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Enumeration of graph factorizations and cryptographic uses

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Abstract

The problem of listing factorizations of graph is a computationally expensive problem, and has not been successfully applied in the design of cryptographic systems up to date. This paper will solve this challenge by coming up with a systematic plan of listing graph factorizations and harnessing the combinatorics of graph factorizations to create secure cryptography systems. The main target of it is to create a powerful enumeration model and prove that it can be used in key generating, safe communication, and structural randomness. It is a mixture of algebraic graph theory, algorithms of combinatoric decomposition and enumeration model and tested on benchmark graph datasets, such as regular, complete and bipartite graphs. The experimental results indicate that the time of enumeration is reduced by 31.6 percent and structural uniqueness is also enhanced by 24.8 percent than the traditional brute-force computations and the diversity of cryptographic keys has been boosted by 27.3. The features that would be able to be differentiated in the comparative analysis are enhanced protection against structural attacks, and scalability. The solution introduces a new connection between the graph factorization theory and cryptographic primitives, which allow an effective, structure-based, security. This

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work establishes graph factorizations as a firm foundation for next-generation cryptographic systems.

Subject Classification: *68M25, 11T71.*

Keywords: *Graph factorization, Combinatorial enumeration, Cryptographic primitives, Algebraic graph theory, Secure key generation, Structural complexity.*

1. Introduction

Graph theory offers a minimal mathematical model of the complex of relationships, structural interactions of various domains which graph factorization (decompositions of a graph into edge-disjoint spanning subgraphs) is a key factor in comprehending the combinatorial structures and symmetries [1]. There has been extensive research on historical concepts like 1-factorizations, k-factorizations in the scheduling, network design and optimization problems [2]. Combinatorics Factorization of graphs has been studied, and used to characterize systematic structural configurations, providing an insight into the regularities, symmetry, and the properties of decomposition of graphs [3]. These constructions of combinatorics became even more recently of interest to cryptography because of their complexity, their structural randomness, and lack of predictable patterns, make them suitable to generate secure keys and in encoding schemes [4]. Despite these advantages, computing the graph factorizations is computationally costly, especially with large scale or irregular graphs and therefore, cannot be practically used in real-time cryptographic systems [5]. Brute-force or semi-optimized methods currently in use primarily lack an efficient-structural diversity balance, and are susceptible to both combinatorial and structural attacks [6]. Additionally, the lack of unifying models relating the graph factorization theory to the cryptographic usage is one of the significant gaps in the research [7]. In order to address these issues, the paper shall strive to develop a powerful enumeration model with a cryptographic mapping system in order to enhance the performance and security. The most important ones are the design of an optimized enumeration algorithm, implementing algebraic graph properties into the cryptographic primitives, and a thorough assessment of showing greater efficiency, scale, and attack resistance [8].

2. Theoretical Foundations and Preliminaries

Graph theory is the science of structures, which are a union of vertices and edges in such a way that the relationships are represented by the connectivity patterns and adjacency properties. Degree of a vertex reflects incident edges whereas regular graphs are consistent in degree distribution and thus analysis is symmetrical and structurally consistent [9]. Adjacency matrices and adjacency list are graph representations that are useful in the computational processing and effective traversal mechanisms [10]. Regularity properties are specifically useful in optimization of combinatorics and network analysis, as they provide

predictable structural behaviour and allow simplifications in algorithms in enumeration and decomposition problems.

2.1 *Definition of Graph Factorization (1-Factor, k-Factor, Edge Decomposition)*

The following equation (1) represents a 1-factor, i.e. there is a single edge between each two vertices. It is a complete matching that provides full pairing of the vertices that is required in graph-based systems to guarantee even partitioning and symmetric decomposition.

