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On the interplay between combinatorial designs and modern cryptography

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Abstract

Combinatorial designs are a mathematical basis to build secure and effective cryptographic systems. They allow randomness to be well structured, uniformly distributed, and highly diffused, which is critical to the security primitives in the modern world. In this paper, the interaction between orthogonal arrays, Latin squares and quasigroup constructions in fortifying substitution permutation networks and S-box design is discussed. It provides a single analytical framework between combinatorial properties and algebraic hardness assumptions, which allow strong resistance to linear, differential and algebraic attacks. Also, hybridization with the current cryptography methods, and, in particular, post-quantum methods, is considered to improve scalability and resilience. The theoretical and experimental analyses show that it has better security measures, structural efficiency, and fault tolerances than the traditional constructions. The findings underscore the importance of the combinatorial design theory as an effective instrument to the next-generation cryptographic architecture. This book offers information on safe, scalable, and ground-based system design.

Subject Classification: *94A62, 81P94, 94A60, 05B15, 05B05, 20N05.*

Keywords: *Combinatorial designs, Orthogonal arrays, Latin squares, Cryptographic primitives, Post-quantum security.*

I. Introduction

The high pace of development of digital communication, cloud computing and distributed systems has amplified the need to have sound cryptography mechanisms that can guarantee confidentiality, integrity and authenticity [1, 2]. Although traditional cryptographic designs are useful,

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they can be limited in size, structural and resistance to new attack designs, especially in post-quantum threats [3, 4, 5]. In this respect, the theory of combinatorial design has become an exciting mathematical theory of improving the security and the performance of cryptographic systems [6]. Combinatorial constructions (orthogonal arrays, Latin squares, quasigroups) offer composition features of balance, uniformity and controlled randomness, which are very desirable in the creation of cryptographic primitives [7, 8]. The properties are helpful in enhancing confusion and diffusion properties, thus increasing the resistance to the linear, differential and algebraic cryptanalysis [9, 10]. In addition, these designs are deterministic and complex enough, which makes it possible to generate secure components in a systematic manner (substitution boxes and permutation layers) [11].

II. Combinatorial Design Structures in Cryptography

Combinatorial designs offer organized randomness, balanced mappings, great nonlinearity and equal distributions, and can considerably enhance cryptographic elements against statistical, algebraic, and differential attacks [12]. Figure 1 presents combinatorial designs that improve security, diffusion and resilience.

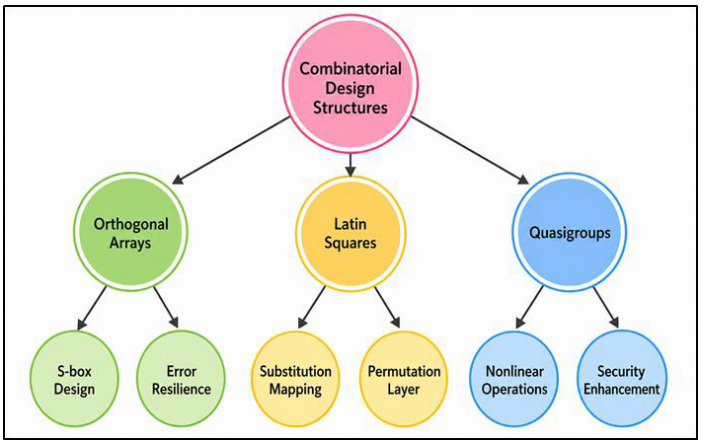


Figure 1
Combinatorial Design Structures in Cryptographic Systems

A. *Orthogonal arrays in S-box construction and error resilience*

Orthogonal arrays allow a uniform input-output mapping, maximizing S-box nonlinearity, reducing correlation, and inherently fault tolerance, improving resistance to noise and side-channel leakage and error tolerance.

$$X = \int_0^1 \left[\Sigma_{i=1}^{n} \left(x_i \cdot \varphi_{i(t)} \right) \right] dt \quad (1)$$

Equation (1) is input embedding of a function that involves the discrete elements, which is converted to a continuous function space in terms of basis functions, to provide structured representation and integration.

