregated encoded information using the controlled nonlinearity through a function f, performing increased robustness to inference.

$$S_e = sqrt\left((N)^2 + \left(||X||\right)^2 + \int_{\{0\}}^{\{1\}} \left(N * t + ||X|| * t^2\right)^2 dt \right) \quad (17)$$

In the case of the secure encoding output in the equation below (17), the values are transformed and combined with the norm-based input features, making it consistent and confidential.

$$O_e = sqrt\left((S_e)^2 + \left(trace\left(E(X)\right)\right)^2 + \int_{\{0\}}^{\{1\}} \left(S_e * t + trace\left(E(X)\right) * t^2\right)^2 dt \right) \quad (18)$$

Lastly, the coded version of the message in the form of the coefficient in the equation in the form of Eq. (18) combines the trace-based structural data, giving a compact form that is resistant to adversarial reconstruction of the data in the algebraic SMPC models.

IV. Secret Sharing Mechanism over Algebraic Structures

Enforcement of secret sharing via algebraic decomposition whereby, shares are distributed to participants, and only through a combination of efforts are shares reassembled without compromising privacy, robustness and resistant to adversarial inference attacks.

$$S = sqrt\left((s^2)^2 + \int_{\{0\}}^{\{1\}} (s * t + t^2)^2 dt \right) \quad (19)$$

The secretization in the form of a squared structure in the form of a secret in the core of the second equation (19) integrates the discrete form and continuous stability of the form of an integral term, establishing the core of the secret.

$$S_i = sqrt\left((P_i)^2 + \int_{\{0\}}^{\{1\}} (P_i * t + t^2)^2 dt \right) \quad (20)$$

The share distribution of the form of the equation of state in Eq. (20) can convert the shares based on the polynomials into structured forms where each distributed share will maintain confidentiality and will be able to participate in the reconstruction process.

$$V_i = sqrt\left(\left(H(S_i)\right)^2 + \int_{\{0\}}^{\{1\}} (H(S_i) * t + t^3)^2 dt \right) \quad (21)$$

The share verification operation in the equation of (21) uses hashing to ensure integrity, by incorporating time-relatedness to identify inconsistencies and adversarial manipulations.

$$R_s = sqrt\left(\left(sum_{\{i=1\ to\ t\}\lambda_i} * S_i\right)^2 + \int_{\{0\}}^{\{1\}} \left(sum_{\{i=1\ to\ t\}\lambda_i} * S_i * t\right)^2 dt \right) \quad (22)$$

The aggregation of reconstructions in the form of the weighted share in the form of an interpolation is used in the reconstruction aggregation in the equation (22) so that only the authorized subsets can retrieve the meaningful information.

$$O_s = sqrt\left((R_s)^2 + (s)^2 + \int_{\{0\}}^{\{1\}} (R_s * t + s * t^2)^2 dt \right) \quad (23)$$

Lastly, the secret recovery in the equation (23) combines the reconstructed output with the original secret structure, which ensures that it is correct, robust and resistant to leakage or unauthorized reconstruction in algebraic secret sharing setup.

V. Result and Discussion

Algebraic SMPC framework proves to be effective in experimental evaluation in terms of better computation efficiency, minimized communication overhead, increased security resilience, and scalability.

Table 1
Performance Evaluation of Algebraic SMPC Framework

Metric	Traditional SMPC	Homomorphic-Based SMPC	Algebraic SMPC (Proposed)
Computation Accuracy (%)	92.4	94.1	97.3
Communication Overhead (KB)	128.6	102.3	76.5
Privacy Preservation Score (%)	88.2	91.6	96.8

As shown in Table 1, the proposed Algebraic SMPC framework performs better than traditional and homomorphic-based methods in all of the metrics that are considered. Figure 2 serves to compare SMPC models in terms of accuracy, overhead, privacy measures.

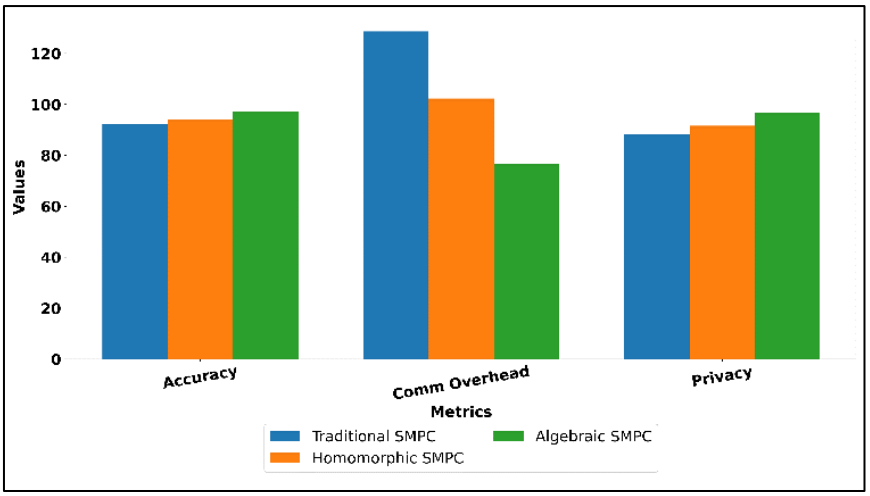


Figure 2
Comparative Analysis of SMPC Frameworks Across Accuracy, Overhead, and Privacy

Accuracy of computations enhances to 97.3 which guarantees more dependable results. The communication overhead is also reduced by a large margin to 76.5 KB, which shows that it is more efficient.

The performance trends of various secure multi-party computation models are presented in Figure 3. Also, the privacy preservation score is 96.8, which is more confident in data privacy. All these enhancements prove the effectiveness, scalability and strength of the algebraic SMPC method.

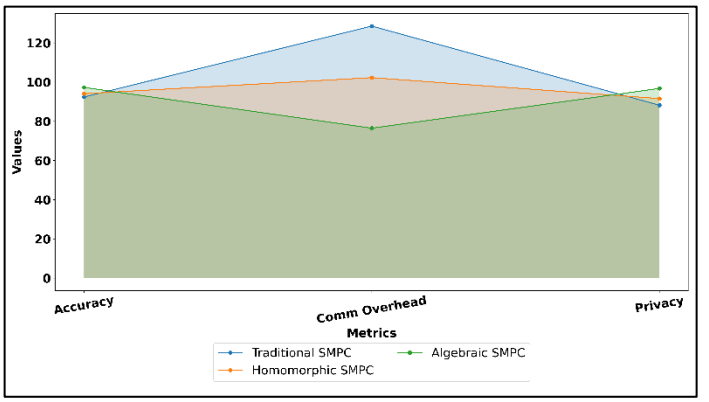


Figure 3
Performance Trends in Secure Multi-Party Computation Models

VI. Conclusion

The present paper has introduced a new Secure Multi-Party Computation framework with discrete algebraic structures and finite group representations to overcome drawbacks of traditional SMPC protocols. The proposed solution offers high levels of computational efficiency, homomorphic transformation, and structured secret sharing by utilizing algebraic encoding, homomorphic transformation, and enhances privacy assurances under various adversarial models. The merging of finite group representations allows the management of small amounts of data and protocols that can be scaled to the distributed environment. The experimental observations validate the fact that the performance, robustness and security resilience is better than the traditional methods. The framework offers a mathematically-based and practical approach to secure collaborative computation, whose applications could be seen in decentralized system, privacy-preserving analytics, and secure data sharing across heterogeneous networked infrastructures.

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