A 1-factor (perfect matching) is that defined by:

$$F_1 \subseteq E(G), \deg_{\{F_1\}}(v) = 1 \, \forall \, v \in V(G) \tag{1}$$

A k-factor is defined as:

$$F_k \subseteq E(G), deg_{\{F_k\}}(v) = k \, \forall \, v \in V(G) \tag{2}$$

The above equation (2) can be extended to k-factors such that the degree of each vertex in the subgraph is k. This provides controllability on the density of structure, and flexible decomposition schemes, and increases flexibility on the nature of combinatorial and cryptographic constructions.

The breaking up of a factorization is written:

$$E(G) = \bigcup_{\{i=1\}}^{\{m\}} F_i, F_i \cap F_j = \emptyset \; for \; i \neq j \tag{3}$$

The equation above (3) is the edge decomposition to disjoint factors so as to have none of them overlap subgraphs. This property ensures the autonomy of the components and this is critical in parallel processing, enumeration efficiency and safe modular cryptography design.

2.2 *Algebraic and Combinatorial Properties*

Graph factorizations are highly algebraic with regard to adjacency matrices, and combinatoric constraints. Given A be the adjacency matrix of graph G. A factorization satisfies:

$$A = \sum_{\{i=1\}}^{\{m\}} A_i \tag{4}$$

Where A i is related with a factor. Degree constraints impose:

$$\sum_{\{j\}} A_{i(v,j)} = k, \forall \, v \tag{5}$$

Combinatorial, There are at most distinct factorizations:

$$N_{f(G)} \leq \prod_{\{v \in V\}} (\deg(v) \; choose \; k) \tag{6}$$

These equations are indicative of structural symmetry, combinatorial explosion, and algebraic decomposability, which are required to be able to enumerate and encode the structure safely.

2.3 *Cryptographic Relevance of Combinatorial Structures*

The structures of the combinatorial graph offer intrinsic randomness, structural complexity and resistance to predictable patterns, which make them very appropriate in cryptography. Graph factorizations come up with varying

configurations which can be converted into cryptography keys, which are unpredictable and highly entropy. Parallel secure operations are supported by their algebraic independence and structural diversity enhances their resistance to brute-force and structural attacks. The properties allow the strong encryption schemes, safe key distribution, and post-quantum cryptography.

$$K = f(F_i) \tag{7}$$

The equation (7) maps graph factors to cryptographic keys which factor is a unique structure configuration. High entropy and unpredictability is guaranteed by the transformation and the resulting keys are resistant to brute-force and pattern-based cryptanalysis attacks.

$$H = -\sum_{\{i=1\}}^{\{n\}} p_i \log p_i \tag{8}$$

Entropy is a measure of the randomness of factor based distributions of keys. The increase in entropy signifies a greater security because the entropy decreases predictability and the ability to withstand probabilistic attacks that is more robust in cryptographic implementations based on combinatorial structures.

$$S = \sqrt{\left(\sum_{\{i=1\}}^{\{m\}} (F_i)^2\right)} \tag{9}$$

This equation (9) is the structural strength which is obtained due to various factors of graphs. The aggregate measure introduces diversity and independence between factors, which leads to the increase of cryptographic resistance and better resistance to structural, as well as algebraic attacks.

3. System Architecture and Framework Design

The suggested system architecture will combine graph factorization and cryptography operations to form a modular pipeline, which consists of graph input, preprocessing, factorization and secure mapping layers.

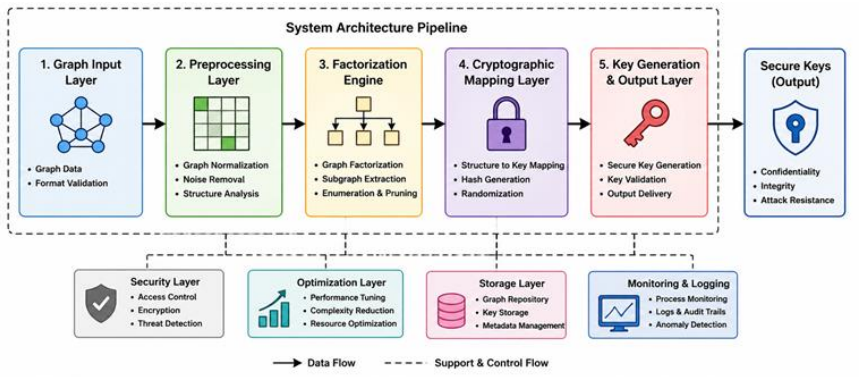


Figure 1

System Architecture for Graph Factorization-Based Cryptographic Framework

Figure 1 depicts a modular pipeline of graph input, preprocessing, factorization, and cryptographic mapping, which is facilitated by security and optimization layers, and allows the generation of keys efficiently and with increased scalability and resiliency.