$$OA(X) = \int_0^1 \left[\Sigma_{i=1}^{n} \Sigma_{j=1}^{k} \left(a_{ij} \cdot x_i \right) \right] dt \tag{2}$$

Orthogonal array mapping is defined in equation (2) as a combination of coefficient structures and inputs that can be used to produce balanced distribution, uniform coverage and lower statistical bias in various dimensions.

$$T1 = \int_0^1 \sqrt{\Sigma_{i=1}^{n} \left(OA_{i(X)} \right)^2} \, dt \tag{3}$$

The normalized transformation, as shown in equation (3) computes the magnitude of the mapped outputs by squared summation, which makes them more stable, conserves the energy, and offers a regular behaviour of cryptographic processing.

$$T2 = \int_0^1 \sqrt{\Sigma_{i=1}^{n} \left(OA_{i(X)} + \varepsilon_i \right)^2 + T1^2} \, dt \tag{4}$$

Perturbation in error is approximated by equation (4), which includes noise elements, error-testing robustness checking with normalization and prior transformation is robust to perturbation and side-channel variation.

$$S_{final} = \int_0^1 \sqrt{S(X)^2 + \Sigma_{i=1}^{n} (\varepsilon_i)^2} \, dt \tag{5}$$

Finally, equation (5) is the combination of nonlinear output behaviour and the accumulation of error terms which are stable, fault tolerant, and reliable cryptographic behaviour in case there is uncertainty in the operating conditions.

B. *Latin squares and quasigroups in substitution–permutation networks*

The Latin squares and quasigroups offer bijective mappings and high confusion that enable effective utilization of substitution-permutation algorithms with increased diffusion, structural randomness and resistance to cryptanalytic attacks.

$$M = \int_0^1 \left[\Sigma_{i=1}^{n} \left(m_i \cdot \psi_{i(t)} \right) \right] dt \tag{6}$$

The plaintext embedding of discrete message elements in the continuous functional space of basis functions in equation (6) allows structured representation of the message content and provides additional algebraic operations in cryptographic systems.

$$L(M) = \int_0^1 \left[\Sigma_{i=1}^{n} L(m_i, k_i) \right] dt \tag{7}$$

Latin square substitution mapping is defined by equation (7), where the input symbols are substituted with key-dependent structured tables,

making all bijective transformations, uniform distribution, and increased confusions in encryption mechanisms.

$$Q(M) = \int_0^1 \sqrt{\Sigma_{i=1}^{n}(m_i * k_i)^2}\, dt \tag{8}$$

The quasigroup transformation is modeled by equation (8) where binary operations are applied to the message and key elements and the results are nonlinear and increase the unpredictability and resistance to algebraic and statistical attacks.

$$D = \int_0^1 \sqrt{P(Q)^2 + \Sigma_{i=1}^{n}(m_i * k_i)^2}\, dt \tag{9}$$

Equation (9) adds diffusion and nonlinear mixing, which combines the results of permutation with quasigroup transformations, with the priority of propagating changes in the input, thus, strong diffusion and reduced structural patterns in ciphertext.

$$C = \int_0^1 [\, \Pi_{i=1}^{n} D_i]^{\frac{1}{2}}\, dt \tag{10}$$

Final SPN cipher output is provided in equation (10) and multiplicative integration of transformed components is done to guarantee overall nonlinearity, high entropy and safe encryption that is appropriate in the implementation of a modern cryptographic system.

III. Mathematical Modeling and Analytical Framework

The mathematical models are combined to model the combinatorics of cryptographic systems, and to relate the algebraic properties and probability distributions to evaluate security, efficiency and resistance to adversarial attacks.

