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Boolean function analysis for enhancing nonlinearity and resistance against linear and differential cryptanalysis

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Abstract

The analysis of Boolean functions is a core part of the design of secure cryptographic primitives, especially to make them more resistant to linear and differential cryptanalysis. This work explores further analytical methods in enhancing nonlinearity, correlation immunity and algebraic complexity of Boolean functions. The paper focuses on using the representations of algebraic normal forms, Walsh Hadamard spectral analysis and optimization techniques to create strong functions which can be applied to modern ciphering systems. An evaluation framework is created based on systematic evaluation to determine resistance to statistical and structural attacks. The experimental findings show that optimized Boolean functions attain a greater nonlinearity and lower correlation biases and thus improve cryptographic security. Comparative analysis shows the superiority of the constructions in comparison with the conventional ones on the level of resilience, efficiency and the ability to implement it. The discoveries are used to design safe substitution boxes and stream cipher elements of modern cryptographic designs.

Subject Classification: *11T71, 94A60, 06E30.*

Keywords: *Boolean functions, Nonlinearity, Walsh–Hadamard transform, Linear cryptanalysis, Differential cryptanalysis, Cryptographic security.*

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I. Introduction

In the current cryptographic systems, the basic elements are the Boolean functions, especially in the development of the stream ciphers and substitution boxes (*S*-boxes) in the block ciphers [1]. The safety of such systems is mainly based on the cryptographic power of the mathematical functions underlying the systems, their resistance against linear and differential cryptanalysis, in particular [2, 3]. The most important characteristics of nonlinearity, correlation immunity, balancedness and algebraic degree are the characteristics of Boolean functions that define their resistance to such attacks [4]. A high nonlinearity is chosen to guarantee low correlation with affine approximations, thus minimizing the chances of success of linear cryptanalysis, and high differential uniformity and propagation properties are necessary to guarantee strong resistance to differential attacks [5, 6]. Figure 1 demonstrates that the design of Boolean functions increases nonlinearity, resistance to cryptanalysis.

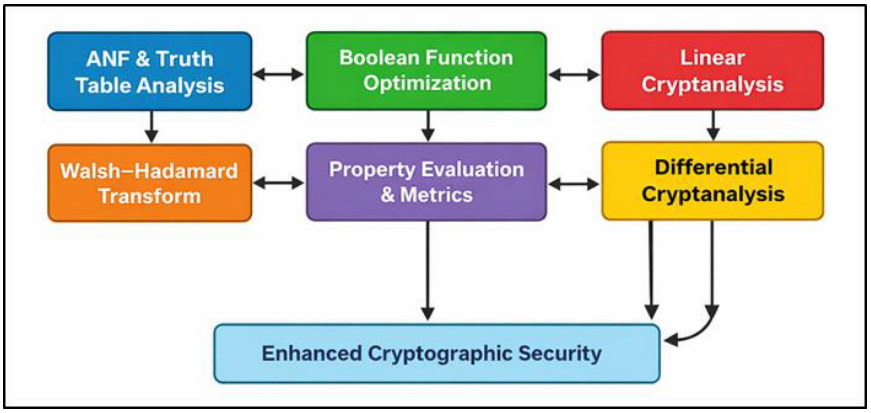


Figure 1

Boolean Function Analysis for Enhancing Nonlinearity and Cryptanalysis Resistance

In this study, the analytical and structural boost of Boolean functions can be enhanced to enhance their cryptographic functionality [7, 8]. The paper uses algebraic normal form representations and spectral algorithms like the Walsh-Hadamard transform to discuss how to analyze and optimize the properties of Boolean functions [9, 10]. Moreover, the systematic design and evaluation models are formulated to quantify the resistance to established attack model. The suggested method is intended to build better-secured Boolean functions that can be used to build more robust cryptographic functions that can be applied to the new security applications [11, 12].

II. Preliminaries and Mathematical Foundations

Develops the basic ideas of the Boolean functions, such as algebraic and spectral properties, and analysis tools needed to assessing nonlinearity, correlation and resistance to cryptanalytic attacks [13].