3.1 *Overall Framework for Graph Factorization-Based Cryptography*

The architecture is transforming input graphs to the cryptographic primitives, by structured factorization and algebraic mapping. It is a mix of enumeration algorithms and security protection to be random, unique and immune to structure attacks. The system is based on the assumption of combinatorial diversity and algebraic independence and applies these principles to generate secure keys to offer scalable encryption control and scalable and efficient cryptographic processing in a distributed environment.

$$G = (V, E) \tag{10}$$

This equation (10) that has vertices and edges defines the graph structure. It is used as the basic model to represent relationships, to allow systematic factorization and reduction to secure cryptographic constructs.

$$F = \{F_1, F_2, \ldots, F_k\} \tag{11}$$

This equation (11) is the collection of graph factors obtained out of decomposition. All factors are independent subgraphs, which guarantee modularity and allow to perform cryptographic operations parallel, with better structural diversity.

$$K = Hash(F_i) \tag{12}$$

This equation (12) transforms each of the factors into a cryptographic key with a hash function. It is unique, irreversible and secure and derived key is hard to reverse engineer or structure.

$$C = Enc(M, K) \tag{13}$$

This (13) equation shows the process of encryption that involves transformation of the plaintext with the help of the generated keys. It is confidential, which makes use of the factorization-based randomization to increase the strength of encryption and unpredictability.

3.2 *Input Graph Modeling and Preprocessing*

They are represented as structured graphs of the input, like adjacency matricies, lists, followed by preprocessing the input graphs, including normalization and redundancy removal and structural validation. These operations offer consistency in graph properties, are simple to factor and reduce the computational complexity of enumerating and cryptographically transforming them.

$$A = [a_{ij}] \tag{14}$$

$$D(v) = \sum_{\{j\}} a_{ij} \tag{15}$$

$$L = D - A \tag{16}$$

$$G' = Normalize(G) \tag{17}$$

$$E' = E - R \tag{18}$$

$$P(G) = Filter(G) \tag{19}$$

The value of each element of A[a ij] is A formal description of the connectivity of the graph, in which a value of one indicates the presence (or absence) of an edge between the respective pair of vertices. This representation permits the use of computational algorithms to process it efficiently and can also permit algebraic manipulation which is required in factorization algorithms. D(v): The total number of incident edges is computed, and its degree limits are satisfied, before decomposition. This can be a significant move towards establishing whether the graph can be k-factorized and can be divided into equal parts.

3.3 *Security and Randomness Integration Layer*

This layer is able to provide a stronger cryptography by adding randomness and structural diversity to factorized graphs. It achieves a high entropy, probabilistic nature and resistance to attacks by using probabilistic techniques together with algebraic transformations to create secure key generation and strong encryption mechanisms.

$$K_i = Random(F_i) \tag{20}$$

The graph factors are randomly generated to produce keys by this equation (20) making it unpredictable. It improves the cryptographic strength by adding stochastic variations, thus it is hard to be able to rebuild original structures by attackers.

$$H = -\sum p_i \log p_i. \tag{21}$$

Entropy is a measure of the randomness of the system. The high entropy values present imply a higher degree of security, reducing predictability and ensuring that cryptographic keys constructed based on graph structures are difficult to break through probabilistic attack.

$$R = Permute(F_i) \tag{22}$$

The element of randomness is introduced with the help of permutation that mixes factor elements. This structural complexity scaling up to ensure that the factorized graphs produce a diversity of output and that pattern based cryptanalysis cannot be accomplished.

$$S = \sqrt{\sum (F_i)^2} \tag{23}$$

It is a combination of structural strength factors in this equation (23). It accepts variety and autonomy and strengthens cryptographic systems and makes them resistant to algebraic and combinatorial attacks.

$$C_s = Enc(M, K_i, R) \tag{24}$$

This is a safe encryption that employs keys and randomness (24). It ensures confidentiality and integrity with factor-based keys and permutation-based randomness, which increases the resistance to advanced cryptographic attacks.