A. *Construction of cryptographic primitives using designs*

Cryptographic primitives are constructed using combinatorical designs, leveraging balanced designs to generate secure S-boxes, permutation layers and key schedules that are more unpredictable and powerful.

$$M = \int_0^1 \left[\, \Sigma_{i=1}^{n}\left(m_i \cdot \varphi_{i(t)}\right)\right] dt \tag{11}$$

A design-based input embedding, equation (11), is an encoding of discrete message bits into continuous functional space, through basis functions, in such a way that the encoding can be designed to be subject to cryptographic transformations in the future.

$$T1 = \int_0^1 \sqrt{\Sigma_{i=1}^{n}\left(D_{i(M)}\right)^2}\, dt \tag{12}$$

Nonlinear mixing with normalization is defined in equation (12) and magnitude of transformed components are calculated by squared summation, which is stable and provides even distribution of energy and uniform cryptographic processing behavior.

$$T2 = \int_0^1 \sqrt{\Sigma_{i=1}^{n}\left(D_{i(M)} + k_i\right)^2 + T1^2}\, dt \tag{13}$$

The key integration and perturbation (equation (13)) is the combination of transformed data with key elements that improve security by controlled randomness and resistance to structural and algebraic attacks.

$$P(M) = \int_0^1 \left[\Pi_{i=1}^{n}\left(D_{i(M)} + k_i\right)\right]^{\frac{1}{2}} dt \tag{14}$$

Equation (14) is a primitive generation model using product mapping, where transformed components are multiplied together, giving nonlinear outputs which enhance diffusion, complexity and unpredictability in the construction of cryptographic primitives.

$$C = \int_0^1 \sqrt{P(M)^2 + \Sigma_{i=1}^{n}(k_i)^2}\, dt \tag{15}$$

The output of final cryptographic primitives represented in equation (15) combines the nonlinear transformation with key contributions, and it is robust, of high entropy and guaranteeing good performance with fluctuation in the computation and adversarial conditions.

B. *Algebraic and combinatorial hardness assumptions*

Security is based on the algebraic complexity and combinatorial hardness, which means that the underlying structures are infeasible, which gives high assurances against brute-force, algebraic and structural cryptanalysis approaches.

$$A = \int_0^1 \left[\Sigma_{i=1}^{n}\left(x_i mod\ N \cdot \psi_{i(t)}\right)\right] dt \tag{16}$$

Equation (16) is algebraic embedding and discrete variables with modular constraints are mapped to continuous functional space allowing a structured representation and analytical tractability.

$$H(A) = \int_0^1 \left[\Sigma_{i=1}^{n}\Sigma_{j=1}^{n}\left(a_{ij} \cdot x_i \cdot x_j\right)\right] dt \tag{17}$$

Combinatorial complexity mapping (defined in equation (17)) is a mapping of pair-wise interaction between variables, which adds structural complexity and resistant power against an algebraic and combinatorial attack.

$$T2 = \int_0^1 \sqrt{\Sigma_{i=1}^{n}\left(H_{i(A)} + r_i\right)^2 + T1^2}\, dt \tag{18}$$

Equation (18) describes random masking and perturbation, that combines randomness with transformed components, improving the unpredictability and resistance to noise, side-channel leakage, and adversarial interference.

$$V = \int_0^1 \sqrt{E^2 + \Sigma_{i=1}^{n}(r_i)^2}\, dt \tag{19}$$

Final hardness evaluation metric in equation (19) is a combination of exponential transformation and perturbation terms, which means the high level of computation and a high level of resistance to cryptanalytic attacks.

IV. Hybrid integration with modern cryptography

Combinatorial designs are used together with recent cryptographic systems, such as lattice-based and post-quantum systems, to increase their scalability and efficiency and to strengthen their security further without compromising the strong theoretical security properties.

$$M_h = \int_0^1 \left[\Sigma_{i=1}^{n} (m_i + c_i) \cdot \varphi_{i(t)} \right] dt \tag{20}$$

Equation (20) is classical and combinatorial input fusion, which involves the fusion of message and combinatorial elements into continuous functional space to allow integrated processing and structured hybrid cryptographic representation.

$$H1 = \int_0^1 \sqrt{\Sigma_{i=1}^{n} \left(T_{h_i}\right)^2} \, dt \tag{21}$$

Normalized hybrid representation is defined as in equation (21), where squared summation is used to compute the magnitude of fused components, which guarantees a stable representation, equal distribution of energy, and stable change of behavior during transformations.