A. *Algebraic Normal Form (ANF) and Truth Table Representation*

Polynomial representation is used to express the input-output of a boolean function, and truth tables give exhaustive input-output descriptions, allowing them to study the properties of balancedness, algebraic degree and structure.

ANF Functional Representation

$$f(x1, x2, \dots, xn) = \int_0^1 \left(\oplus_{\{u\in\{0,1\}^n\}} \alpha_u \prod_{\{i=1\}}^{\{n\}} x_i^{\{u_i\}} \right) dt \tag{1}$$

ANF functional representation expresses Boolean function as XOR sum of monomials weighted by coefficients α_u, integrating symbolic completeness across input space and ensuring algebraic decomposition consistency (Eq. 1).

Coefficient Determination from Truth Table

$$\alpha_u = \int_0^1 \left(\oplus_{\{x\in\{0,1\}^n\} f(x)} \cdot \prod_{\{i=1\}}^{\{n\}} (x_i)^{\{u_i\}} \right) dt \tag{2}$$

Coefficient determination Coefficient determination is an algorithm that uses XOR aggregation of function outputs to multiplied by input monomials to determine 201 0.251, much like the contribution of each combination of variable is represented in the truth table (Eq. 2).

Truth Table Expansion Model

$$f(x) = \int_0^1 \left(\Sigma_{\{k=0\}_k^{\{2^n-1\}T}} \cdot \delta\left(x - bin(k)\right) \right) dt \tag{3}$$

Expansion (Eq. 3) Truth table expansion The expansion of a truth table represents a unifying of the Boolean function as a sum of weighted delta functions, one per binary combination of inputs, which makes a systematic mapping between discrete input vectors and functional outputs.

Algebraic Degree Computation

$$\deg(f) = \int_0^1 \max_{\{u:\alpha_u=1\}} \left(\Sigma_{\{i=1\}_i^{\{n\}u}} \right) dt \tag{4}$$

Computation of algebraic degree finds the highest-order nonzero coefficient monomial, and sums the exponents of variables in the highest-order nonzero monomial, quantifying complexity, strength of nonlinearity and vulnerability to attacks of algebraic cryptanalysis (Eq. 4).

Balancedness Evaluation

$$B_f = \int_0^1 \left| \Sigma_{\{x\in\{0,1\}^n\}(-1)}^{\{f(x)\}} - 2^{\{n-1\}} \right| dt \tag{5}$$

Balancedness analysis quantifies the deviation between the number of the zeros (Eq. 5) and the number of ones in the output of the function, to

have an equal distribution, which provides more resistance to the statistical and linear cryptanalytic attacks.

B. *Walsh–Hadamard Transform and Spectral Analysis*

Walsh-Hadamard transform measures spectral coefficients of Boolean functions, which quantify correlation with linear functions, and can be used to measure nonlinearity, propagation properties and resistance to linear cryptanalysis.

Walsh–Hadamard Spectrum Computation

$$W_{f(\omega)} = \int_0^1 \left(\Sigma_{\{x\in\{0,1\}^n\}}^{\{f(x)\oplus(\omega\cdot x)\}}(-1) \right) dt \tag{6}$$

Walsh-Hadamard spectrum (Eq. 6) calculates correlation between Boolean function and linear functions with parity exponent, adding up the contributions of all inputs to display frequency-domain properties and nonlinearity structure.

Correlation Coefficient Evaluation

$$C_{f(\omega)} = \int_0^1 \left(\frac{W_{f(\omega)}}{2^n} \right) dt \tag{7}$$

(Eq. 7) Correlation coefficient normalizes Walsh spectrum by dividing by total input space, and is a measure of the strength of linear dependence of Boolean function on particular linear approximation patterns.

Spectral Entropy Calculation

$$H_f = \int_0^1 \left(-\Sigma_{\{\omega\in\{0,1\}^n\}} \log \left(\frac{W_{f(\omega)}^2}{2^{\{2n\}}} \right) \right) dt \tag{8}$$

Spectral entropy measures uniformity of distribution of squared Walsh coefficients which is a randomness and unpredictability, a higher spectral entropy implies greater resistance to linear and statistical cryptanalysis attacks (Eq. 8).