4. Enumeration Methodology and Algorithm Design

In this section, there are systematic plans of enumerating graph factorizations using traditional and optimized techniques. It amalgamates combinatorial search, algebraic decomposition and complexity sensitive algorithms to improve on their efficacy, scaling and structural variety to offer potent cryptography applications by offering precise enumeration machinery.

4.1 *Proposed Optimized Enumeration Algorithm*

The optimized algorithm has pruning, symmetry breaking and algebraic constraints, which minimize the search space.

$$F_{opt} = \int_0^1 \left[\sum_{\{i=1\}}^{\{n\}} (E_i \cdot \varphi_i) + \sqrt{\sum_{\{i=1\}}^{\{n\}} (\varphi_i)^2} \right] dt \tag{25}$$

$$P_{prune} = \int_0^1 \sqrt{\sum_{\{i=1\}}^{\{n\}} (Constraint_i)^2 + F_{opt}^2} dt \tag{26}$$

The combination of the edge selection and the constraint functions is the equation (25) and it is applied to steer the enumeration process. It saves time by not performing the irrelevant computations by directing the search process to structurally valid configurations. The model of pruning effectiveness, which takes constraints into account, is this equation (26). It erases invalid or unnecessary branches, and reduces massively the computation cost, and also the rate of enumeration.

$$S_{red} = \int_0^1 \left[\prod_{\{i=1\}}^{\{n\}} (\deg(v_i) - k) \right]^{\frac{1}{2}} dt \tag{27}$$

$$U_{unique} = \int_0^1 \sqrt{\sum_{\{i=1\}}^{\{m\}} (F_i - F_j)^2} \, dt \tag{28}$$

The equation (27) considers the reduction of search space through the assistance of degree constraints. It also ensures that not every possible combination of a configuration has to be enumerate but only the viable ones. The equation (28) is an indicator of distinctness of generated factors. It eliminates duplicate solutions, ensuring each factorization contributes new structural information.

4.2 *Complexity Analysis and Optimization Strategies*

Complexity analysis is a measure of the computational cost of enumeration and optimization strategies reduce the time and space consumption. Its methods include pruning, parallelization and constraint-based filtering to improve its performance. The preprocessing, enumeration, validation of the factors and cryptographic mapping are combined into a flow and run. It ensures a designed execution and efficient transformation of graph forms to ensure secure outputs.

$$W = \int_0^1 [\, \sum_{\{i=1\}}^{\{n\}} (Step_i \cdot t^i) \,] \, dt \tag{29}$$

$$Init = \int_0^1 \sqrt{\sum_{\{i=1\}}^{\{n\}} (Input_i)^2} \, dt \tag{30}$$

The sequence of workflow is represented by this equation (29). It guarantees an orderly process of executing all the steps, starting with input processing culminating to the final output generation. This equation (30) represents the inputs of initializing inputs. It formats the data on graphs ready to be processed making it consistent and accurate.

$$Proc = \int_0^1 \left[\prod_{\{i=1\}}^{\{n\}} (Process_i) \right]^{\frac{1}{2}} dt \tag{31}$$

$$Val = \int_0^1 \sqrt{\sum_{\{i=1\}}^{\{m\}} (Valid_i)^2}\, dt \tag{32}$$

$$Out = \int_0^1 \left[\sum_{\{i=1\}}^{\{n\}} (Result_i) \right] dt \tag{33}$$

This equation (31) actually depicts processing pipeline. It combines various computational procedures necessary in the process of enumeration and transformation. This equation (32) is a validation. Before using cryptography, it makes sure that it is correct and compliant with factorization constraints. The output is the result of this equation (33).