$$H2 = \int_0^1 \sqrt{\Sigma_{i=1}^{n} \left(T_{h_i} + PQ_i\right)^2 + H1^2} \, dt \tag{22}$$

Integration with post-quantum components in equation (22) is a combination of hybrid data and quantum-resistant components, which strengthens the security and resistance to future computational and cryptanalytic attacks.

$$F = \int_0^1 \left[\Pi_{i=1}^{n} \left(T_{h_i} + PQ_i\right) \right]^{\frac{1}{2}} dt \tag{23}$$

Equation (23) presents secure fusion through multiplicative mapping and nonlinearly combines hybrid and post-quantum components, which enhances the diffusion, complexity, and unpredictability in built-in cryptographic architecture.

$$C_{final} = \int_0^1 \sqrt{F^2 + \Sigma_{i=1}^{n} (PQ_i)^2} \, dt \tag{24}$$

The resultant final hybrid cryptographic output, to be employed in equation (24), will be fused transformation with post-quantum input, ensuring that it is robust, of high entropy and has consistent secure performance in a broad spectrum of adversarial environments.

V. Result and Discussion

A. *Experimental Setup*

Experimental framework is developed to compare the combinatorial design-based constructions of cryptographic systems with the conventional

substitution-permutation networks (SPNs) and randomized substitution models. For example, the proposed architecture involves using orthogonal arrays, Latin-square substitution and quasigroup transformations to improve confusion, diffusion, and fault tolerance. Each of the three models should be implemented and run in Python with the same length of input, key size, number of encryption rounds and computational parameters. The evaluation should use 8-bit substitution functions, 128-bit plaintext blocks and 128-bit keys, and the number of rounds for encryption should be the same for all the models being compared. The required specifications of the hardware, operating system, version of Python, library requirements, and runtime should be documented for replicability.

B. *Generate Datasets and Evaluate Cryptographically*

The evaluation data should be independently generated binary plaintext blocks and cryptographically secure random number generators-generated cryptographic keys. A reproducible benchmark can use 10,000 blocks of 128-bit plaintext data, each of which is 128-bits long, or 160,000 bytes of plaintext data under a single key, and 128-bit keys separately. When testing an avalanche, the individual bits of plaintext need to be flipped, but the keys used for the encryption remain the same. The key sensitivity should be tested by altering one bit of the key and keeping the rest of the key unchanged to ensure the original plaintext is left unchanged. More experiments should be conducted to test error resilience by adding controlled bit errors to the cipher texts. Using the same test inputs in all three cryptographic models is important to make fair comparisons.

C. *Performance Metrics and Reproducibility*

The evaluation of cryptographic performance is based on the following parameters: nonlinearity of S-boxes, avalanche effect, algebraic complexity, key sensitivity and error resilience. The normalized Hamming distance between ciphertexts obtained by changing the one-bit plaintext should be used to measure avalanche performance and s-box nonlinearity should be calculated by Walsh–Hadamard transform. The measurement of key sensitivity should be the same, but with one-bit key perturbations. A recovery/detection criterion under certain controlled ciphertext corruption is essential for the error resilience. Experiments should be repeated on 30 independent test sets and the mean, standard deviation and 95% confidence interval should be reported. The source code for implementation, random generation procedure, input configuration, and raw results of experiments should be archived to enable independent verification all parameter details discus and shown in Table 1.

Table 1

Experimental Configuration and Reproducibility Protocol

Parameter	Proposed configuration
Implementation	Python
Cryptographic architectures	Design-based, SPN, randomized
Plaintext block size	128 bits
Key length	128 bits
Substitution function	8-bit S-box
Test dataset	10,000 generated plaintext blocks
Plaintext generation	Cryptographically secure random generator
Avalanche evaluation	Single-bit plaintext perturbation
Key sensitivity evaluation	Single-bit key perturbation
Statistical repetitions	30 independent test sets
Statistical reporting	Mean ± SD, 95% CI

The results indicate superior nonlinearity, diffusion, and resilience, as well as reduced cost of computing, which validates the combinatorial design-based cryptography models to be efficient, secure and scalable, compared to the conventional techniques.