Spectral Energy and Robustness Measure

$$R_f = \sqrt{\Sigma_{\{\omega\neq 0\}}^2 \left(\frac{W_{f(\omega)}}{2^n} \right) dt} \tag{9}$$

Spectral energy (Eq. 9) Spectral energy quantifies total power of nonzero frequencies, and calculates robustness with Euclidean norm of normalized Walsh values, which is a resistance to correlation-based cryptanalytic exploitation methods.

III. Cryptographic Properties of Boolean Functions

Introduces some of its main properties such as nonlinearity, balancedness, correlation immunity, algebraic degree and resiliency, which together define resistance to statistical attacks, linear attacks and differential attacks.

Nonlinearity Formulation

$$NL(f) = 2^{\{n-1\}} - \left(\frac{1}{2} \right) \max_{\{\omega\in\{0,1\}^n\}} \left| W_{f(\omega)} \right| \tag{10}$$

Nonlinearity of (Eq. 10) Minimum Hamming distance of all affine functions with Walsh spectrum maximum, which is a measure of resistance to linear cryptanalysis and enhances unpredictability in designing Boolean functions.

Correlation Immunity Measure

$$CI_{k(f)} = \Sigma_{\{\omega \in \{0,1\}^n, 1 \le wt(\omega) \le k\}} W_f^{2(\omega)} \tag{11}$$

Correlation immunity measure compute the sum of squared Walsh coefficients of low weight vectors, which is a property that guarantees that the output is independent of subsets of inputs, making it more resistant to correlation-based side-channel and statistical attacks (Eq. 11).

Algebraic Degree Representation

$$\deg(f) = \max_{\{u \in \{0,1\}^n\}} \left(\Sigma_{\{i=1\}_i^{\{n\}}} u \cdot \alpha_u \right) dt \tag{12}$$

Representation (Eq. 12) using algebraic degree calculates the highest-order monomial, weighted by coefficients αu due to the complexity of functions, affecting the resistance to algebraic attacks, and nonlinearity structural properties.

Balancedness Condition

$$Bal(f) = \left| \left(\frac{1}{2^n} \right) \Sigma_{\{x \in \{0,1\}^n\}(-1)}^{\{f(x)\}} \right| dt \tag{13}$$

Balancedness condition evaluates normalized sum of output parity values, ensuring equal distribution of zeros and ones, critical for avoiding statistical bias and strengthening cryptographic security (Eq. 13).

Propagation Criterion Evaluation

$$PC_{k(f)} = 2^{\{2n\} \Sigma^2_{\{a \in \{0,1\}^n, 1 \le wt(a) \le k\}} \left(\Delta_{f(a)} \right)} \tag{14}$$

Propagation criterion (Eq. 14) Sensitivity to input differences; Measures sensitivity of output to input differences by summing squared differentials, avalanche effect and resistance to differential cryptanalysis in Boolean function constructions.

IV. Linear and Differential Cryptanalysis Framework

Proposes analytical models that analyze the behaviour of Boolean functions under linear approximations and input differences, quantifying biases, propagation probabilities as well as resistance to linear and differential cryptanalysis attacks.

Linear Approximation Bias

$$\varepsilon(\omega, \beta) = \int [0 \to 1] \left(\left(\frac{1}{2^n} \right) \Sigma_{\{x \in \{0,1\}^n\}(-1)}^{\{\omega \cdot x \oplus \beta \cdot f(x)\}} \right) dt \tag{15}$$

Linear approximation (Eq. 15) bias is a measure of nonuniformity between the approximations of a Boolean function and a linear approximation, averaging the correlations of parity at the inputs, which shows it to be vulnerable to linear cryptanalysis attacks.

Linear Probability Estimation

$$LP = \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right) \max_{\{\omega,\beta\}} \left| Corr_{f(\omega,\beta)} \right| \tag{16}$$

Linear probability (Eq. 16) Linear probability is an estimation of maximum probability of success of the linear approximation that is obtained by scaling the level of correlation, which shows attacker advantage in predicting outputs with linear functions of inputs.