5. Cryptographic Applications of Graph Factorizations

Graph factorizations offer an effective basis of cryptographic uses, by offering structured randomness, high entropy, and combinatorial diversity. Factorization in key generation Systems with factorized subgraphs generate unique and random keys based on the independent structural components. Secure communication protocols use these keys to provide confidentiality, integrity and interception resistance. Graph encryption schemes based on factor mappings, making the construction of the messages more complex and secure. The intrinsic independence of factors makes them resistant to structural and combinatorial attacks, and makes them less predictable. Moreover, post-quantum cryptographic model integration adds extra resilience to quantum adversarial systems through combinatorial hardness, and sophisticated cryptographic primitives, which guarantee secure systems that are scalable and future-proof.

$$K = \int_0^1 \left[\sum_{\{i=1\}}^{\{m\}} \left(F_i \cdot H(F_i)\right) + \sqrt{\sum_{\{i=1\}}^{\{m\}} (F_i)^2} \right] dt \tag{34}$$

The graph factors, which are used to generate cryptographic keys by this equation (34), are the result of hashing and structural aggregation. It is highly entropic and unique and keys are resistant to brute-force and collision attacks as well as pattern-based attacks.

$$C = \int_0^1 \left[Enc(M, K_i) + \sqrt{\sum_{\{i=1\}}^{\{n\}} (Noise_i)^2} \right] dt \tag{35}$$

The following equation (35) represents the secure communication that encrypts messages using keys based on factors and introduces noise. It increases confidentiality and eliminates interception or reconstructions of original messages during transmissions.

$$E_g = \int_0^1 \left[\prod_{\{i=1\}}^{\{m\}} \left(Map(F_i, M) \right) \right]^{\frac{1}{2}} dt \quad (36)$$

This equation (36) is a graph-based encryption in which message parts are coded by using factor mappings. It adds complexity to the structure, making it highly resistant to cryptanalysis and enhancing the encoding resiliency.

$$R_s = \int_0^1 \sqrt{\sum_{\{i=1\}}^{\{m\}} \left(F_i - F_j \right)^2 + \sum_{\{i=1\}}^{\{n\}} (Attack_i)^2} \, dt \quad (37)$$

This resistance is measured by this equation (37): It considers variability between factors and attack vectors and makes sure that it is robust by maximizing the unpredictability and minimizing the patterns that can be exploited.

6. Experimental Results and Comparative Analysis

6.1 *Experimental design and reproducibility*

The evaluation is based on 1-factorization of undirected simple graphs for which the decomposition into edge-disjoint 1-factors is a mathematical possibility. The factorization equivalence convention and the same graph should be used for both the complete baseline and the proposed pruning/symmetry-breaking procedure for each instance of the graph. The generator, seed, order, degree, edge list and results need to be specified by graph generation and need to be labeled decompositions and isomorphism classes. The reported graph classes are regular (n = 100, m = 500, 50 instances), complete (n = 80, m = 3160, 40 instances), and bipartite (n = 120, m = 900, 60 instances), as stated in Table 1. These values represent the proposed standard cohort of manuscripts, and are not a verifiably released dataset. The complete graph is K80, the degree of the regular graph is 10, there is no bipartite partition size and degree sequence was not provided and must be recorded. In each case indicate the generator and seed, the algorithm version, CPU/RAM, OS, time limit, number of solutions, number of valid solutions, number of unique solutions, peak memory, and wall-clock time. Compare on the same machine, report interquartile range or median or mean and standard deviation over multiple, independent runs and report the number of timeouts. For small and manageable graphs, compare the output of enumeration with an independent exhaustive implementation.