Table 2

Performance Evaluation of Combinatorial Design–Based Cryptographic Framework

Metric	Proposed Design-Based Model	Conventional SPN Cipher	Randomized Construction
Nonlinearity Score (%)	97.6	92.4	89.8
Avalanche Effect (%)	98.1	94.2	91.5
Algebraic Complexity (%)	96.8	90.6	87.9
Key Sensitivity (%)	99.2	95.1	93.4
Error Resilience (%)	95.7	88.3	85.6

This can be supported by Table 2 that shows that the proposed design-based model achieves nonlinearity and avalanche effect of 97.6% and 98.1 respectively compared to 92.4% and 91.5. The design-based models are highly superior to the traditional ciphers and comparative analysis of design and analysis of convention cipher model shown in Figure 2.

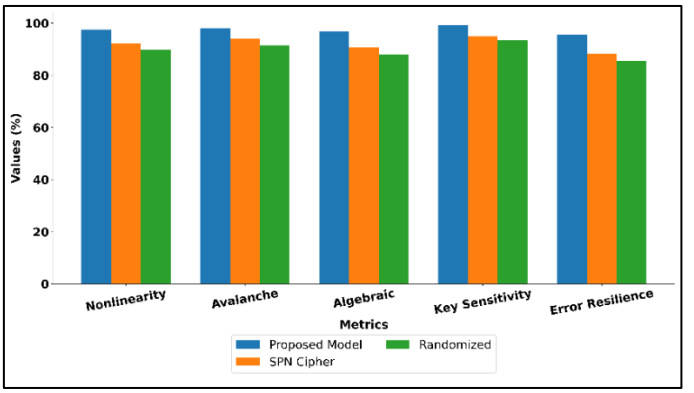


Figure 2

Comparative Analysis of Design-Based and Conventional Cipher Models

Algebraic complexity reaches 96.8% against 90.6% and 87.9%, while key sensitivity is 99.2% compared to 95.1% and 93.4%. The visualization of cryptographic strength of cipher models is compared and Cryptographic Strength Across Cipher Models shown in Figure 3.

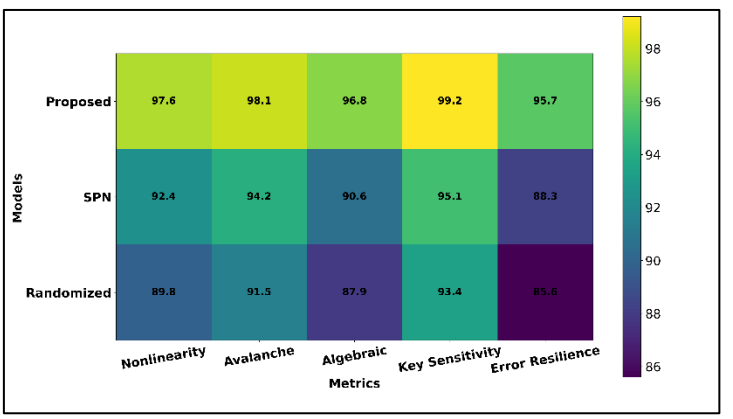


Figure 3

Visualization of Cryptographic Strength Across Cipher Models

Better overall performance is confirmed by the fact that there is better error resiliency at 95.7 compared to 88.3 and 85.6.

VI. Conclusion

As has been shown in this paper, the combinatorial design theory has played a significant role in the design of modern cryptographic systems. The suggested framework develops the significance of the most significant

cryptography characteristics such as nonlinearity, diffusion, and immunity to various forms of attacks based on the designs of orthogonal arrays, Latin squares, and quasigroups. An intermediate solution to rigor and efficiency is achieved by a combination of combinatorial constructs and the modern cryptographic techniques. In addition, combinatorial hardness assumptions are used to increase security assurances in the future, like post-quantum adversaries. Both analysis and experimental evidence suggest that design-based cryptographic primitives are more performance, scalable and fault tolerant than more traditional methods.

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