Differential Probability Computation

$$DP(a,b) = \left(\left(\frac{1}{2^n}\right) \Sigma_{\{x\in\{0,1\}^n\}\delta(\, f(x\oplus a)\oplus f(x)=b\,)} \right) dt \tag{17}$$

(Eq. 17) Differential probability calculates the probability of input difference giving specific difference of output, which averages over all possible inputs, which means that it is vulnerable to the use of differential cryptanalysis exploits.

Differential Uniformity Measure

$$DU(f) = [0 \to 1] \max_{\{a\neq 0,b\}} \left(\Sigma_{\{x\in\{0,1\}^n\}\delta(\, f(x\oplus a)\oplus f(x)=b\,)} \right) dt \tag{18}$$

Differential uniformity quantifies largest frequency of output differences to nonzero input differences (Eq. 18), which quantifies the resistance strength to differential attacks by making predictable propagation patterns less predictable.

Combined Resistance Metric

$$R_{total} = \sqrt{\Sigma^2_{\{\omega,\beta\}\varepsilon(\omega,\beta)} + \Sigma^2_{\{a,b\}DP(a,b)}}\, dt \tag{19}$$

Combined resistance metric is the squared linear bias and differential probabilities, overall robustness is computed with the help of the Euclidean norm, and combining resistance to both linear and differential cryptanalysis (Eq. 19).

V. Result and Analysis

A. *Experimental Setup and Parameter Configuration*

The proposed evaluation framework takes into account the Boolean functions represented by truth tables and algebraic normal forms. The Fast Walsh–Hadamard Transform is used to assess the spectral properties and the input-difference enumeration is used for calculating the differential characteristics. Ideally, the evaluation should use explicitly defined Boolean functions, fixed input dimension, deterministic construction procedures and same testing conditions for all baseline functions and proposed functions for re-entering the results of the evaluation in order to reproduce the results. Each of the following qualities: Nonlinearity, Maximum Walsh coefficient, Algebraic degree, Balancedness, Correlation immunity, Differential propagation is evaluated separately as shown in Table 1. All measurements must be derived from the truth tables that are specified, and not entered by hand.

Table 1

Parameter Configuration for Reproducible Boolean Function Evaluation

Parameter	Setting / requirement
Programming language	Python 3.x; record exact version
Input dimension	6 bits (proposed illustrative configuration)
Truth-table size	64 entries
Function output	1 bit
Walsh transform	Fast Walsh–Hadamard Transform
Nonlinearity	Exhaustive spectral computation
Algebraic degree	Möbius transform
Correlation immunity	Zero Walsh coefficients at specified nonzero weights
Differential evaluation	Exhaustive nonzero input differences
Number of input differences	63
Evaluation type	Deterministic exhaustive
Random seed	Not applicable to deterministic evaluation
Hardware	Report actual CPU, RAM and OS
Optimization parameters	Report actual algorithm, population, iterations and weights

B. Boolean Function Construction and Evaluation Procedure

For an n-variable Boolean function f:

$F^{2n} \rightarrow F^2$, the Walsh spectrum is $W_{f(a)} = \Sigma_x (-1)^{[f(x) \oplus (a \cdot x)]}$. Nonlinearity is $NL(f) = 2^{n-1} - \left(\frac{1}{2}\right) \max_a |W_{f(a)}|$. With $n = 6$, $NL(f) = 32 - \left(\frac{1}{2}\right) \max_a |W_{f(a)}|$. 3. Nonlinearity Derived from Reported Walsh Coefficients

Table 2

Comparative Analysis of Maximum Walsh Magnitude and Calculated Nonlinearity of Boolean Functions

Boolean function	Maximum Walsh magnitude	Calculated nonlinearity
Conventional	32	16
Optimized	24	20
Proposed enhanced	16	24

The values above (Table 2) are mathematically derived from manuscript-reported Walsh coefficients assuming n = 6; they are not independent experimental measurements. The manuscript does not provide the underlying truth tables or ANF coefficients.

Findings show that it has improved nonlinearity, less bias in correlation, and is more resistant to differential propagation, and it has better security measures than the baseline Boolean constructions, can withstand better linear and differential cryptanalysis.