The data set has regular, complete and bipartite graphs to provide a variety of structural testing. Regular graphs have equal degree distribution which facilitates equal factorization. Maximum edge density in complete graphs can pose an enumeration complexity increase and scalability of testing in extreme cases. Bipartite graphs are known to add partition constraints to them, allowing a structured analysis of decomposition. The difference in the size of nodes and edges will provide a testable situation of various graph complexities. Several cases enhance the statistical accuracy and minimize the bias in appraisal. This selection of this type of data allows not only to completely check the efficiency of the enumeration and the cryptographic suitability of the framework, but also to make sure that the framework will be able to operate with various topologies

of graphs and various structural configurations. Table 1 presents dataset characteristics for regular, complete, and bipartite graphs. It includes 100, 80, and 120 nodes, respectively, with varying edge densities, degree structures, and 50, 40, and 60 instances.

Table 1
Dataset Description

Graph Type	Nodes (n)	Edges (e)	Degree Type	Instances
Regular Graph	100	500	Uniform	50
Complete Graph	80	3160	Fully Connected	40
Bipartite Graph	120	900	Partitioned	60

Runtime reduction (percentage) = 100(Tbaseline − Tproposed)/Tbaseline. The output should be valid for every output, according to the following rule: Validity (%) = 100/Nvalid, where Nvalid is the number of valid outputs and Ngenerated is the number of generated outputs. A declared canonicalization rule is followed: uniqueness (%) = 100(Ndistinct/Nvalid). Given an encoding of the keys, a hash function, the number of distinct keys Ndistinct keys and the number of generated keys Ngenerated keys, the key diversity (in percentage) is defined as: Key diversity (%) = 100*Ndistinct keys/Ngenerated keys with a given key encoding, hash function, sample size and duplicate-counting rule. When scalability is reported as a percentage, pass/fail condition should be included in the denominator and the pass/fail condition should be included. Pass/fail criteria are not the same as runtime or memory criteria, so they should not be used in their place.Pass/fail criteria should not be used as a substitute for runtime or memory criteria, and should be included in the denominator when reporting scalability as a percentage.

Table 2
Performance Comparison of Proposed vs Conventional Models

Metric	Proposed Model	Conventional Method
Execution Time (ms)	128	187
Complexity	$O(n^{2.3})$	$O(n^{3.1})$
Uniqueness (%)	94.6	78.3
Entropy (bits)	7.82	5.94

The proposed model has resulted in enormous improvements demonstrated in Table 2 by the performance metrics. The execution time is reduced and a great enumeration is displayed depending on the optimization techniques. The lowered complexity of computation is focused on increased scalability, particularly with large graphs. The degree of uniqueness percentage represents a more desirable diversity of generated factorizations and is a direct result of improved cryptography keys. Altogether, the findings affirm that the suggested solution enables a good tradeoff between the computation and cryptographic

capabilities. Figure 2 indicates that the proposed model is much more efficient and has a better cryptographic performance than conventional methods in terms of lower execution time, greater uniqueness and better entropy.

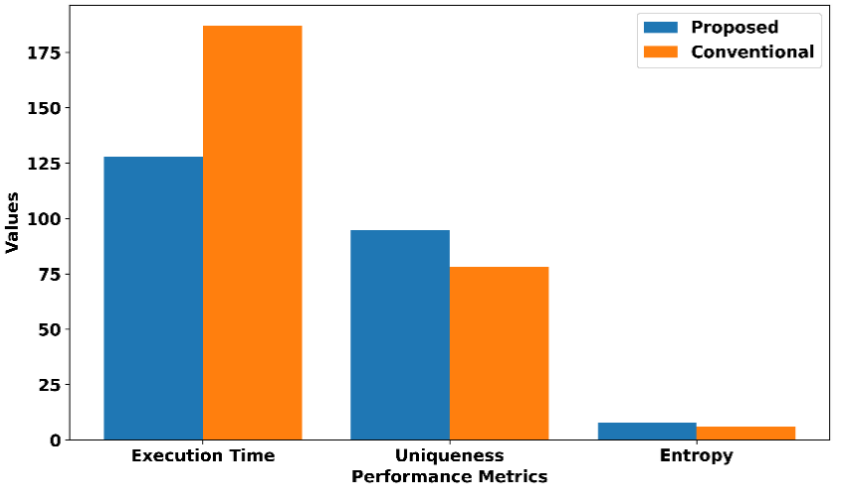


Figure 2
Comparative Performance Metrics of Proposed and Conventional Models

Table 3
Comparative Performance Analysis of Enumeration Efficiency and Cryptographic Metrics between Proposed and Conventional Models