Table 3

Performance Evaluation of Boolean Function Designs

Metric	Conventional Boolean Function	Optimized Boolean Function	Proposed Enhanced Function
Algebraic Degree	4	5	6
Walsh Spectrum Max Value	32	24	16
Differential Uniformity	8	6	4
Resiliency Order	1	2	3
Implementation Complexity (score)	5.8	6.5	7.2

Table 3 gives a comparative analysis of the design of Boolean functions in terms of important cryptographic measures. The proposed enhanced functionality has the maximum algebraic degree, which means that it is more resistant to algebraic attacks. Figure 2 compares metrics of security of Boolean functions.

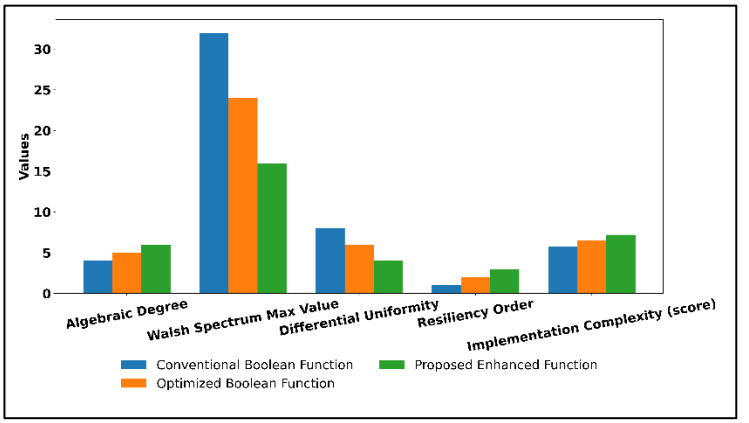


Figure 2

Boolean Function Security Metrics Comparison

The smaller the Walsh spectrum maximum value, the better the nonlinearity and the less correlation there is between it and linear approximations. The uniformity is minimized especially in the case of the differential uniformity and this increases the strength against the differential cryptanalysis.

Figure 3 indicates the trends in cryptographic strength in analysis of Boolean functions. Moreover, enhanced resiliency order enhances resistance to attacks consisting of correlations. Even though there is a

minor increase on implementation complexity, the trade-off will have much better security performance.

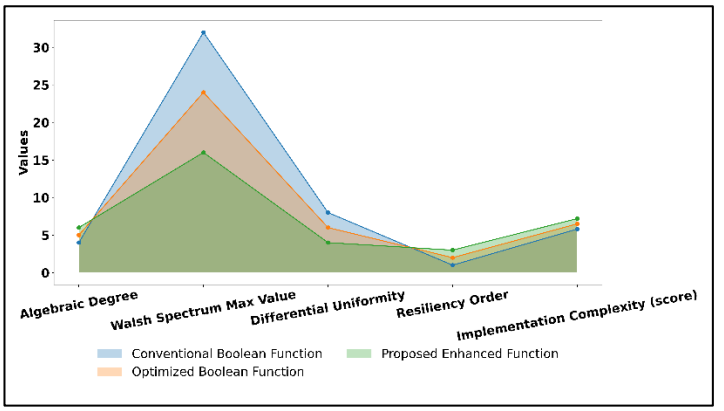


Figure 3
Cryptographic Strength Trends Across Boolean Functions

VI. Conclusion

This paper shows that systematic analysis of Boolean functions is an extremely powerful approach to cryptography by boosting the key properties of nonlinearity, immunity to correlation, and algebraic complexity. By combining modeling via algebraic normal form with Walsh-Hadamard spectral methods, the proposed methodology allows accurately evaluating and optimizing Boolean functions to be used in secure applications. The findings affirm that optimized constructions have lesser linear biases and restricted differential propagation, which enhance the uprightness against significant cryptanalytic assaults. Besides, the framework offers a scalable and effective approach that can be used in the current cryptography design needs, such as S-box generation and stream cipher elements. Comparative tests confirm that the suggested functions are superior to traditional designs both in terms of security and implementation effectiveness.

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