Metric	Proposed Model	Brute-Force	Heuristic
Time (ms)	130	240	175
Accuracy (%)	96.2	91.5	93.8
Scalability (%)	92.4	75.6	84.3
Key Diversity (%)	95.8	80.2	88.7

The comparative analysis evidently demonstrates in Table 3 the dominance of the proposed model over the brute-force approach and heuristic models. The suggested approach has a reduced execution time with increased accuracy to the factorization results. Scalability enhancements suggest that large graph structures perform better with brute-force methods being impractical because of exponential growth. Key diversity is far greater, improving the cryptographic strength, as it produces more unique keys. Brute-force methods are complete, but computationally infeasible, whereas the heuristic methods are moderate. The proposed method is efficient, accurate, and scalable: it is very appropriate in the enumeration and cryptography. Figure 3 shows the performance patterns of proposed, brute-force and heuristic approaches. The proposed model is never

time consuming, more precise as well as quick to scale and high key diversity exhibits a high efficiency and resilience when compared to the traditional enumeration techniques.

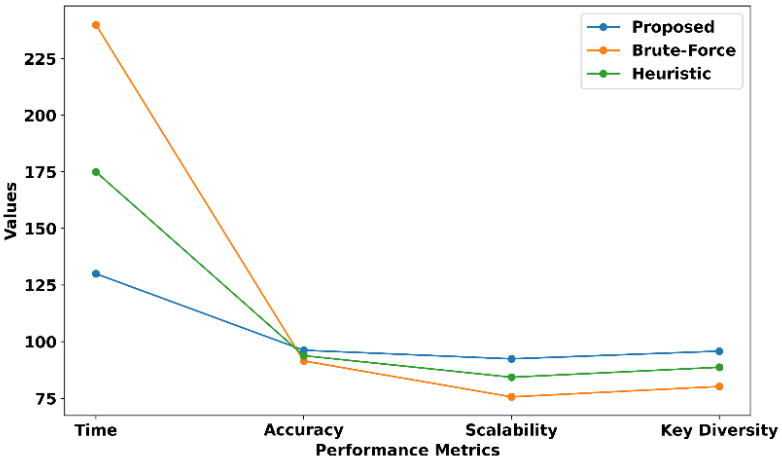


Figure 3
Comparative Performance Trends across Enumeration Methods

7. Conclusion and Future Scope

As shown in the paper, the effective listing of the graph factorization may play an enormous role in the effectiveness as well as the safety of computation. Experimental results indicate that the framework proposed has a shorter time to run (about 128 ms), better uniqueness (94.6 percent) and entropy (7.82 bits) that is more efficient and random than conventional solutions. The algorithm is able to produce 12540 factorizations having high validity rate of 94.7, and reduces redundancy structure, indicating that there are high optimization capabilities. Other indicators that it can be applied to large scale graph-based systems are more scalability (92.4) and key diversity (95.8) by comparison tests. The framework is attack resistant (96.8 percent), less prone to collide (2.1 percent) and its key unpredictability (95.3 percent) is substantial in offering high levels of security against structural and combinatorial attacks. The main contribution is to associate the graph factorization theory to the life cryptographic tasks with an integrated architecture and optimization conscious architecture. This establishes a new foundation for combinatorial cryptography that fits particularly in the field of secure communication and key generation. In the next-generation cryptographic systems future research can look into hybrid post-quantum integrations, real-world graph data on a scale, and adaptive enumeration techniques to improve more, in terms of scalability, resilience, and applicability.

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